

SCATTERING OF RAYLEIGH WAVES BY SURFACE IRREGULARITIES*

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Scattering of surface waves by mass defects is estimated as a function of frequency and defect depth. Attenuation of surface waves of microwave frequencies on polished surfaces is discussed. Most of the energy is scattered into other surface waves, rather than body waves, so that energy of surface waves is retained, and leaks into the interior relatively slowly. This may explain the duration of seismic signals on the moon after the crash of the lunar excursion module during the Apollo 12 mission.

Surface elastic waves or Rayleigh waves are not only important in seismology, but have recently become of interest in solid-state electronic applications at microwave frequencies. Since a real surface is rough, and since the material just below the surface contains many imperfections, surface waves are scattered, and it is of interest to calculate the amount of scattering to be expected.

The calculation is analogous to the scattering of lattice waves or elastic waves by model defects.¹ Defects cause a local perturbation of the Hamiltonian, either because of a local mass excess or deficiency, or because of local changes of the elastic constants. The effects of the former are easier to calculate, and the model defect adopted here is an additional mass Δm situated at a point either at the surface or at some depth z below it. The perturbation Hamiltonian is thus $H' = (\Delta m/2)(du/dt)^2$, where \vec{u} is the displacement of the solid at the defect site. Two kinds of scattering must be considered; from the primary Rayleigh wave into Rayleigh waves of different propagation direction, and from the Rayleigh wave into body waves. These processes correspond to terms in the Hamiltonian containing $u_R u_R$ and $u_R u_B$, respectively, where u_R and u_B are displacements due to individual Rayleigh and body waves.

The displacement field of a Rayleigh wave has the standard form²

$$\vec{u}(\vec{r}) = u_{z0}(i\xi(z), 0, -\eta(z)) \exp[i(\vec{k} \cdot \vec{r} - \omega t)], \quad (1)$$

where u_{z0} is the amplitude of the z component at the surface ($z=0$), the x axis is chosen in the propagation direction \vec{k} , and ξ, η describe the polarization as a function of depth. One can show for an isotropic elastic continuum that

$$\xi(z) = \frac{k}{q_t} \left[\frac{k^2 + q_t^2}{k^2 - q_t^2} \exp(-q_l z) - \frac{2q_l q_t}{k^2 - q_t^2} \exp(-q_t z) \right],$$

$$\eta(z) = \frac{k^2 + q_t^2}{k^2 - q_t^2} \exp(-q_l z) - \frac{2k^2}{k^2 - q_t^2} \exp(-q_t z).$$

The two decay constants q_l, q_t are given by

$$q_l^2 = k^2(1 - v^2/v_l^2)$$

$$q_t^2 = k^2(1 - v^2/v_t^2),$$

where $v = \omega/k$ is the velocity of the Rayleigh wave, while v_l, v_t are the velocities of longitudinal and transverse body waves.

The body waves must be chosen so as to satisfy the boundary conditions of the perfect surface. Strictly speaking these are linear combinations of groups of these waves, as discussed for example by Sommerfeld,³ of wave vector (\vec{q}_0, \vec{q}_z) , $(\vec{q}_0, -\vec{q}_z)$, and $(\vec{q}_0, -\vec{q}_z')$ when either the first two waves are transverse and the third longitudinal or vice versa, with q_z' chosen so that all waves have the same frequency. Since the perturbation theory is sensitive only to the enumeration of scattered waves, we simplified the problem, replacing the body waves by combinations (\vec{q}_0, \vec{q}_z) , $(\vec{q}_0, -\vec{q}_z)$, i.e., waves standing in the z direction. However, transverse waves hitting the surface at an angle to the normal of $\theta > \theta_0$, where $\sin \theta_0 = v_t/v_l$, cannot convert into longitudinal waves. This means that the boundary conditions cannot be satisfied exactly at the surface by such transverse waves, and their field must vanish at the surface. We have taken account of this in a crude manner by summing only over transverse waves $0 < \theta < \theta_0$, but over all directions of longitudinal waves.

We thus obtain the following expression for the relaxation rate of the primary Rayleigh wave:

$$\frac{1}{\tau} = \frac{c(\Delta m)^2}{8\rho^2 B^2 v^4} \omega^5 [A_1(kz) + A_2(kz) + A_3(kz)], \quad (2)$$

where ρ is the density of the medium, c is the concentration of defects per unit surface area, while A_1, A_2, A_3 , and B are dimensionless parameters. The coefficient B is a fixed function

of the Poisson ratio ν and varies slowly from 3.20 when $\nu=0.250$ to 3.44 when $\nu=0.29$ to 5.04 when $\nu=0.500$. The term A_1 describes scattering into Rayleigh waves, A_2 into longitudinal body waves, A_3 into transverse body waves. These coefficients decrease rapidly with z , the depth of the defect, as A_1 is a function of ξ^4 and η^4 , while the other terms depend essentially on ξ^2 and η^2 , and the ξ and η decrease exponentially with increasing z . When $z=0$, and for $\nu=0.29$, $A_1=2.20$.

One may define a partition ratio γ , which is the rate of scattering into other Rayleigh waves relative to scattering into body waves:

$$\gamma = (\tau_R)^{-1} / (\tau_B)^{-1} = A_1 / (A_2 + A_3),$$

$$1/\tau = 1/\tau_R + 1/\tau_B, \quad (3)$$

where τ_R and τ_B are the relaxation times if all the scattering were either to other Rayleigh waves or to body waves, respectively. In Fig. 1 γ is plotted versus kz . Clearly γ is very sensitive to impurity depth. When the impurity is less than a wavelength below the surface, Rayleigh waves scatter almost exclusively into other Rayleigh waves of the same frequency but different direction.

Equation 2 predicts that for surface irregularities ($z=0$) scattering varies as the fifth power of frequency. For defects at a depth z below the surface, scattering will increase more slowly with frequency eventually reaching a maximum for $\omega \sim 4\nu/z$ and decreasing at higher frequencies not monotonically but with further oscillations.

The ability to propagate very high-frequency surface waves depends on being able to fabricate sufficiently smooth surfaces. Consider a surface which has been polished with a powder of micron size. Assuming this leaves highs and lows of $0.1 \mu^3$ with a density of $10^8/\text{cm}^2$, the attenuation length $\nu\tau$ would be of the order of 10^3 cm at 1 GHz, but only 10^{-2} cm at 10 GHz. Surface waves of 2 to 3 GHz have been successfully propagated on crystal surfaces; the present considerations would indicate that one is approaching the limit of practicability unless ways are found of preparing smoother surfaces.

The fact that γ ranges typically from 10 to about 60 for surface defects means that surface waves are scattered into other surface waves of different direction but are lost to the interior only relatively slowly, provided that the defect concentration is sufficiently great so that other attenuation mechanisms can be neglected. It is

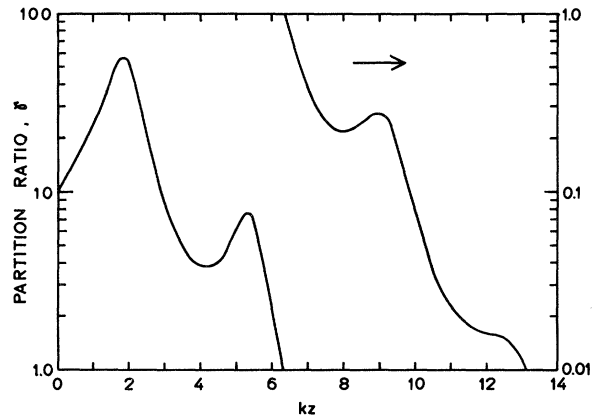


FIG. 1. The energy partition ratio γ versus relative depth of impurity kz for Poisson ratio $\nu=0.29$.

tempting to speculate that this trapping of surface-wave energy is the reason why the seismograph on the moon reported a prolonged "ringing" after the seismic disturbances produced by the nearby crash of the lunar excursion module during the Apollo 12 mission. The radiation in the surface waves spreads slowly outward in a random walk manner. The energy per unit area in a spectral interval inside this swarm of scattered surface waves at time t is

$$U_s = \frac{E_0}{\pi v^2} \frac{\gamma}{\gamma+1} \frac{1}{t\tau} \left\{ \exp\left[-\frac{t}{\tau(1+\gamma)}\right] - \exp\left[-\frac{t}{\tau}\right] \right\}, \quad (4)$$

where E_0 is the energy initially released as surface waves in this spectral interval. We expect γ to be between 20 and 40, depending on the depth of the major obstacles in relation to the wavelength. From press reports it appears that $\gamma\tau$ would have to be of the order of 2×10^8 sec; during this time an unscattered wave would have travelled perhaps 6000 km, but multiple surface scattering would have confined this energy to a radius of say $\frac{1}{6}$ of that, i.e., $1/\sqrt{\gamma}$, or say 1000 km.

It now remains to estimate the roughness parameter

$$R = \sum_i c_i (\Delta m_i)^2$$

summed over obstacles of all sizes. If we assume, for argument's sake, that τ is about 50 sec at frequencies of 30 Hz, we would require a roughness parameter $R=10^8$ kg/m². Individual obstacles of the order of 1000 tons mass distributed at an average frequency of one every 10 000 m² would account for this scattering. This amount of roughness is not inconsistent with our

picture of the moon's surface. It is equivalent to a crater 20 m in diameter and 6 m deep which, according to the Ranger photographs, should occur on the average with a frequency of one crater per 10 000 m².⁴

Since the energy partition ratio γ as well as τ depends on kz , a detailed analysis of the seismograph response with time as a function of frequency should give information about the distribution in depth of the principal obstacles and tell whether one is dealing with surface craters or heavy meteorites embedded at some depth. That roughness could have a bigger effect on surface waves on the moon than on the earth is of course not unreasonable: The absence of atmospheric weathering and the larger number of embedded meteorites would make the moon's surface much rougher on the scale of tens of meters.

An alternative explanation of the duration of seismic signals is that the original disturbances

set off secondary events due to instabilities of the lunar surface. The trapping of surface-wave energy proposed by us would also tend to enhance the period during which secondary disturbances could occur. In that case, however, the spectral decay should depart significantly from Eq. (4).

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ELECTRONIC AND MAGNETIC STRUCTURE OF SmB₆

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A new electronic configuration involving localized 6s electrons is hypothesized for the Sm ions in SmB₆. The model is successful in explaining earlier results of susceptibility measurements and new Mössbauer spectroscopy data reported here.

Recently, a number of reports¹⁻³ discussing the unusual electric and magnetic properties of SmB₆ have appeared. This paper presents new data, obtained by Mössbauer-effect measurements on Sm¹⁴⁹, which help establish the electronic structure of the Sm ions. The results of the Mössbauer experiments are combined with the previously published information to develop a new model which seems to explain the observed properties.

Briefly outlining the results reported in Ref. 1, the electrical characteristics of the material appear to be semiconducting, with resistance sharply increasing down to 3°K, and thereafter increasing only very slowly. The system does not order magnetically down to below 0.35°K, and the temperature dependence of the susceptibility is complex and not consistent with what would be expected from either Sm²⁺ or Sm³⁺ ions. The earlier work attributed the unusual behavior to a thermally excited electronic transition; the Sm ions were considered to be divalent at low temperature and trivalent at high temperature.

This electronic transition has been used as the basis for recent theoretical articles^{2,3} discussing the temperature dependence of the conductivity. We present here a different analysis of the SmB₆ electronic structure.

The Mössbauer isomer shift measures the density of s electrons at the Sm nucleus. In rare-earth ions, a change in the number of 4f electrons results in a changed shielding of the s electrons (especially the two 5s electrons), so that an increase in the number of 4f's increases the shielding and decreases the net s density at the nucleus. This shielding effect is much larger than the direct change in s density obtained by a change in 6s configuration; thus the measured isomer shift provides a direct measurement of the number of 4f electrons. The results of our measurements of isomer shift as a function of temperature are shown in Fig. 1(a). Comparison of the results with those for Sm in known valence states shows that the Mössbauer line falls at neither the Sm²⁺ (4f⁶) nor Sm³⁺ (4f⁵) valence position, but in between. Since rare-earth ions,