NUCLEAR FERROMAGNETISM IN NEUTRON STARS CALCULATED WITH REALISTIC FORCES*

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Using a highly realistic effective interaction, it is shown that nuclear ferromagnetism in neutron-star matter is not possible for densities of $k_F < 1.8\ \mathrm{fm}^{-1}$.

It now seems fairly clear that pulsars are rotating neutron stars,[1] and although several different models have been proposed to account for the "searchlight" effect, all require magnetic fields of 10^9-10^{14} G.[2-4] At nuclear and subnuclear densities neutron-star matter, which we like to call "celestial nuclear matter," consists of a neutral mixture of neutrons, protons, and electrons (muons and hyperons only become significant components at somewhat higher densities), and it is in the electrons that most workers have sought the precise origin of these intense magnetic fields. In particular, a newly proposed mechanism leading to a stable ferromagnetic state in a degenerate electron gas would appear to be promising.[5]

An alternative possibility that has been considered in two recent papers[6,7] is that of nuclear ferromagnetism. It is suggested that because of the strong repulsion in the short-range part of the nucleon-nucleon (N-N) force, it might be energetically preferable for the degenerate nucleons in celestial nuclear matter to go over into a partially polarized configuration, the intrinsic magnetic moments of the nucleons then giving rise to a net field. Polarization certainly leads to an increase in the kinetic energy, but it also increases the number of nucleon pairs in a triplet state and hence, through the Pauli principle, the mean separation of the nucleons. Thus, it is argued, there will be a reduction in the net repulsion, which may be sufficient for a ferromagnetic transition to occur.

However, at ordinary nuclear densities the real N-N force gives rise to a net attraction,[8] so a priori it is not at all obvious that the proposed mechanism will be operative in reality. In fact, whether or not the transition takes place will depend sensitively on the nature of the cancelation between the repulsive and attractive parts of the force, and it would seem that the interactions used in Refs. 6 and 7 are far too simplified to permit any firm conclusion. (Both papers use spin-independent hard-core interactions, and while the latter has an attractive tail as well, unlike the former, its relation to reality is not clear.) In the present note, therefore, we re-examine the problem by calculating celestial nuclear matter in perturbation theory with a highly realistic effective interaction that we have developed.[9]

This interaction, which we label SP1, gives the correct binding energy and density of ordinary nuclear matter at saturation, calculated in first-order perturbation theory. Also, a good value is obtained for the symmetry coefficient, which implies that a reliable extrapolation to large neutron excesses can be made. Furthermore, the calculated compressibility agrees well with the value of 210 MeV that we have suggested,[10] thus indicating that the extrapolation to higher densities can be made with confidence. Finally, our interaction goes over in its long-range part to the one-boson exchange potential, thereby guaranteeing, in the spirit of the separation method,[11] a meaningful relation with the real N-N force.

To determine whether or not a ferromagnetic state is possible at any particular density that is likely to prevail in a neutron star[12] the total energy per nucleon has to be minimized with respect to the proton ratio $\gamma = Z/A$, the neutron polarization $\xi_n = (N_+ - N_-)/N$, and the proton polarization $\xi_p = (Z_+ - Z_-)/Z$. Here N represents the number of neutrons in a small element of volume Ω, Z the number of protons, and $A = N+Z$ the number of nucleons. Also N_+ and Z_+ are the numbers of neutrons and protons, respectively, pointing in one direction, while $N_- = N - N_+$ and $Z_- = Z - Z_+$ are the corresponding numbers of nucleons pointing in the opposite direction. The Fermi momenta of these four groups of nucleons are given by

$$\kappa_{n\pm} = \{2(1-\gamma)(1\pm\xi_n)\}^{1/3} k_F, \quad \kappa_{p\pm} = \{2\gamma(1\pm\xi_p)\}^{1/3} k_F, \tag{1}$$

where $k_F = (3\pi^2 A/2\Omega)^{1/3}$ is the usual Fermi momentum.

The total energy per nucleon may be written as

$$e(k_F, \gamma, \xi_n, \xi_p) = \bar{t}_{\mathrm{nucl}} + (\Omega/A)\bar{T}_e + v_{\mathrm{nucl}} + \{M_n(1-\gamma) + (M_p + m_e)\gamma\}c^2. \tag{2}$$

Here, assuming complete nucleon degeneracy,

$$\bar{t}_{\mathrm{nucl}} = \frac{3\hbar^2}{40M}\frac{1}{k_{\mathrm{F}}^3}(\kappa_{n+}{}^5 + \kappa_{n-}{}^5 + \kappa_{p+}{}^5 + \kappa_{p-}{}^5) \tag{3}$$

is the mean nucleon kinetic energy, while $\overline{T}_e$ is the electron (degenerate) kinetic energy per unit volume, and has been given by Chandrasekhar[13] (it is completely determined by k_{F} and γ, since neutrality is assumed).

The potential energy per nucleon is given in first-order perturbation theory by

$$\bar{v}_{\mathrm{nucl}} = \frac{3}{(4\pi)^4}\frac{1}{k_{\mathrm{F}}^3}\sum_{\sigma}\sum_{\substack{m_i m_j \\ \tau_i \tau_j}}\sum_{ST} C^2(\tfrac{1}{2}\tfrac{1}{2}S, m_i m_j)C^2(\tfrac{1}{2}\tfrac{1}{2}T, \tau_i\tau_j) \times \int d\vec{k}\,d\vec{K}\langle e^{i\vec{k}\cdot\vec{r}}\chi_S{}^{m_i+m_j}|V_{ST}{}^{\sigma}|\{e^{i\vec{k}\cdot\vec{r}} - (-)^{T+S}e^{-i\vec{k}\cdot\vec{r}}\}\chi_S{}^{m_i+m_j}\rangle, \tag{4}$$

where $V_{ST}{}^{\sigma}$ denotes the various types of force (central, vector, or tensor as $\sigma = 0$, 1, or 2, respectively) in the different spin-isospin states ST. The matrix elements of the direct tensor term and of both the direct and exchange vector terms vanish identically. Furthermore, the tensor exchange term vanishes when integrated over all directions of $\vec{k}$, so that only the central force can contribute. (One of us recently[14] made the incorrect statement that the tensor force could contribute in first order to a polarized configuration.) Finally, the integration over the c.m. momentum $\vec{K}$ and the relative momentum $\vec{k}$, which is constrained by the four κ_i, can be reduced analytically to an integration over k. Then

$$\bar{v}_{\mathrm{nucl}} = \frac{1}{\pi k_{\mathrm{F}}^3}\sum_{\substack{m_i m_j \\ \tau_i \tau_j}}\sum_{ST} C^2(\tfrac{1}{2}\tfrac{1}{2}S, m_i m_j)C^2(\tfrac{1}{2}\tfrac{1}{2}T, \tau_i\tau_j)\Big[2\kappa_<{}^3\int_0^{\kappa_-} k^2 I_{ST}{}^0(k)dk \times \int_{\kappa_-}^{\kappa_+}\{k^5 - \tfrac{3}{2}(\kappa_i{}^2 + \kappa_j{}^2)k^3 + (\kappa_i{}^3 + \kappa_j{}^3)k^2 - \tfrac{3}{16}(\kappa_i{}^2 - \kappa_j{}^2)^2 k\}I_{ST}{}^0(k)dk\Big], \tag{5}$$

where $\kappa_- = \frac{1}{2}|\kappa_i - \kappa_j|$, $\kappa_+ = \frac{1}{2}(\kappa_i + \kappa_j)$, $\kappa_<$ is the lesser of κ_i and κ_j, and

$$I_{ST}{}^0(k) = \sum_l \{1 - (-)^{l+S+T}\}(2l+1)\int_0^\infty j_l{}^2(kr)[V_{ST}{}^s(r) + 2f(k^2)V_{ST}{}^d(r)]r^2 dr. \tag{6}$$

In this latter expression, we are admitting an arbitrary velocity dependence into the central force: $V_{ST}{}^0 = V_{ST}{}^s(r) + \{f(p^2/\hbar^2)V_{ST}{}^d(r) + \mathrm{H.c.}\}$. The explicit values of the Clebsch-Gordan coefficients appearing in (5) depend upon which of the ten different pairwise groupings of nucleons is being considered, and the resultant expression, being rather complicated, is not shown here.

To seek ferromagnetic states we set $\xi_n = \xi_p = 0$ and minimized e with respect to γ at the density in question. Then keeping this value of γ fixed we computed e as a function of ξ_n and ξ_p, using (2) and (5). For all densities up to $k_{\mathrm{F}} = 1.8\ \mathrm{fm}^{-1}$, e increased monotonically with the polarization. (Special attention had to be paid to the possibility of very shallow minima occuring for very small polarization, since a ξ_n as small as 10^{-6} would give rise to enormous fields.)

We thus conclude that for $k_{\mathrm{F}} < 1.8\ \mathrm{fm}^{-1}$ there is no nuclear ferromagnetism in celestial nuclear matter. For higher densities it is essential to take account of the presence of muons and hyperons, and here our knowledge of the baryonic forces is quite insufficient. Indeed, even the N-N force becomes very uncertain and there is no reason for believing that our effective interaction will be valid at very high densities. On the other hand, there is no firm reason for believing in the famous hard core, either—if it does exist then nuclear ferromagnetism will certainly set in at high enough densities.

For completeness, we show in Fig. 1 the equilibrium values of e and γ over the range of medium densities. In view of the realistic nuclear-matter characteristics of our interaction, we believe that these curves can be treated with some confidence. It is interesting to note that the proton-electron concentration reaches a maximum of ~5% at the density of ordinary nuclear matter.

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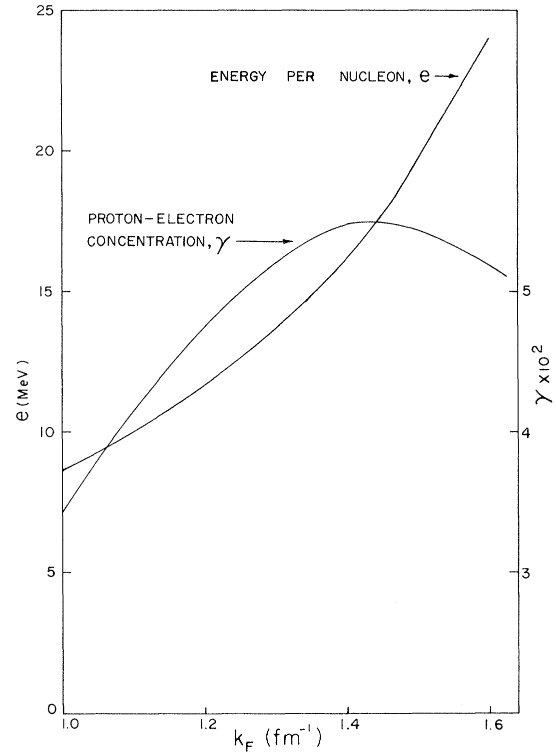

FIG. 1. Energy per nucleon and proton-electron concentration of celestial nuclear matter, calculated with interaction SP1.

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CONFIRMATION OF A NEW $\Lambda\pi$ RESONANCE IN THE REACTION $K^-n \to \Lambda\pi^+\pi^-\pi^-$ †

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Evidence is presented to confirm the existence of a new $\Lambda\pi$ resonance in the $\Lambda\pi^+\pi^-\pi^-$ final state from K^-d interactions at 4.5 GeV/c. A mass of 1642 ± 12 MeV and a width of 55 ± 24 MeV are obtained. Upper limits on $\Sigma\pi$ and $\Lambda(1405)\pi$ branching ratios are obtained for the first time. The branching ratios are found to be consistent with an octet assignment.

In this paper evidence is presented for the production of a $\Lambda\pi$ resonance of mass 1642 ± 12 MeV and width 55 ± 24 MeV in the reaction

$$K^-d \to p_s\Lambda^0\pi^-\pi^-\pi^+. \quad (1)$$

It is observed to be produced with a cross section of 18 ± 3 μb. This confirms the original observation of this state in the same reaction by Crennell et al.[1]

The data were obtained from an analysis of a