GRAVITATIONAL COLLAPSE WITH ASYMMETRIES

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Two idealized collapse models, involving a magnetic dipole and a gravitational quadrupole, are analyzed, treating departures from sphericity as small perturbations. Radiative leakage (largely downwards through the Schwarzschild radius) causes externally observable asymmetries to decay to zero in an oscillatory fashion, with a period of the order of the Schwarzschild characteristic time $2Gm/c^3$. These results have significant consequences for astrophysics; they imply in particular that a "black hole" cannot be a source of synchrotron radiation.

Every static nonspherical perturbation of Schwarzschild's exterior field due to gravitational or electromagnetic sources within the stationary lightlike surface $g_{00}=0$ becomes singular on this surface.[1-3] Stationary perturbations of Kerr's rotating solution appear to have a similar property.[4] Assuming these results to be applicable to the asymptotically stationary exterior field of a collapsing star, one is led to conjecture that all externally detectable asymmetries,[1] including magnetic fields,[2] must somehow decay, leaving behind Schwarzschild's vacuum field (or, in the case of nonvanishing angular momentum, Kerr's field) as the sole external manifestation of the collapsed object.

To examine these questions, we have carried out a dynamical analysis of two idealized collapse models, one involving a magnetic dipole, the other a gravitational quadrupole. Our results support the foregoing conjecture and reveal the decay mechanism to be a rapid radiative leakage of the asymmetric perturbing field, largely downwards through the event horizon.

We cast the Schwarzschild metric into the form $(ds^2)_{\mathrm{Schw}}=\alpha dx dy+r^2 d\Omega^2$, where $\alpha=1-1/r$, and the retarded and advanced time coordinates $-x, y$ are related to the standard Schwarzschild coordinates by $x,y=(r-1)+\ln(r-1)\mp t$. Lengths are measured in units of the Schwarzschild radius: $2m=1$.

Both of our models can be considered as linearly perturbed variations of the following basic situation (Fig. 1). A thin, hollow spherical shell of mass $m=\frac{1}{2}$ is initially static with radius $R_0 \gg 1$; at time $t=-\frac{1}{2}x_0 \equiv -(R_0-1)-\ln(R_0-1)$, it suddenly begins to collapse at the speed of light (history of surface $y=0$). (This model, adopted for mathematical simplicity, is highly artificial from an astrophysicist's point of view, but does not violate any of the principles of relativity theory. Moreover, our main interest is in the asymptotic behavior of the external field as $t\to\infty$, and we do not expect this to depend too sensitively on the precise structure of the source or the initial conditions.)

In our first ("magnetic collapse") model, we suppose a static magnetic dipole of moment μ placed at the center of the shell. (It is assumed that $\mu^2 \ll 1$, which means gravitational effects of the magnetic energy density can be neglected for $r\geq 1$.) Our second ("quadrupole") model assumes a weak gravitational quadrupole of moment q superimposed on the spherical background field and caused by unevenesses in the surface density of the shell.

Since news of the onset of collapse cannot reach the interior ahead of the shell itself, the

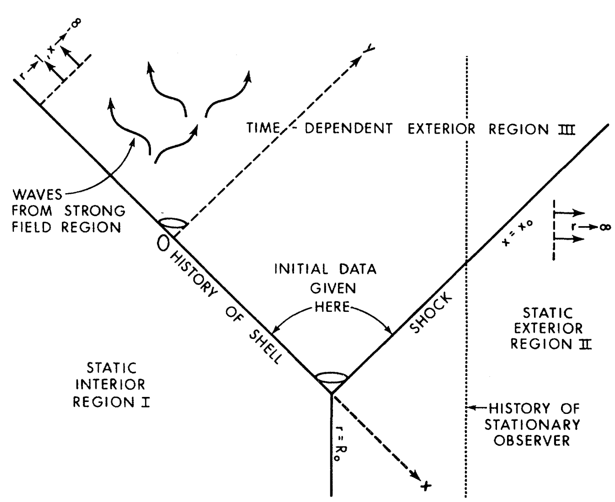

FIG. 1. Space-time diagram for collapsing shell model.

initial static interior field (region I in Fig. 1) remains unchanged in both models. The exterior field, however, becomes time dependent after passage through a shock front at $x=x_0$. The problem is thus to find the perturbing field in the time-dependent region $x \leq x_0$, $y \geq 0$ (region III in Fig. 1), given Cauchy data on the characteristics $y=0$, $x=x_0 \gg 1$. (It is unnecessary to distinguish, at this level of approximation, between null hypersurfaces of the Schwarzschild background field and of the gravitationally perturbed field.)

To formulate the magnetic-collapse problem, let us write for the covariant azimuthal component of four-potential $A_\varphi = \psi(x,y)\sin^2\theta$. The electromagnetic field equations on a Schwarzschild background then yield

$$\psi_{xy} = f(r)\psi, \tag{1}$$

where $f=\alpha/2r^2$ and the subscripts indicate partial differentiation. The appropriate initial static solutions are $\psi=\mu/r$ for the flat ($\alpha=1$) interior domain $r \leq R_0$; and for $r \geq R_0$,

$$\psi = \text{const}\times r^2 \int (r-1)^{-1} r^{-3} dr \approx \mu/r \quad (R_0 \gg 1).$$

(The remaining components A_ν vanish.) The characteristic initial conditions supplementing (1) in the time-dependent region are thus $r\psi=\mu$ on both $x=x_0$ and $y=0$. (The jump conditions for the electromagnetic field require continuity of A_φ across a characteristic surface.)

Figure 2 shows some results of a numerical integration of this characteristic initial-value problem. To a stationary external observer the field appears nearly constant ($r\psi \approx \mu$) for a period about equal to the Newtonian free-fall time, i.e., down to $x \approx 0$. The epoch $x \approx 0$ is marked by the fairly sudden onset of an oscillatory decline towards zero with period $\approx 4\pi$. On the horizon $r=1$, ψ displays a similar damped oscillatory behavior as a function of y. A free-falling observer close to the collapsing body sees no decline of the field, but we can find no support (at least in this idealized model) for a suggestion of Ginzburg,[2] based on a quasistatic analysis, that the field becomes infinitely compressed against the body.

To see what becomes of the external magnetic-field energy, we integrate the identity

$$\partial_x(\psi_y^{\,2} + f\psi^2) = \partial_y(\psi_x^{\,2} + f\psi^2), \tag{2}$$

which is an immediate consequence of (1), over region III of Fig. 1. In the limit $x_0 \to \infty$, we find $I_1 + I_2 = \frac{1}{4}\mu^2$, where $I_1 = \frac{2}{3}\int_{-\infty}^{\infty}\psi_x^{\,2}(x,\infty)dx$ and $I_2 = \frac{2}{3}\int_0^{\infty}\psi_y^{\,2}(-\infty,y)dy$. Physically, (2) is just the explicit form of the law of energy conservation $\partial_\alpha[(-g)^{1/2}T^{\alpha\beta}\xi_\beta]=0$, where ξ_β is the timelike Killing vector and $T^{\alpha\beta}$ the electromagnetic energy tensor. It follows that I_1 and I_2 represent the electromagnetic energy radiated out to infinity and in through $r=1$, respectively. Our numerical integrations show that $I_1 = 0.010\mu^2$ and $I_2 = 0.240\mu^2$, so that about 96% of the field energy falls in through the Schwarzschild radius. (The difference between the magnetostatic energy $\frac{1}{3}\mu^2$ initially present outside $r=1$ and the total radiated energy $\frac{1}{4}\mu^2$ is exactly accounted for as the work done by the difference of the electromagnetic stresses on the two sides of the shell, and goes into increasing the kinetic energy of the collapsing body.)

Turning now to gravitational perturbations, we note that, in the coordinate gauge of Regge and Wheeler,[4] a generic spherical harmonic component of any linearized vacuum perturbation of the

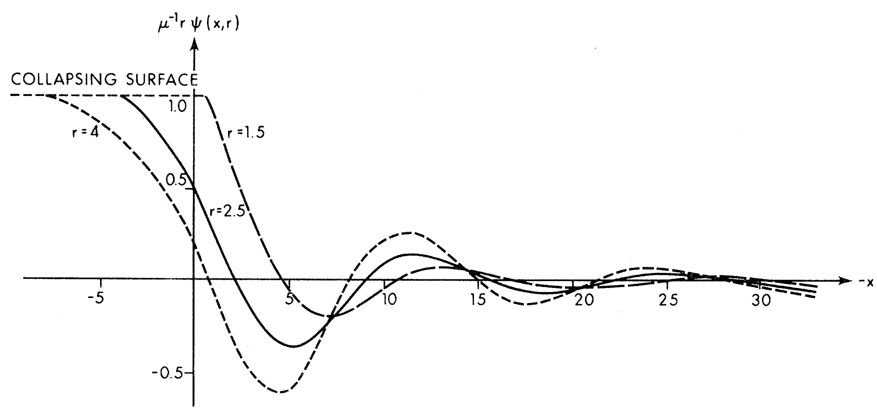

FIG. 2. $r\psi$ as a function of retarded time ($-x$) for stationary observers with radial coordinates equal to 1.5, 2.5, and 4 Schwarzschild radii. The vector potential $\vec{A} = \vec{e}_{(\varphi)} r^{-1}\psi\sin\theta$, where $\vec{e}_{(\varphi)}$ is a unit azimuthal vector.

Schwarzschild metric may be written

$$ds^2-(ds^2)_{\rm Schw}=[\tfrac{1}{2}\alpha^2(\eta dx^2+\xi dy^2)+r^2Kd\Omega^2]\times P_l(\cos\theta),$$

where K, ξ, and η are functions of x and y. (In terms of the quantities H and H_1 used by Regge and Wheeler, $\alpha\eta=H-H_1$, $\alpha\xi=H+H_1$.) The vacuum-field equations for the perturbations, which, as usually given,[5] have a complicated appearance, assume the following neatly separated form in terms of $\xi(x,r)$, $\eta(y,r)$:

$$\alpha r^2\eta_{rr}+2r^2\eta_{ry}+6r\eta_y+r(3-\alpha)\eta_r-l(l+1)\eta=0; \qquad (3)$$

$\xi(x,r)$ satisfies an equation of the same form with y replaced by x; and

$$K_r(y,r)=2\eta_y+\alpha^{-1}(\alpha^2\eta)_r,$$
$$K_r(x,r)=2\xi_x+\alpha^{-1}(\alpha^2\xi)_r. \qquad (4)$$

Testing a perturbation for regularity as $r\to 1$ or $r\to\infty$ is complicated by the occurrence of coordinate singularities of the Regge-Wheeler gauge.[5] Examination shows that the geometry is regular on the future event horizon $r=1$, $t=+\infty$ if K, η, and $\alpha^2\xi$ are bounded there. At future null infinity, assuming outgoing radiation with a Bondi news function[6] $c(x,\theta)=f(x)P_l^{(2)}(\cos\theta)$, the condition for asymptotic flatness is

$$K\to -2f'(x),\quad \eta/r\to -4f''(x),\quad \xi=O(r^{-2})$$
$$(r\to\infty,\ x \text{ fixed}).$$

For our quadrupole collapse model, we set $l=2$. The static quadrupole field superimposed on the flat interior of the shell (replace α in the preceding formulas by 1) is given by $\xi=\eta=K=2qr^2$ $(qR_0^3\ll 1)$, while the initial static external field is

$$\xi=\eta=b\alpha r^2\int_r^\infty \alpha^{-3}r^{-6}dr \quad (r\geq R_0, x\geq x_0),$$

where the constant $b\approx 10qR_0^5$ is fixed by continuity of K across $r=R_0$.

The appropriate junction conditions (continuity of K,η and of K,ξ across $y=0$ and $x=x_0$, respectively) now yield

$$\eta=K=2qr^2,\quad \eta_y(r,y)=0 \text{ on } y=0,\ r<R_0,$$
$$\xi\approx K\approx 2qR_0^5/r^3,\quad \xi_x(r,x)=0 \text{ on } x=x_0,\quad r>R_0\gg 1, \qquad (5)$$

as characteristic initial conditions for the time-dependent region.

To make the problem definite, let us assume $f''(x_0)=0$, i.e., that gravitational radiation is absent initially. Then the expression

$$\eta=2qr^2-\tfrac{1}{16}qr^{-3}y^3(y^2+5x_0y+5x_0^2) \qquad (6)$$

solves (3)-(5) uniquely for $\alpha=1$, and therefore approximates the initial form of the actual solution for $x\gg 1$, when r is large throughout the exterior region.

To determine whether the exterior quadrupole field eventually damps out, we eliminate y from (3) by a Laplace transform. Writing $r^{-3}Z(s,r)$ for the Laplace transform of $2qr^2-\eta(y,r)$ [also a solution of (3)], we obtain the ordinary differential equation

$$r^2(r-1)Z_{rr}+(2sr^2-4r+7)rZ_r-15Z=0, \qquad (7)$$

and, from (6), $Z(s,\infty)=\frac{15}{2}qs^{-6}+O(s^{-5})$. There is a unique solution of (7) which takes an assigned value at $r=\infty$ and remains finite at $r=1$. Now, a WKB analysis combined with numerical integration shows that

$$\lim_{s\to 0}[Z(s,r)/s^5Z(s,\infty)]=(4/15)r^5$$

for any solution of (7) which is bounded at $r=1$. We thus obtain $sr^{-3}Z\to 2qr^2$ as $s\to 0$, which indicates that $\eta(r,y)\to 0$ as $y\to\infty$ for a stationary observer with $r\geq 1$. It follows that K and ξ also tend to zero. We have confirmed these conclusions by a direct numerical integration of (3); again, the decline towards zero has an oscillatory character.

Our results have an important bearing on the question of the observability of collapsed stars and their distinguishability from neutron stars. Unlike neutron stars, which can remain intensely magnetized for periods exceeding 10^6 years, a collapsed object cannot be a source of synchrotron or other radiation requiring the agency of magnetic fields. However, accretion of interstellar material in a favorable environment, e.g., in close binary systems, could produce a strong source of thermal x-ray bremsstrahlung.

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Detailed accounts of this work are in preparation.

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TESTS FOR EIGHTFOLD-WAY OCTETS IN THE BARYON SPECTRUM

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A search is reported for octets among baryons of spin $\leq \frac{5}{2}$, using the Gell-Mann–Okubo mass formula as a selection criterion. The likely number of unphysical "chance" octets so selected is estimated by another search using unphysical mass formulas. The findings discourage reliance on mass formulas for classification purposes, and tend somewhat against the existence of further octets.

Belief in SU(3) as an approximate symmetry of the strong interaction rests mainly and understandably on the eightfold way's success with the "stable" baryon and meson octets, and its spectacular predictions for the baryon-resonance decuplet.[1] [We take the eightfold way here to include assumptions about the symmetry violation which are sufficient to give the Gell-Mann–Okubo (GMO) mass formula.] However, there does not appear to have been any wider investigation of its power to put in order the whole spectrum of known hadrons. Some unexpected initial findings on this question seem worth summarizing.

In what follows, we have restricted ourselves to the N^*, Σ^*, Λ^*, and Ξ^* baryon resonances of spin $\frac{5}{2}$ or less, and to search for further octets only. (It should be noted that, for convenience but contrary to convention, we allow the asterisked symbols to represent the stable baryons also.) It seemed profitable to test the assumption that some at least of these candidates should rightly be assigned by SU(3) to octets. An entire absence of further octets would mean, for example, that all the Λ^* are unitary singlets or members of the 27-plet, since the decuplet has no $T=0$, $Y=0$ member; being forced to this conclusion would itself be interesting.

Now, suppose we adopt some linear mass relation, and then select octets on the criterion that this relation be satisfied to within, say, 2%. Then, provided its coefficients are not too outlandish, we would expect such a relation to "select" a certain number of octets (each constant in spin and parity) just by chance. To find the expected number of such chance octets is hardly a soluble statistical problem, but we can estimate this number from those selected by a set of mass formulas lacking any known physical justification.

On the assumption above, that some true assignments to octets exist, GMO would of course have a head start over any unphysical mass relation, since it would "select" these physical octets in addition to its chance quota; indeed, the stable octet quarantees it a start of at least one octet. This procedure, then, gives a test of the classifying power of GMO. (The assumption here that "physical" and "chance" octets are distinct is easily seen to be justified, as long as the number of "physical" octets must be small compared with the number of possible octets; this is always the case below.)

We also consider a criterion built far more integrally into the whole SU(3) scheme: the constancy of spin and parity within a multiplet. We first ignore this requirement and take any octet satisfying the mass formula (we call these "offered" octets). Suppose there are N possible combinations into octets, of which only m satisfy the constancy rule. Then imposing this rule would, by pure chance, give us an expected fraction (m/N) surviving as "selected" octets; and, by reasoning similar to the above, the eightfold way can show its superiority by exceeding this expectation.

The particle spectrum examined was taken from the data of Barash-Schmidt et al.[2] The $N(1860)$ and $\Sigma(1385)$ were omitted since no $\frac{3}{2}^+$