tained data similar to that shown in Fig. 3 for a lead film with $R_\square = 8\ \Omega$, also agreeing well with AL near the transitions. We think their results may be related to the disconnected character of their films as discussed by R. S. Thompson, M. Strongin, O. F. Kammerer, and J. E. Crow, Phys. Letters 29A, 194 (1969), for the AL conductivity. Similar considerations applied to the Maki term give $\sigma_M'/\sigma \approx (2\tau_0/\tau)[\xi(0)^2/d^2\delta]$ when the last factor is $\ll 1$.

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PIEZOELECTRIC POLARON-CYCLOTRON RESONANCE IN THE QUANTUM LIMIT IN *n*-CdS

Kenneth J. Button, Benjamin Lax,* and Daniel R. Cohn*
Francis Bitter National Magnet Laboratory,† Massachusetts Institute of Technology,
Cambridge, Massachusetts 02139

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The zero-temperature cyclotron resonance of the electron splits into two groups as the temperature is increased. One group moves rapidly toward very small mass, the other toward larger mass. This splitting has not been predicted by previous theories. Moreover, the first member of the small-mass group can be accounted for only qualitatively by these theories.

We have observed a spectacular splitting of the electron-cyclotron-resonance absorption line in *n*-CdS as a function of temperature. We believe this to be the first observation of the temperature-dependent behavior of the piezoelectric polaron in the quantum limit. The cyclotron resonance was observed at wavelengths of 337 and 195 μm by using an HCN-DCN laser as a source of radiation.[1] At low temperatures and high frequencies where the quantum condition is generously satisfied, namely, $k_B T/\hbar\omega < \frac{1}{4}$, a single sharp absorption line is seen as shown at the lower left of Fig. 1. It appears to be the true cyclotron-resonance electron effective mass in CdS of $(0.159 \pm 0.002)m_0$ at 337 μm and $(0.172 \pm 0.002) \times m_0$ at 195 μm. It is observable only in this limit and is less than the "bare" mass value[2] of $0.2m_0$. We believe that this is due to electron-phonon interaction and, in this case, it corresponds to the emission of phonons. As the temperature is increased to the range $\frac{1}{4} < k_B T/\hbar\omega < \frac{1}{2}$, the single absorption line becomes an unresolved doublet with center near $0.17\ m_0$ which is the mass previously reported.[3,4] Finally, at higher temperatures, the two lines become two groups of lines. These

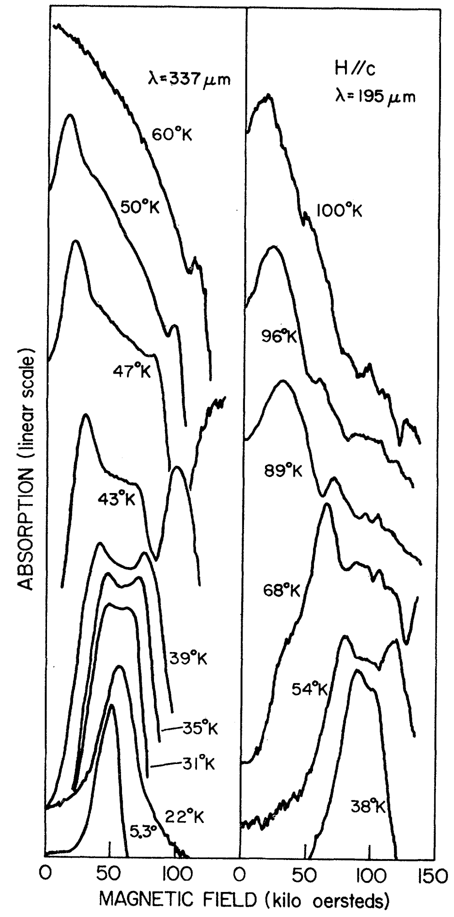

FIG. 1. Tracings of cyclotron-resonance absorption lines in *n*-CdS for several temperatures and at two different wavelengths.

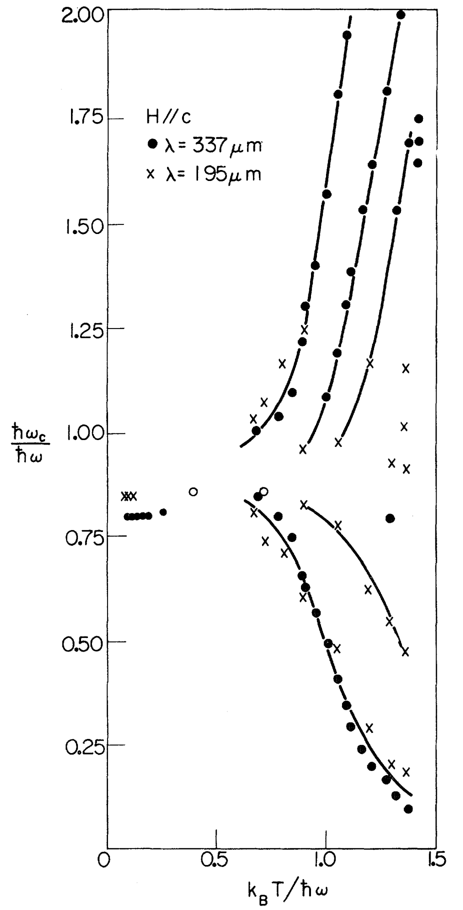

FIG. 2. The equivalent of the resonance magnetic field versus temperature where the cyclotron resonance energy $\hbar\omega_c$ and thermal energy k_BT have been normalized by the photon energy. A "bare" effective mass of $0.2m_0$ has been used in calculating $\hbar\omega_c$ in order to illustrate the depression of the low-temperature mass by electron-phonon interaction. The microwave results of Baer and Dexter (left open circle) and Sawamoto (right open circle) are shown.

groups can be seen more clearly in Fig. 2 where the positions of the resonance peaks of Fig. 1 have been plotted as the ratio of the cyclotron-resonance effective mass to the "bare" mass[2] of $0.2m_0$.

The cyclotron-resonance absorption was recorded by transmitting unpolarized, monochromatic, continuous-wave laser radiation through the specimen at a fixed temperature. The change in transmission was observed as the applied magnetic field intensity was swept continuously up to 130 kOe as shown in Fig. 2. The magnetic field was applied parallel to the c axis of the CdS and also parallel to the direction of propagation as shown in Fig. 3. A water-cooled solenoid magnet was used. The specimen was cooled by the circulation of liquid helium outside of the evacuated light pipe. Helium exchange gas was admitted to the section of the $\frac{3}{8}$-in.-i.d. pipe containing the 1 cm-long specimen in order to cool it uniformly. A heating coil was provided to vary the temperature and a carbon-resistance thermometer was used to measure the temperature to an accuracy of about ±10%.

The laser beam diverges when hole coupling is used and the beam is best collimated by a TPX lens[5] to avoid excessive losses in the light pipe. The laser beam is chopped at 30 Hz and phase-sensitive detection is employed. The monochromatic wavelength is changed, when necessary, by changing the molecular gas that is admitted to the flow-through laser. A few microwatts of cw power is incident on the specimen.

The n-CdS crystals were selected from Eagle-Picher Lot No. 345 by J. E. Powderly, who measured resistivities and mobilities in the range of 8 Ω cm and 250 $\text{cm}^2\ \text{V}^{-1}\ \text{sec}^{-1}$, respectively. During our experiments we noted moderate car-

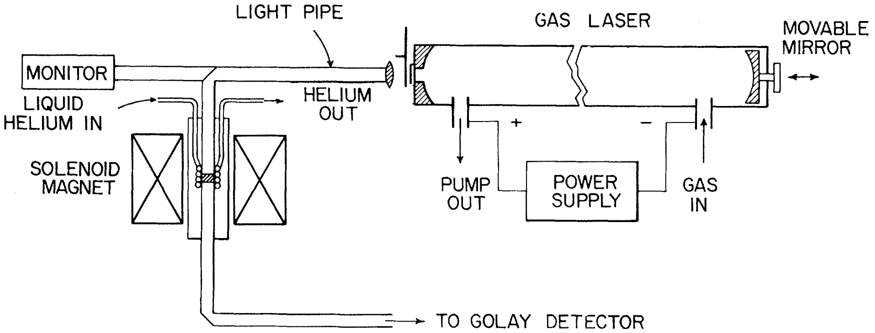

FIG. 3. Diagram of submillimeter molecular-gas–laser spectrometer. The laser may be operated cw using H_2O, D_2O, DCN, or HCN molecules. The specimen may be held at any temperature above 5°K. The magnet has a 1-in. bore.

rier freeze out as the temperature was reduced to about 35°K, but the residual carrier concentration at lower temperatures was estimated to be nearly constant at about 5×10^{14} cm^{-3}.

Theoretical interest in the piezoelectric polaron has been considerable in recent years[6-12] but the relative paucity of experimental data has hindered the complete development of the theory. Hence the analysis of our observation of cyclotron resonance over a wide range of temperature and magnetic field should prove to be of considerable value in the assessment of current piezoelectric-polaron theory.

The lower set of curves in Fig. 2 may be qualitatively accounted for by quantizing the momentum in the zero-field second-order perturbational treatment of the piezoelectric polaron.[7] In this case the lowest two curves would correspond to transitions between $0\rightarrow1$ and $1\rightarrow2$ Landau levels, respectively, but the curves may be partially fitted only if α, the coupling constant, is assumed to be substantially larger than the accepted value. The upper set of curves, however, cannot be understood on the basis of any previous theoretical model.

The data suggest that each of the Landau levels may be split due to the emission and absorption of an acoustic phonon. If this is true, then the splitting should increase with temperature due to the increase in phonon population. Thus, for example, the dominant light-mass peak would correspond to a transition between the lower $n=0$ Landau level and the higher $n=1$ Landau level. The dominant heavy-mass peak would correspond to a transition between the upper $n=0$ Landau level and the lower $n=1$ Landau level. At higher temperatures the lower $n=1$ level becomes populated and transitions from $n=1$ to $n=2$ are possible giving rise to the smaller peaks in Fig. 1.

If, in fact, our data do correspond to the splitting of the levels, one might consider several additional possibilities. We shall mention two: One of these might involve possible phonon-induced degeneracies of adjacent Landau levels analogous to the case of the electron-optical-polaron interaction.[13] Since there is a continuous spectrum of acoustic phonons, however, this mechanism does not seem very probable. Using this approach, we have also invoked the possibility of a multiphonon process, but we have been unable so far to demonstrate the splitting mathematically. Another possible model for splitting is that, at lower magnetic field intensities, net emission of phonons dominates, which would tend to make the cyclotron-resonance mass lighter, whereas at higher magnetic field intensities, net absorption is favored. In effect, we are speculating that there may be a difference in a level when it absorbs or emits a phonon. In fact we believe that, on a classical basis, such a phenomenon may involve some form of geometrical resonance.[14] However, again we have been unable to derive this quantitatively from either a classical or a quantum-mechanical model. A rationale for this model of splitting rests partially upon the statement by Miyake that the strongest interaction of electrons with phonons is within each magnetic sub-band rather than between magnetic subbands. His theory does not demonstrate a possible splitting, however.

The resonance condition might be derived from a classical model but the temperature dependence of the threshold for splitting shown in Fig. 2 cannot emerge explicitly in this way. We can derive the temperature dependence from a quantum-mechanical model and demonstrate singularities which resemble the condition for geometrical resonance.

Unfortunately, both the existing theoretical treatments as well as our recent attempts to modify them to account for the splitting have been inadequate to account quantitatively for the results. Consequently it appears that additional theoretical work will be necessary to achieve this.

Of particular interest are the sharp absorption lines whose positions are independent of temperature at low temperatures indicated by the points and crosses in Fig. 2 where $0.05<k_BT/\hbar\omega<0.2$. When the zero-temperature approach of Mahan and Hopfield involving second-order perturbation theory is applied, it can be shown that the zero-point vibrations of phonons add a fixed term to the first transition in the quantum limit. This suggests that these determinations of two masses at two different frequencies will enable us to estimate the "bare" effective mass from the expression $m_0{}^*/m_0=e(H_1-H_2)/m_0c(\omega_2-\omega_1)$ where 1 and 2 are the two frequencies and magnetic fields for which we have data. The two values of field are 50.7 and 89.7 kOe. This yields a calculated effective mass of 0.17 m_0. Since the calculations at zero temperature are relatively uncomplicated, it appears to be profitable to make additional low-temperature measurements at other frequencies to compare with zero-temperature calculations. The purpose of this is to provide a good quantitative measure of the coupling constant α.

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*Also Physics Department, Massachusetts Institute of Technology, Cambridge, Mass.

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