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GROWTH AND DAMPING OF WAVE-PARTICLE INTERACTIONS IN COUNTERSTREAMING ELECTRON BEAMS*

M. Murakami† and L. M. Lidsky
Research Laboratory of Electronics
and Department of Nuclear Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
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Experimental observations have been made of wave-particle interactions in counter-penetrating electron beams in a static magnetic field. A new technique is used to measure the electron-distribution function at every stage of wave growth, saturation, and decay. The particular instability discussed here occurs at one half the cyclotron frequency and has a density threshold for onset. As the wave grows, trapping of resonant particles in the wave-potential troughs becomes significant and the velocity distribution is appreciably broadened. Finally, "finite-temperature" stabilization prevails and the wave decays. The experimentally observed resonance width is in agreement with the predictions of strong turbulence theory.

We present experimental observations of a wave-particle interaction in counterstreaming electron beams of small radius in a magnetic field. There is some controversy concerning the nature and stability range of this one-half cyclotron frequency instability: Etievant and Perulli[1] and Maxum and Trivelpiece[2] investigated the instability growth range and explained the limits by nonlinearities resulting in thermalization which stabilizes the instability when the wavelength λ becomes comparable with the Debye length λ_D:

$$\lambda_D \gtrsim \lambda, \text{ stable.} \qquad (1)$$

Recently Motz and Rumsby[3] compared their experimental observations with solutions of a dispersion equation for finite temperature beams and concluded that the beam temperature remains equal to the original cathode temperature. The controversy seems to be settled if we can determine the distribution function, which has not been measured in these three experiments.

We developed recently a gated electron trap[4] which allows us to measure the distribution function at arbitrary times during development of the instability. We observe that broadening of the distribution function stabilizes the instability, and that the stability condition approximately satisfies the condition, Eq. (1). The broadening results from strong wave-particle interaction because the phase velocity of the unstable wave is very close to the electron streaming velocity. To explain the mechanism of the broadening, it is necessary to take into account effects of trapping of resonant particles in the wave-potential troughs.

The experimental arrangement is shown schematically in Fig. 1. Two negatively biased electrodes form an electron trap in a uniform magnetic field of 100 G. During injection of an electron beam of 1000 eV by a pulsed electron gun, a positive high-voltage pulse is applied to elec-

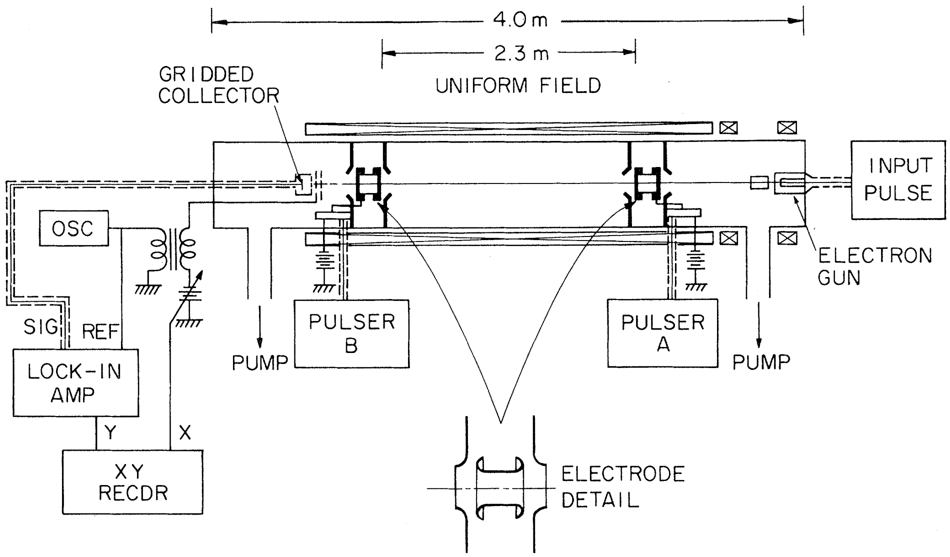

FIG. 1. Schematic of the experimental arrangement.

trode A to null the electrode potential. Before the reflected electrons return to electrode A after a two-way transit of the trap (340 nsec), the positive pulse is turned off. Now the trap is completely filled with electrons which have a distribution function described by two peaks at $\pm v_0$, where v_0 is the injection velocity. If there is any perturbation, these peaks are broadened. After a preset time, a positive high-voltage pulse is applied to electrode B, and the electrons impinging on the gridded Faraday cage are energy analyzed. Direct measurements of the parallel-energy distribution function are made by modulating the retarding grid potential and monitoring the beam-current component at the modulation frequency by a lock-in amplifier. To prevent electrons rejected at the grid from undergoing further perturbations during the measurement interval, the positive pulse applied to electrode B has a duration shorter than a two-way transit for any trapped electrons. To clear the system, a positive pulse is finally applied to electrode A. The whole cycle is repeated every 1/60 sec, which choice obviates effects due to ripple of the magnetic field.

The beam instability is detected by an electrostatic probe located near the beam. Oscillations are observed only when the electron beam is trapped between two trapping electrodes, i.e., when counter-streaming exists. When the beam current is below 40 μA, there are no oscillations, and the initial distribution does not change substantially for about 100 μsec, when broadening of the distribution due to collisions with background neutrals becomes apparent (estimated momentum-transfer collision time at the operating vacuum of 4×10^{-7} Torr is 250 μsec). When the beam current is above 40 μA, oscillations are observed and the distribution of the electrons suffers radical deformation on a much shorter time scale.

In Fig 2 is shown the time evolution of the parallel-energy distribution function for beam current of 120 μA. The original narrow distribution is broadened during the first 1.5 μsec and splits into two peaks; thereafter the lower energy peak increases its population, while the higher energy one is decaying, and finally the two peaks merge at about 6 μsec. The resulting distribution is much wider than the original. The probe-detected oscillations occurring during this process are shown in Fig. 3. The upper oscillogram is a display of the output signal of a tuned amplifier whose center frequency is 140 MHz, which is one-half the cyclotron frequency in the static field. The oscillation grows during the first 1 μsec, and saturates and then starts decaying at about 1.5 μsec.

In Fig. 4 is shown the frequency at which the tuned amplifier output is highest as a function of the axial magnetic field. The error flags indicate approximate spread in the frequency measurements. The observed oscillation frequency is equal to one half the cyclotron frequency. The observed one-half cyclotron-frequency wave easily accounts for the strong resonant wave-particle interaction, because the phase velocity of the unstable wave is very close to the stream

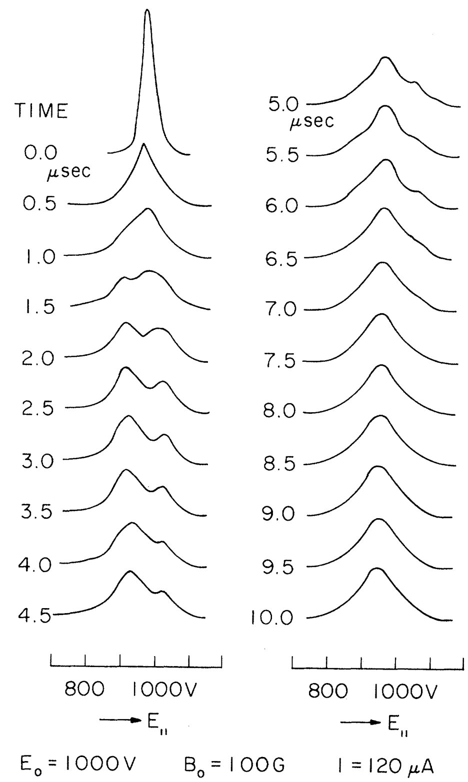

FIG. 2. Time development of the parallel-energy distribution function in the presence of the instability. The beam energy is 1000 eV, the static magnetic field is 100 G, and the beam current is 120 μA.

velocity of the beam.[5] Other possibilities (background plasma, cavity modes, two-stream instability, etc.) were investigated and dismissed as being either greatly disparate in frequency, not velocity resonant, or both.

In counterstreaming beams of small cross section, the radial electric component of space-charge waves on one beam couples unstably with the fast cyclotron wave on the oppositely drifting beam. Consideration of matching polarization directions leads to an absolute instability for coupling between the slow space-charge wave and backward cyclotron wave, and a convective instability for coupling between the fast space-charge wave and backward cyclotron wave. In this experiment, a grounded electrode near each trapping electrode may reflect the convective wave, thus the convective instability may be important, as well as the absolute instability.

Although the "zero-temperature, quasistatic" dispersion equation predicts that all finite cross-section beams are unstable, random motion of electrons (including perpendicular motion) tends to stabilize the instability. The enhancement of

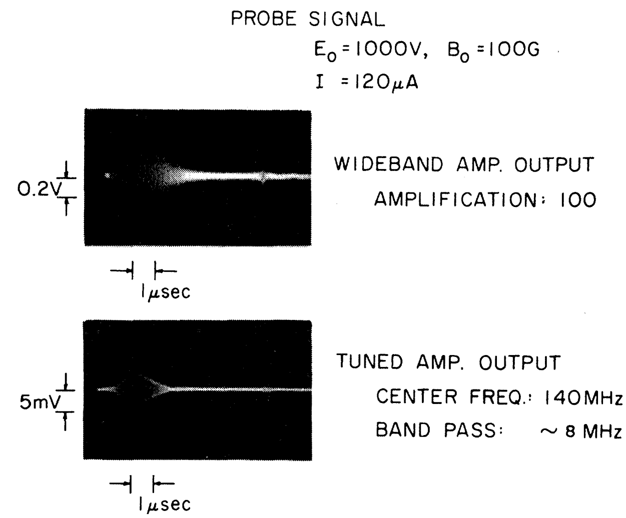

FIG. 3. Oscilloscope display of electrostatic probe signal amplified by a wideband amplifier and by a tuned amplifier whose center frequency (140 MHz) is half the cyclotron frequency.

the random energy can be considered as an increase in the "temperature," and a corresponding increase in the stream Debye length. The stability condition, Eq. (1), can be rewritten as

$$\frac{2\omega_p}{\omega_c} \lesssim \frac{v_{\rm th}}{v_0}, \text{ stable,} \tag{2}$$

where ω_p is the plasma frequency, ω_c is the cyclotron frequency, and $v_{\rm th}$ is the equivalent "thermal velocity." Therefore for instability, the distribution should be narrow and the beam density (beam current) should be larger than

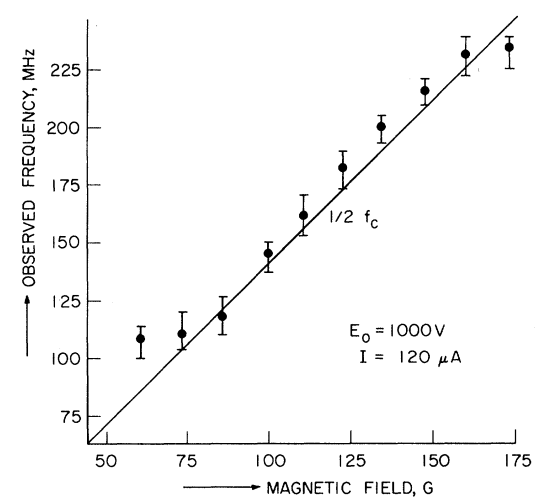

FIG. 4. Observed frequency at which the output of the tuned amplifier is highest versus static magnetic field. The error flags indicate dispersion in the frequency measurement.

some threshold.

The threshold current intensity predicted by Eq. (2) for the measured distribution width of 20 eV and estimated beam diameter 2.5 mm is 45 μA, which is in excellent quantitative agreement with the experimentally observed threshold 40 μA. Figure 3 shows that the unstable wave amplitude saturates at $t \approx 1.5$ μsec when the measured particle distribution function has an equivalent temperature of 80 eV (i.e., $v_{\text{th}} \approx 5.3\times10^8$ cm/sec). The thermal velocity required for stabilization computed by using Eq. (2) and the observed instability threshold of 40 μA is 4.6×10^8 cm/sec. The wave saturates, but the saturated oscillation continues to diffuse particles in velocity space. The resulting decay of the wave is thus a complicated function of time. It appears from Fig. 3 that the damping rate decreases slowly with time, as expected.

The next relevant question concerns the process that actually broadens the distribution function during the first 1.5 μsec. The threshold conditions for both the absolute and convective instabilities derived under the "zero-temperature" approximation provide the linearly possible spectrum width

$$\frac{|v_0-\omega/k|}{v_0} \lesssim \frac{1}{2}\frac{\omega_p{}^2}{\omega_c{}^2} \tag{3}$$

which is much smaller than the observed width. Therefore, it does not seem possible to explain the broadening by diffusion according to a weak turbulence model,[6] even if we take into account mode-mode couplings (which seem implausible in this short time scale). A better explanation is given by strong turbulence theory,[7] which takes into consideration the trapping of resonant particles in wave-potential troughs. The resonance width of strong turbulence is given by

$$W = b\left(\frac{q\overline{E}_z}{mk_z}\right)^{1/2} = b\left(\frac{q\overline{\varphi}}{m}\right)^{1/2}, \tag{4}$$

where b is a numerical factor of order unity, $\overline{E}_z$ is the rms axial electric field, k_z is the axial wave number, $\overline{\varphi}$ is the rms wave potential, and the second equality results from the "quasistatic" approximation.

To estimate the resonance width, we must know the rms wave potential. Direct measurement of the wave amplitude was not possible because of the unknown coupling coefficient to the probe. Instead we estimate it from the observed distribution functions. Integration of the observed distribution functions indicates that the total parallel energy decreases approximately 1% in the first microsecond and that thereafter the decrease is saturated. We assume that the amount of energy equal to this deficit has been equally divided between wave energy and particle perpendicular energy (particles are accelerated mainly perpendicularly because of $|E_r| \gg |E_z|$, where E_r is the radial electric field). Then the estimated rms wave potential is 2.6 V, which gives the corresponding resonance width $w = 1.7$ eV$^{1/2}$ for $b=1$. The uppermost velocity v_0+w corresponds to 1090 and the lowermost velocity v_0-w corresponds to 930 V; thus the resonant particles can diffuse between 1090 and 930 eV. This range is in excellent agreement with the experimentally observed distribution function.

Thus, the history of the instability seems to be: If the beam current is intense enough for the linear growth to exceed the "finite-temperature" damping effect, then the unstable coupling of the counterstreaming beams sets up a growing wave at one half the cyclotron frequency. Since the phase velocity of the unstable wave is very close to the electron streaming velocity, a strong resonant wave-particle interaction takes place. As the wave grows, the trapping of the resonant particles in wave-potential troughs become significant, resulting in broadening of the distribution function, or increase in the beam "temperature." Finally the "finite-temperature" stabilization prevails and the wave damps.

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†Present address: Oak Ridge National Laboratory, Oak Ridge, Tenn. 37830.

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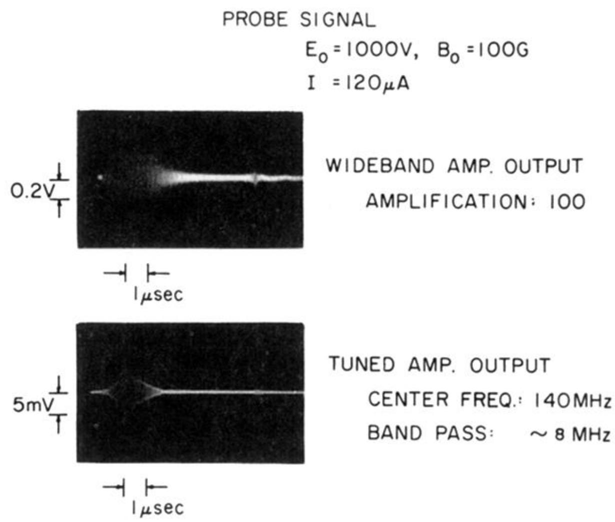

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