that in the range $0 \leq \rho \leq 8.42$ the lower bound on $A(s,t)/A(s,0)$ is always negative and less in magnitude than the upper bound $U(s,t)$ varying monotonically from -0.162 to -0.138 as ρ varies from 0 to 11.5. Therefore in the range $0 \leq \rho \leq 8.42$ the upper bound on $[A(s,t)/A(s,0)]^2$ is simply $[U(s,t)]^2$.

Further details and other upper and lower bounds in the physical and unphysical regions will be reported in a later detailed paper.[3]

[1]S. W. MacDowell and A. Martin, Phys. Rev. 135, B960 (1964).

[2]A. Martin, Phys. Rev. 129, 1432 (1963).

[3]V. Singh and S. M. Roy, "Unitarity Upper and Lower Bounds on the Absorptive Parts of Elastic-Scattering Amplitudes" (to be published).

COMMENTS ON THE LETTER "EVIDENCE OF QUARKS IN AIR-SHOWER CORES"*

Hans Frauenfelder, U. E. Kruse, and R. D. Sard
Department of Physics, University of Illinois, Urbana, Illinois 61801
(Received 17 November 1969)

The "quark tracks" observed by McCusker and co-workers in cloud chambers can be explained by making reasonable assumptions about two processes: fluctuations in the number of droplets in a cloud-chamber track, and the relativistic rise of the ionization. However, before a firm conclusion can be drawn, more experimental data on the drop-count distribution of a large sample of tracks in the experiment are needed.

McCusker and collaborators have recently performed an ingenious experiment to search for quarks in the cores of large air showers.[1,2] Among 5×10^4 tracks in delayed-expansion cloud chambers, they have found five with about one-half the ionization of comparison tracks in the same or near-by pictures; they interpret these as being due to quarks with charge $\frac{2}{3}$. The quark flux computed on the basis of this experiment is in mild disagreement with the result of searches in terrestrial matter.[3,4] This disagreement can be explained away because the chemical behavior of quarks is not known. Nevertheless it has led us to re-examine the evidence presented by the Sydney group.

In estimating the probability of a few deviant cases in a large sample, one is faced with the difficulty that the result is very sensitive to the value of the assumed standard deviation of the parameter measured. Only an experimental determination of the frequency of tracks as a function of drop count will permit a definitive distinction between a subsidiary peak at low ionization and the low-ionization tail of a broad peak. Meanwhile, consideration of what is known at present about drop-count statistics and the increase of ionization beyond the minimum suggests that the observed effect is not necessarily due to particles of reduced charge.

The increase in ionization beyond the minimum is well established[5]; the corresponding increase in drop count in a cloud chamber has been observed in many gases[6]; in argon it reaches 20 % at $\gamma = 20\text{-}40$ and 40 % at $\gamma = 100\text{-}400$ (γ is the energy in units of the rest energy). Shower particles rarely appear at minimum ionization in a cloud chamber.[7] In an air shower of 10^6 charged particles at sea level, typical[8] electron energies near the core are 0.5 GeV ($\gamma = 1000$); the muons have average energy exceeding 5 GeV ($\gamma = 50$). Under lead the electron energies are lower, but it is safe even here to assume that the average ionization is at least $1.2\times I_0$, where I_0 is the minimum ionization. To be conservative, we shall assume for the following estimates that particles with energies corresponding to ionization between I_0 and $1.3\times I_0$ are present, and that the number of particles is uniformly distributed over this range. Then the average ionization of the comparison tracks is $1.15\times I_0$ and the expected ionization of a charge $\frac{2}{3}$ quark at its minimum is $(4/9)/1.15 = 0.39$ times the average of the comparison tracks.

The fluctuation in the number, N_d, of droplets in cloud-chamber tracks of particles of fixed charge and velocity is not given by $(N_d)^{1/2}$, but is considerably larger.[9-11] Three processes are involved: the primary ionization by the fast particle; the secondary ionization by ejected electrons; and the formation of photographable drops on diffused ions. The primary ionization is indeed Poissonian, but the mean number involved[12] is about $0.55 \times N_I$, where N_I is the total number of ion pairs along the track, excluding blobs and delta rays.[6,13,14] The secondary ionization is determined mainly by the energy lost in the primary collision. The elimination in drop counting of track segments containing blobs or delta rays reduces fluctuations in this ionization cascade and also makes it impossible to apply the calculations of Landau[15] or Blunck and Leisegang.[16] Experimental data on the ionization fluctuations will be discussed below. The fluctuations in the third process result from the inefficiency of drop formation. Operation of a cloud chamber at full condensation efficiency gives excessive background (fog); when ethyl alcohol and water are used as condensants, one usually tries to operate at full efficiency for positive ions and partial efficiency for negatives.[17,9,10] Since track segments containing agglomerations of droplets are excluded, one is justified in assuming that the vapor supply is adequate to make condensation on different ions statistically independent. The fluctuation in the drop count N_d is thus given by

$$\left(\frac{\sigma(N_d)}{\langle N_d\rangle_{\rm av}}\right)^2 = \left(\frac{\sigma(N_I)}{\langle N_I\rangle_{\rm av}}\right)^2 + \frac{\epsilon_+(1-\epsilon_+)+\epsilon_-(1-\epsilon_-)}{(\epsilon_++\epsilon_-)^2}\frac{1}{\langle N_I\rangle_{\rm av}}.$$

Here $\sigma(N_d)$ is the standard deviation of N_d and $\sigma(N_I)$ is that of N_I; ϵ_+ (ϵ_-) is the average efficiency for forming a drop on a positive (negative) ion. The first term, giving the fluctuation in the number of ion pairs, takes account of the primary and secondary ionization Only if there is no secondary ionization is $\sigma^2(N_I)$ equal to $\langle N_I\rangle_{\rm av}$; in reality it is always larger. The second term in the formula is the contribution from the inefficiency in the drop formation.

Published data on drop-count statistics are summarized in Table I. The most informative paper is that of Wilson and co-workers.[9] They present histograms of the number of drops per centimeter for minimum-ionizing tracks in oxygen without magnetic field, both for about 1600 segments of 1 cm length and for 101 tracks of average length 16 cm. The positive and negative columns were separated, and only the positive column was counted with $\epsilon_+ = 1.0$. For the full-length tracks the distribution is approximately Gaussian, with a mean of 40.2 drops/cm and a standard deviation of 3.6 drops/cm. The fractional error is thus 9.0%, somewhat larger than $16^{-1/2}\times$ the fractional width for 1-cm segments. This suggests that there is, as expected,[15] positive correlation between drop counts in adjacent cells. The fractional ionization errors quoted in the different experiments are shown in column 7. For comparison they have been normalized in column 8 to the same mean number of ion pairs (500). Assuming that the $I = I_0$ tracks in the Syd-

Table I. Cloud-chamber drop-count statistics.

(1) Ref.	(2) Chamber gas[f] (atm)	(3) ϵ_+	(4) ϵ_-	(5) Magnetic field (G)	(6) $\langle N_I\rangle_{\rm av}$ for track length used	(7) $\sigma(N_I)/\langle N_I\rangle_{\rm av}$	(8) Expected fractional error in N_I if $\langle N_I\rangle_{\rm av} = 500$[g]	(9) Expected fractional error in N_d if $\langle N_I\rangle = 500$, $\epsilon_+ = 0.5$, $\epsilon_- = 0$.
a	He, 1.4	1.0	0.6	7500	500	0.074	0.074	0.086
b	O_2, 1.0	1.0	···	0	640	0.090	0.102	0.111
c	Ar, 0.21 He, 0.21	1.0	0.4	8000	715	0.050	0.060	0.075
d	Ar, 0.44 He, 0.44	1.0	0.54	0	5.63[h]	0.68[h]	0.072[h]	0.085[h]
e	Xe, 0.25	1.0	0.25	6200	840	0.094	0.122	0.13

[a]Fretter, Friesen, and Lagarrigue, Ref. 10.
[b]Ghosh, Jones, and Wilson, Ref. 9.
[c]L. F. Hansen and W. B. Fretter, Phys. Rev. 118, 812 (1960).
[d]Louttit, Ref. 14.
[e]Rousset, Lagarrigue, Musset, Rançon, and Sauteron, Ref. 11.
[f]In addition to 4-5 cm Hg of ethyl-alcohol water.
[g]Computed on assumption that fractional error goes as $(\langle N_I\rangle_{\rm av})^{-1/2}$.
[h]Data given only for 2-mm segments, excluding those with more than 28 drops. There is some correlation between adjacent segments, and the calculated errors in columns 8 and 9 are therefore underestimates.

Table II. Expected number of tracks with low drop count in a sample of 5×10^4 tracks for various fractional fluctuations in drop counts. The quantity α is the ratio of droplets on a low-count track to the number of droplets on an average track.

Fractional fluctuation in drop count of minimum tracks (%)	α 0.45-0.50	0.50-0.55	0.55-0.60
9	0.006	0.11	1.3
11	0.4	3.3	18
13	5.1	20	72

ney experiment[1,2] have $\langle N_i\rangle = 500$, and that $\epsilon_+ = 0.5$, $\epsilon_- = 0$, we derive from the formula above the fractional drop-count fluctuations shown in column 9.

We now assume that the reference tracks are distributed in velocity as indicated above, and that the drop-count distribution for a particular γ is Gaussian with variance proportional to the mean. It is straightforward to compute the number of tracks appearing with N_d equal to some fraction α of the average value for the standard tracks. The results are shown in Table II. The different rows correspond to different assumed values for the drop-count error on minimum tracks. In order to decide which value is most appropriate for the Sydney data, we compare the conditions of this experiment (1.4-atm argon, no magnetic field) with those presented in Table I. The only two experiments performed without magnetic field are those of Refs. 9 and 14. Of these the latter very likely gives an underestimate of the error on an extended track, because the track segments used were only 2 mm long and the error has been calculated on the assumption of no correlation between adjacent cells. Louttit's data indicate that there is positive correlation and such correlation is expected to increase the width.[15,16] The conditions of Ref. 9 (data for full-length tracks) seem to resemble most closely those of the Sydney experiment, and we use 11% as our best estimate of the drop-count error. In this case the expected number of tracks in a sample of 5×10^4 is 0.4 in the range $\alpha = 0.45$-0.50, 3.3 in the range 0.50-0.55, and 18 in the range 0.55-0.60. However, even with the smallest error in Table II, the expected values are uncomfortably close to those observed in the Sydney experiment. Careful experiments on fluctuations, with a known spectrum of incident velocities, are therefore needed before a definite statement on the presence or absence of quarks in air-shower cores can be made.

*Work supported by the National Science Foundation under Grant No. NSF GP 9312 and by the U. S. Atomic Energy Commission.

[1]I. Cairns, C. B. A. McCusker, L. S. Peck, and R. L. S. Woolcott, Phys. Rev. (to be published). R. D. S. is grateful to Dr. McCusker for providing a copy of this technical report, with half-tones.

[2]C. B. A. McCusker and I. Cairns, Phys. Rev. Letters 23, 658 (1969).

[3]W. Chupka, J. Schiffer, and C. Stevens, Phys. Rev. Letters 17, 60 (1966).

[4]D. D. Cook, G. DePasquali, H. Frauenfelder, R. N. Peacock, F. Steinrisser, and A. Wattenberg, Phys. Rev. (to be published).

[5]R. Sternheimer, in Methods of Experimental Physics, edited by C. S. Wu and L. C. L. Yuan (Academic Press, Inc., New York, 1961), Vol. 5A, p. 1.

[6]R. C. Kepler, C. A. D'Andlau, W. B. Fretter, and L. F. Hansen, Nuovo Cimento 7, 71 (1958).

[7]W. E. Hazen, Phys. Rev. 67, 269 (1945).

[8]K. Greisen, Ann. Rev. Nucl. Sci. 10, 63 (1960).

[9]S. K. Ghosh, G. M. D. B. Jones, and J. G. Wilson, Proc. Phys. Soc. (London), Ser. A 67, 331 (1954).

[10]W. B. Fretter, E. W. Friesen, and A. Lagarrigue, Nuovo Cimento Suppl. 4, 569 (1956).

[11]A. Rousset, A. Lagarrigue, P. Musset, P. Rançon, and X. Sauteron, Nuovo Cimento 14, 365 (1959).

[12]M. Cosyns, Nature 138, 284 (1936); G. W. McClure, Phys. Rev. 90, 796 (1953).

[13]R. H. Frost and C. E. Nielsen, Phys. Rev. 71, 864 (1953).

[14]R. I. Louttit, thesis, Washington University, 1958 (unpublished).

[15]L. Landau, J. Phys. (U.S.S.R.) 8, 201 (1944).

[16]I. Blunck and S. Leisegang, Z. Physik 128, 500 (1950).

[17]C. E. Nielsen, thesis, University of California, Berkeley, 1941 (unpublished).