thods in Many-Body Systems, translated by A. J. Meadows (Pergamon, New York, 1967).

[12]Although this is no longer strictly true when the photon momentum is nonzero, any optical excitation arising from the direct absorption of photons is expected to be negligible.

[13]The next nonzero set of terms is second order in A, T, and U, and comprises a "volume" contribution to the photoemission. The formalism for this set of terms will be described elsewhere. There is no interference between the surface and volume terms.

[14]H. E. Bennett, private communication.

[15]Directionality effects in the EDC's here and in the interacting-electron gas should not be taken seriously, first because all bulk processes are being neglected, second, because the effects are very sensitive to the detailed form of $T(\vec{k},\vec{k}')$.

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[18]No further approximations, other than a finite grid size, were made. The computation requires about 30 min on an IBM 360/91 computer.

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ANGULAR DEPENDENCE OF SURFACE SCATTERING*

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Calculations of the rf surface impedance of a thin metal plate as a function of a dc magnetic field applied parallel to the faces of the plate reveal that one can measure experimentally the critical angle of collision with the surface, if such an angle exists, below (above) which electron surface scattering is mostly specular (diffuse).

The influence of surface scattering on the response of thin metal plates to external fields has been described by a phenomenological model, first proposed by Fuchs in the study of the static conductivity of thin films,[1] in which a single specularity parameter p is supposed to describe the surface-scattering mechanism. In this model, p is the probability that an electron will be specularly scattered at the surface (the component of the velocity normal to the surface changing sign upon reflection), while $1-p$ is the probability for diffuse scattering. Diffuse scattering means that for any given angle of incidence the angle of reflection is random so that the drift velocity of the electron after collision with the surface is zero on the average and the subsequent contribution of that electron to the conductivity vanishes.

It is desirable to generalize the Fuchs specularity parameter into a specularity function[2,3] $S(\theta)$ which will depend on θ, the angle of collision with the surface (see Fig. 1), for the following reason. Even if the surface is rough on the atomic scale only, electrons with large angles of collision will be expected to be diffusely scattered as their wavelength normal to the surface is comparable with the scale of roughness. On the other hand, electrons with small angles of collision have wavelengths normal to the surface which can be much larger than the scale of surface roughness. These "grazing incidence" electrons are expected not to "see" the details of the surface and thus have a high probability of being specularly reflected.[4] On the basis of these arguments we have extended the path-integral solution of the Boltzmann equation[5] to include specularity functions $S(\theta)$ which will be nearly equal to unity (zero) below (above) a critical angle θ_0. The correct functional form of $S(\theta)$, if indeed this phenomenological description of surface scattering is at all valid, will have to be settled by experiment in the absence of a detailed microscopic theory of surface scattering.

In this Letter we propose the following simple experiment by which one can probe the angular dependence of surface scattering. An rf coil wrapped around a metal plate of thickness d sets up (antisymmetrically) surface-current layers, $\vec{j}=j(z)e^{-i\omega t}\hat{y}$, of effective thickness δ just inside the two plate surfaces which are normal to the z axis.[6] An external dc magnetic field is applied in the plane of the plate and in the transverse direction with respect to the rf current, $\vec{H}=H\hat{x}$. This magnetic field curves the trajectories of

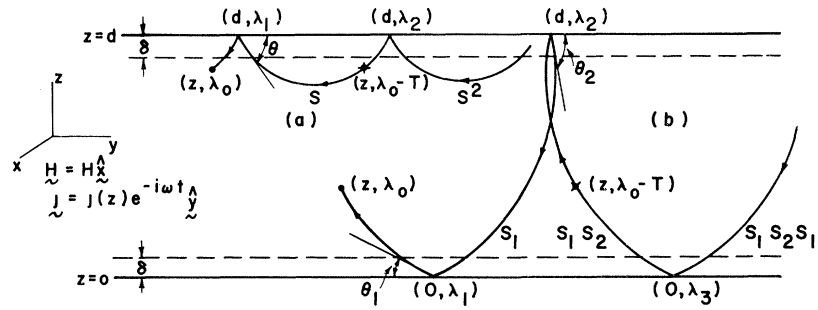

FIG. 1. (a) Electronic surface "skipping" trajectories, of period $T=\lambda_{n-1}-\lambda_n$, colliding with the $z=d$ surface at the instants λ_n, $n=1,2,\cdots,\infty$, with the common angle of collision with the surface θ and with the probability for specular scattering being reduced by the factor $S=S(\theta)$ after each collision. (b) Electronic trajectories colliding with both surfaces $(T=\lambda_{n-1}-\lambda_{n+1})$, the probability for specular reflection being reduced by the factor $S_1=S(\theta_1)$ $[S_2=S(\theta_2)]$ after each collision with the surface $z=0$ $[z=d]$.

the electrons in a way dictated by the dispersion law, $\epsilon=\epsilon(p)$, where ϵ and $\vec{p}$ are the electronic energy and quasimomentum. To avoid complicated dispersion effects, which are not central to this investigation, we assume a cylindrical Fermi surface for the electrons, the axis of the cylinder being along $\vec{H}$ and the height Δp_x being adjusted so as to give the correct electron density $n=2\pi p_F{}^2\Delta p_x/(2\pi\hbar)^3$ for a typical metal. In this idealized model, the electrons describe circular trajectories with the common cyclotron frequency $\Omega=|e|H/mc$. The condition $\omega\ll\Omega$ will ensure a nearly static field for electrons returning to the surface layers. The velocity of electrons on the Fermi surface is simply given by

$$\vec{v}\equiv v_F\vec{u}=v_F(\hat{y}\cos\varphi+\hat{z}\sin\varphi), \tag{1}$$

where v_F is the Fermi velocity and φ is the dimensionless "orbit" or "phase" variable, $\dot{\varphi}=\Omega$, used in place of the real time t.[7]

Maxwell's equations for this problem reduce to

$$\partial^2E(z)/\partial z^2=-i(4\pi\omega/c^2)j(z), \tag{2}$$

where the current density is given by the path-integral formula of Chambers[5] (at low T the energy integration is restricted on the Fermi surface):

$$j(z)=\sigma_0/(\pi\Omega\tau)\int_0^{2\pi}d\varphi\,\mu_y(\varphi)I(\varphi,-\infty); \tag{3a}$$

here

$$I(\varphi,-\infty)=\int_{-\infty}^{\varphi}d\varphi'\,\exp[-\gamma(\varphi-\varphi')]\mu_y(\varphi')E\Big(z-\Omega^{-1}\int_{\varphi'}^{\varphi}d\varphi''v_z(\varphi'')\Big) \tag{3b}$$

and

$$\gamma=(1-i\omega\tau)/\Omega\tau, \tag{3c}$$

the collisions of electrons with an imperfect, infinite lattice being described phenomenologically by a mean free path $l=v_F\tau$, in which case the static conductivity is equal to $\sigma_0=ne^2\tau/m$.

For a specimen of finite thickness, where size effects are important, the integral of (3b) over the history of an electron which at "time" φ is at the point z of the conductor is easily generalized to

$$I(\lambda_0,-\infty)=I(\lambda_0,\lambda_1)+\sum_{n=2}^{\infty}\Big\{\prod_{l=1}^{n=1}S(\theta_l)\exp[-\gamma(\lambda_{l-1}-\lambda_l)]\Big\}I(\lambda_{n-1},\lambda_n), \tag{4a}$$

where

$$I(\lambda_{n-1},\lambda_n)=\int_{\lambda_n}^{\lambda_{n-1}}d\varphi'\exp[-\gamma(\lambda_{n-1}-\varphi')]\,\mu_y(\varphi')E(\xi_n(\varphi')), \tag{4b}$$

and

$$\xi_n(\varphi')=\xi_{n-1}(\lambda_{n-1})-\Omega^{-1}\int_{\varphi'}^{\lambda_{n-1}}d\varphi''v_z(\varphi''),\quad\text{with }\xi_0(\lambda_0)\equiv\xi_0(\varphi)=z, \tag{4c}$$

for electrons which collided with the surface $z=0$ at the "instants" λ_n (previous to φ) defined by

$$\xi_n(\lambda_n)=0,\quad\lambda_{n+1}\le\lambda_n\le\varphi, \tag{4d}$$

or at the "instants" λ_n defined by

$$\xi_n(\lambda_n)=d, \quad \lambda_{n+1}\le\lambda_n\le\varphi, \tag{4e}$$

for electrons colliding with the surface $z=d$. For electrons not colliding with either surface the integral (3b) remains unmodified. The angle of the nth collision with either surface is given by

$$\theta_n=\arcsin[\mu_z(\lambda_n)], \tag{5}$$

and the probability that an electron will continue on a specularly reflected path is reduced by the factor $S(\theta_n)$, $0\le S(\theta_n)\le 1$, after the nth collision. Using the periodicity of the circular trajectories, we find that the complicated sum of (4a) reduces to a simple integral over the period $T=\lambda_{n-1}-\lambda_n$ for the "skipping" surface trajectories of Fig. 1(a) characterized by the same angle of incidence θ for all collisions:

$$I(\lambda_0,-\infty)=\{I(\lambda_0,\lambda_1)+S(\theta)\exp[-\gamma(\lambda_0-\lambda_1)]I(\lambda_1,\lambda_0-T)\}[1-S(\theta)\exp(-\gamma T)]^{-1}, \tag{6}$$

with a similar expression for the trajectories of Fig. 1(b).

Substituting (3a) and (4a) into (2) yields an integrodifferential equation which we solve for the unknown $E(z)$, subject to the antisymmetric boundary condition, $E(0)=-E(d)=1$, using the numerical model described by the author in an earlier paper.[7] From the calculated $E(z)$ we obtain the surface impedance

$$Z(H)=i(4\pi\omega/c^2)E(z)/E'(z)\,|_{z=0^+}=(4\pi\omega/c^2)(R+iX), \tag{7}$$

whose real part R we exhibit as a function of H in Fig. 2 for $d=1.56\times10^{-2}$ cm, $l/d=1$, $\omega/2\pi=1$ MHz, and for the specularity function

$$S(\theta)=1, \text{ for } \theta\le\theta_0, \quad =0, \text{ for } \theta>\theta_0, \tag{8}$$

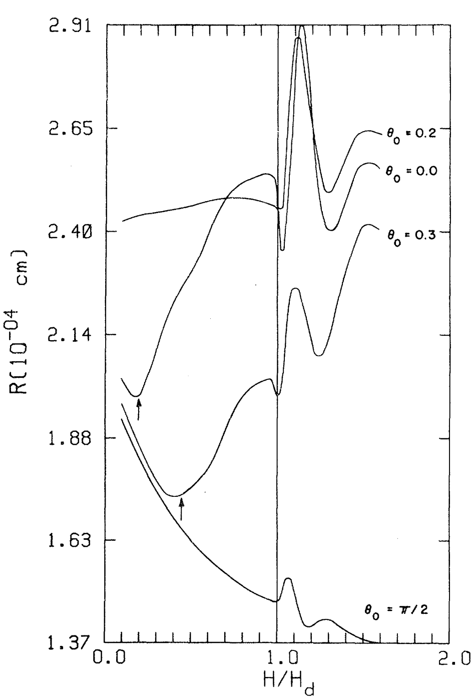

FIG. 2. The surface resistance R as a function of magnetic field for different choices of the critical scattering angle θ_0, below (above) which the surface scattering is all specular (all diffuse).

with four different choices of θ_0: (i) $\theta_0=0$ (diffuse scattering for all angles of collision), (ii) $\theta_0=0.20$ rad, (iii) $\theta_0=0.30$ rad, and (iv) $\theta_0=\pi/2$ rad (specular scattering for all angles of collision).

The thickness of the plate dictates the onset of the rf size effect (RFSE) signal which begins at $H/H_d=1$, where $H_d=2(c\hbar/e)(k_F/d)$ is the critical field for which the diameter of the electron trajectories spans the plate exactly once, and which has a width, $\Delta H/H_d=2\delta/d$, defined by the second important length of the problem, the effective skin depth δ.[7] The sharp changes in the slope of R that appear between $H/H_d=0$ and $H/H_d=1$ (and are denoted by arrows in Fig. 2) are defined in terms of the scattering angle θ_0 and the effective penetration depth δ by the following simple expression:

$$H_\delta/H_d=(d/2\delta)(1-\cos\theta_0). \tag{9}$$

The field H_δ/H_d signals the transition from a situation ($H<H_\delta$) in which <u>all</u> of the surface trajectories that remain completely within δ are of the sort shown in Fig. 1(a) (i.e., they can describe an infinite number of "skips" since they are specularly scattered at the surface) to a situation ($H>H_\delta$) in which more and more such trajectories are diffusely scattered after just part of one "skip" (from λ_0 to λ_1). In contrast to the RFSE line, which reflects changes in the effectiveness of the coupling between the orbital motion of the "skimming" electrons (those which describe complete circular trajectories) and the rf field in both skin layers, the anomaly discussed here, which is due to the existence of a critical θ_0, reflects the changes in the effectiveness of the coupling between the "skipping" surface trajectories and the rf field in each skin layer separately; in the latter case, there is no interference between the transmitted and the driving rf currents. The anomaly that should exist (if a θ_0 exists) at $H/H_d = (d/2d)(1-\cos\theta_0)$, i.e., when the skipping surface trajectories span the whole plate rather than each skin layer alone, has not yet been investigated in detail.

If a critical angle θ_0 exists, then one should be able to measure it experimentally from a measurement of H_δ/H_d and the width of the first dip in the RFSE line corresponding to the depth of the first rapid decay of the rf field just inside the surface, rather than the larger depth which describes the total extent of the spatial inhomogeneity of the rf field in anomalous skin-effect conditions.[7] In order to make sure that the anomaly at H_δ/H_d is due to the existence of such a scattering angle θ_0, we have verified the invariance of the product $2(H_\delta/H_d)(\delta/d) = 1-\cos\theta_0$ as a function of ω ($\delta \propto \omega^{-1/3}$ for anomalous skin-effect conditions), a check which can also be done experimentally. In addition, as can be seen from the position of the two arrows in Fig. 2, corresponding to the critical angles $\theta_{0,1} = 0.2$ and $\theta_{0,2} = 0.3$, one can easily verify that $H_\delta(\theta_{0,1})/H_\delta(\theta_{0,2}) \approx (1-\cos\theta_{0,1})/(1-\cos\theta_{0,2})$.

In closing, we note that the nonmonotonic behavior in the rf surface impedance of gallium in low magnetic fields[8] could be attributed to a size effect of the sort discussed in this Letter, provided the complicated Fermi surface of gallium does not obscure the interpretation.

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