approximation explained by Smith[5] may be used to show that $\sigma$ is composed of a smoothly varying and an oscillatory part for $H_{fi}'$ passing through an extremum.

In conclusion, we observe that the theoretical prediction of the oscillatory structure of the charge-transfer cross sections in qualitative agreement with experiment is possible with the present method. However, the possibility of existence of excited states of ions and atoms that may influence the experimental results is not considered in the present theory. Further analysis of the experimental results will depend upon the accuracy of the approximate atomic wave functions and other approximations used in the theory.


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# OBSERVATION OF A PLASMA POLARIZATION SHIFT FOR THE RESONCE LINE OF IONIZED HELIUM*


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The He II 304-Å line emitted from a $N_e \approx 3\times 10^{17}$-cm$^{-3}$, $T_e \approx 4$-eV plasma in a $T$-type electric shock tube shows a blue shift of ~0.05 Å. This shift is interpreted as due to the negative space charge from the polarization of plasma regions occupied by positive ions.


Spectral lines from one-electron systems (hydrogen, ionized helium, etc.) are generally assumed not to suffer any shifts from interactions between the radiating atoms or ions with the charged particles in a plasma. Rather, they are considered to be broadened in an almost symmetrical way through the usually quasistatic action of ions and by electron collisions, whose effects can normally be treated in the impact approximation. Symmetrical and unshifted profiles calculated[1,2] on this basis have been found to be in very satisfactory agreement with many experiments,[3] and no substantial shifts have been reported. There was but one exception.[4] The profile of the He II 4686-Å line appeared to be shifted to shorter wavelengths by about 25% of its full width. This was explained as due to the partial screening of the nuclear charge by plasma electrons, whose average charge is not entirely neutralized by plasma ions in the presence of Coulomb interactions between radiators and plasma particles. A quasiclassical estimate[4] of the resulting line shift indeed agreed with the shift first reported, but additional measurements[5] with improved resolution showed that the shift had actually been simulated by some unresolved Si III lines on the short-wavelength side of the He II line, and later theoretical work[6] suggested that quantum corrections reduce the estimated shift by as much as an order of magnitude.

Before presenting our new experimental evidence for a plasma polarization shift, we briefly review the theoretical situation, beginning with the observation that all standard line-broadening calculations utilize a multipole expansion for the electrostatic interaction energy of radiators and perturbers but omit the first (monopole) term in this expansion. This is justified by saying that the interactions corresponding to this term are the same for the two radiator states responsible for a line, and that their difference, which is relevant for the line-broadening problem, therefore vanishes. However, if perturbers come to within the range of radiator wave functions, the monopole terms also depend on the internal radiator state, and their effect must be included, e.g., by considering[6] the additional electrons to be in (mostly doubly excited) bound or free states of neutral helium. Also, the standard calculations of the electron broadening assume that the radiator can be taken as unperturbed initially, i.e., only account for fluctuations in the perturber charge density. This is appropriate for neutral radiators, for which the time-averaged perturber charge density indeed vanishes (as do the combined electron and ion monopole effects), but not for, e.g., singly ionized radiators in whose vicinity classical statistics gives a time-averaged electron density

$$N(r) = N\exp(e^2/rkT) \tag{1a}$$

with $N$ being the electron density averaged over the whole plasma volume. For radii $r$ as small as a Bohr radius $a_0$, the corresponding quantum-mechanical distribution would of course be smoothed out considerably, and Eq. (1a) should be replaced by

$$N(r) \approx N\exp(E/kT), \tag{1b}$$

where $E$ will be close to the ionization energy of neutral helium. For perturbing ions, a relation analogous to Eq. (1a) remains reasonably valid, so that their average density is entirely negligible near $r \approx a_0$, and the time-averaged perturber charge within the $n$th Bohr orbit of the ion is accordingly

$$\Delta z \approx -N\exp\left(\frac{E}{kT}\right)\frac{4\pi}{3}\left(\frac{n^2a_0}{2}\right)^3. \tag{2}$$

Using $E_n = z^2/n^2$ Ry for the energy levels of the radiator, the (relative) wavelength shift of the line in question is finally estimated as

$$\frac{\Delta\lambda}{\lambda} = -\frac{10\pi}{3}a_0^3N\exp\left(\frac{E}{kT}\right) \tag{3}$$

from this simple model.

Our measurements of the shift of the ionized-helium resonance line were made using a $T$-type electric shock tube. The filling gas was pure helium, and the tube was driven by a 1-$\mu$F capacitor charged to 45 kV. To allow observation of the vacuum-ultraviolet radiation, the Pyrex $T$ tube is joined to an aluminum tube with the same 17-mm internal diameter. At a total distance of approximately 13 cm from the electrodes, two stainless-steel slits are let into the wall of the aluminum expansion tube, diametrically opposite each other. These slits are 0.3 mm wide and protrude into the expansion tube far enough to expose a 5-mm slit length to the plasma. Behind one slit is a small cavity, closed with a Pyrex glass window to permit visible observations; behind the other slit is a 2-m grazing-incidence spectrograph-monochromator. (Putting the slit and expansion chamber in front of the glass window reduced deposits on the window to a negligible level even during a run of several hundred discharges.) The vacuum monochromator has a 20-$\mu$ entrance slit, resulting in a near Gaussian instrument profile with full half-width 0.13 Å at $\lambda = 304$ Å. Profiles of the helium-ion resonance line were obtained at any time in the duration of the $T$-tube plasma by making measurements at different wavelengths on a shot-to-shot basis. The position of the unshifted line center was then found by scanning the same line in the emission from a Tanaka type capillary discharge lamp operating in helium. Using this lamp, absolute wavelength measurements were found to be reproducible within ±0.01 Å.

Measurements of the visible spectrum were made simultaneously on two $\frac{1}{4}$-m Ebert monochromators and a $\frac{1}{2}$-m monochromator. These instruments observed continuum radiation near 5200 Å, the total intensity of the helium ion line at 4686 Å, and (on a shot-to-shot basis) the profiles of He I $\lambda$5876 Å and He II $\lambda$4686 Å. Thus the first two observations verified the reproducibility of electron density and temperature, while the magnitudes of electron density and electron temperature were found from the width of He I $\lambda$5876 Å and the ratio of total line intensities, respectively.

Operating the $T$ tube at a fill pressure of approximately 1 Torr created a very reproducible plasma at the point of observation. Then two quite different plasma conditions could be studied: (1) the incident plasma and (2) the reflected plasma. It was not possible to make these measurements on the same discharge, however, because for (2) the reflector was positioned less than 1 mm beyond the observation point. In case (1) the electron density was $\sim 3\times 10^{16}$ cm$^{-3}$, the temperature ~4 eV, and the profile of the helium-ion resonance line was given by the apparatus function with no measurable shift ($\lesssim 0.02$ Å). In case (2) the electron density was $\sim 3\times 10^{17}$ cm$^{-3}$, the temperature ~4 eV, and the helium-ion resonance line (see Fig. 1) showed a blue shift of 0.05 Å, to be compared with 0.06 Å from Eq. (3) and using $E = 24$ eV. [Note that the comparison between cases (1) and (2) rules out Doppler effects as a dominant cause of the shift.] Further measurements at higher electron density and lower temperature, though less reporducible, indicated

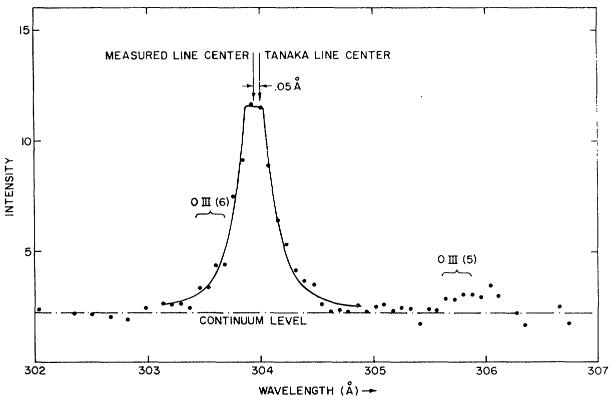


FIG. 1. Profile of the ionized-helium resonance line. The measured points are averaged over ten or more discharges; the curve is a best-fit profile of the form $A\{1-B\exp[-C(\lambda-\lambda_0-\Delta\lambda)^{-2}]\}$.

larger blue shifts consistent with Eq. (3). (Measurements on the line He II λ256 Å were inconclusive because of the near equality of the intensities of O III lines with the helium ion line and the strong continuum level.)

Also indicated in Fig. 1 are the multiplets (5) and (6) of O III, which is the most abundant impurity. However multiplet (6), which overlaps the helium-ion line, is even weaker[7] than multiplet (5) and can thus be eliminated as a cause of the measured blue shift.

The observed shift (believed to be accurate to ~±20%) is actually much larger than Doppler and Stark widths (of absorption or emission coefficients), because the emission-linewidth is mostly due to the large optical depth of the plasma. Still, the wings of the measured profile can be explained by Stark-broadening calculations along the usual lines, except for the blue shift discussed here. For most other lines from singly charged ions, plasma polarization shifts should be less important, mainly because for them the analog of the exponential factor in Eq. (3) is much smaller. According to our theoretical model, relative wavelength shifts for resonance lines of higher members of the one-electron sequence should scale as $(2/z)^4$, if the exponential factor and electron density remain the same. Their shifts may therefore also be observable, e.g., in spark sources with electron densities higher than ours by two or three orders of magnitude, since the ratio of ionization and thermal electron energies would be about the same.

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# EXACTLY SOLUBLE QUANTUM-STATISTICAL SYSTEM*

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The partition function for a system of bosons or fermions interacting via pairwise harmonic forces is evaluated exactly. It is found that the partition function has a structure similar to that of a system of independent bosons or fermions in an external harmonic well with, however, an important modification which implies a greatly reduced density of states.

In view of the paucity of exactly soluble models of interacting particles[1] a new example of such a system is of great interest. We consider here a system of $N$ interacting bosons or spinless fermions with the Hamiltonian

$$H=-\sum_{l=1}^{N}\frac{\partial^2}{\partial\vec{x}_l^{\,2}}+\frac{\omega^2}{N}\sum_{l<j}(\vec{x}_l-\vec{x}_j)^2. \tag{1}$$