INTERPRETATION OF EXPERIMENTS ON FEEDBACK CONTROL OF A "DRIFT-TYPE" INSTABILITY

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Previous experimental results on the feedback stabilization of a "drift-type" instability are interpreted in terms of a nonlinear theory of the Van der Pol type. This theory predicts a variation of the instability amplitude and its frequency as a function of the gain and the phase angle in the feedback loop. Comparison between these predictions and the experimental results shows remarkably good agreement.

Recently there have been a number of papers[1-3] which have shown feedback suppression of various "drift-type" plasma instabilities. In particular, in the work of Ref. 3, some success has been achieved in the interpretation of the results by the use of a "feedback" source term included in a linearized theory. However, in order to allow for the "positive" feedback case where the instability signal is amplified, a nonlinear theory must be employed which limits the final signal level to a finite value. In the last few years there has been considerable interest in the nonlinear mechanisms which determine the saturation level of plasma instabilities. In fact, it has been shown that the Van der Pol type of equation[4] gives a good description of various kinds of nonlinear phenomena occuring in some plasma instabilities. These phenomena include mode locking and mode competition,[5,6] periodic pulling,[7] frequency entrainment or "synchronization,"[8] and "asynchronous quenching" effects.[9] Further, it has been shown theoretically by Stix,[10] when considering finite-amplitude collisional drift-wave oscillations, that a solution may be obtained "which saturates in a manner similar to the Van der Pol solutions." Consequently, as it has been shown that this type of differential equation gives a good description of finite-amplitude collisional drift waves, the phenomenological approach has been adopted, in which the Van der Pol equation is taken to describe the density oscillations in the plasma. The equation in its simplest form for the unperturbed case is

$$d^2n_1/dt^2-(\alpha-\beta n_1^2)dn_1/dt+\omega_0^2n_1=0, \tag{1}$$

where n_1 is the density perturbation, ω_0 is the drift wave frequency, α is the linear growth rate $(\alpha/\omega_0 \ll 1)$, and β is a nonlinear saturation coefficient which limits the final amplitude of the unperturbed oscillation. This final amplitude (a_0) in the unperturbed case is given by

$$a_0=(4\alpha/3\beta)^{1/2}. \tag{2}$$

Now consider a signal $gn_1(\tau)$ which is fed back into the system where g is an absolute gain in density perturbations and τ represents a delay time (here $\omega_0\tau=\varphi$, the phase shift). Then the equation is given by

$$\frac{d^2n_1}{dt^2}-(\alpha-\beta n_1^2)\frac{dn_1}{dt}+\omega_0^2n_1+g\omega_0^2n_1(\tau)=0. \tag{3}$$

This equation is a simple example of a difference-differential equation,[11] and can be rearranged in the form

$$d^2n_1/dt^2+\omega^2n_1=(\omega^2-\omega_0^2)n_1+(\alpha-\beta n_1^2)dn_1/dt-g\omega_0^2n_1(\tau)=\sum H. \tag{4}$$

If a solution of the form $n_1=a\sin\omega t$ is assumed, it can be shown that Eq. (4) can be brought into the form

$$d^2n_1/dt^2+\omega^2n_1=F(a(t),\omega)\cos\omega t+f(a(t),\omega)\sin\omega t+\text{harmonics}+\cdots. \tag{5}$$

If the calculation is limited to the fundamental frequency ω the solution in the first approximation is

$$a(t)=\frac{1}{2\omega}\int_0^t F(a(t),\omega)d\epsilon;\;\; 0=\frac{1}{2\omega}\int_0^t f(a(\epsilon),\omega)d\epsilon. \tag{6}$$

For the transient condition the first part of Eq. (6) gives

$$da/dt = F(a, \omega)/2\omega, \tag{7}$$

and for the stationary state one has

$$F(a, \omega) = 0,\ f(a, \omega) = 0. \tag{8}$$

Therefore, if one calculates the coefficients of $\cos\omega t$ and $\sin\omega t$ from the expression for $\sum H$ in Eq. (4), the following conditions are obtained for the stationary state:

$$a_0^2 - a^2 = -g(\omega_0^2/\alpha\omega)a_0^2 \sin\varphi, \tag{9}$$

$$\omega^2 = \omega_0^2(1 + g\cos\varphi). \tag{10}$$

Equation (9) shows that as the gain g is increased from $g = 0$ the amplitude (a) will increase or decrease according to the sign of $\sin\varphi$. Optimum supression is achieved with

$$\sin\varphi = -1 \text{ (i.e., } \varphi = -90° \text{ or } +270°); \tag{11}$$

then suppression occurs when $g = \alpha/\omega_0$ (since $\omega = \omega_0$ at $\varphi = -90°$).

The apparatus was the same as that employed in Ref. 2, and further experiments have been performed to test the above theory. Summarizing, the plasma used was a hollow cathode arc discharge running in argon, with an electron temperature ~5.0 eV, a peak density ~10^{13} cm^{-3}, and an inverse scale length $n_0^{-1/2}\partial n_0/\partial r = 0.70 \pm 0.05$ cm^{-1} in an axial magnetic field of 1 kG. The instability was predominantly an $m = +1$ instability with an axial wavelength λ larger than the apparatus ($\lambda > 200$ cm), and under these conditions its frequency was 7.0 kHz. It was identified as a collisional-type drift wave.[12]

In these experiments, ion density perturbations were detected on an ion-biased probe, and a signal proportional to these perturbations was fed back via a wideband amplifier with variable gain, a phase shifter (variable over 450°), and a power amplifier onto a plate in the plasma. This plate was in the same axial plane as the detecting probe and could be moved radially across the plasma. As in Ref. 3, minimum gain was required for suppression when the plate was situated at the radius (≈1.1 cm) corresponding to maximum instability amplitude. However, experiments were usually performed with the plate further out of the plasma ($r = 1.6$-2.0 cm), so that it caused less disturbance to the density profile. The effect was observed on a further ion-biased probe which could be moved axially and radially, and the output was displayed on a spectrum analyzer.

The phase angle φ in the feedback loop was varied until a minimum was obtained in the instability level, and then the amplitude a was measured as a function of the gain G in the wideband amplifier. Equation (9) predicts that the square of the signal level (a^2) should fall linearly as a function of the absolute gain g (or relative gain G) in the feedback loop. This variation is shown in Fig. 1, where $(a/a_0)^2$ is plotted against the increasing gain G in the amplifier; this is shown for the suppressor plate set at two different radii $r = 1.6 \pm 0.1$ cm, and $r = 2.0 \pm 0.1$ cm. It is seen that a good linear relationship is obeyed.

Further experiments were performed with the plate kept at a radius $r \approx 1.8 \pm 0.1$ cm. The gain G was left set at its value for suppression ($G = 25.2$) and in this case the phase angle φ was varied through 360°, and the relative amplitude a/a_0 measured. This is shown plotted in Fig. 2(a) as $(a/a_0)^2$ versus phase angle φ. It is seen that optimum suppression is achieved when $\varphi = -90°$, or +270°, as predicted by Eq. (11). Figure 2(b) shows $(a/a_0)^2$ plotted versus $\sin\varphi$, and a good linear relationship is seen to be obeyed as predicted by Eq. (9). Other gain values of $G = 12.6$ and 7.9 are shown plotted in both Figs. 2(a) and 2(b). The absolute gain g of the system was obtained with the aid of Eq. (10), since, with the gain set at $G = 25.2$ (for optimum suppression), theory gives $g = \alpha/\omega_0 \ll 1$. So if $\omega - \omega_0 = \Delta\omega$,

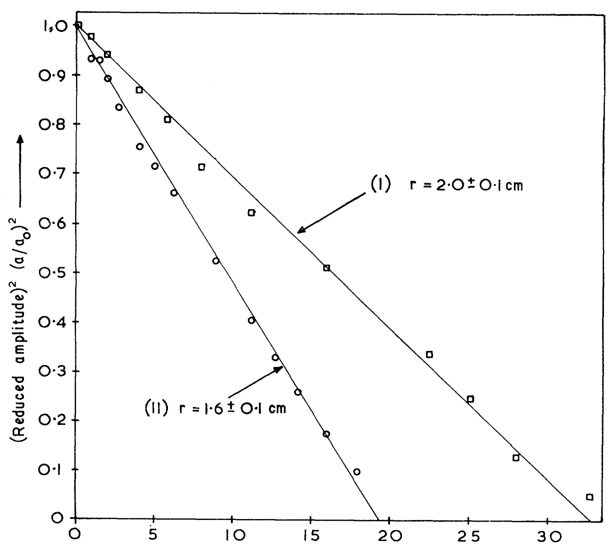

FIG. 1. The square of the reduced amplitude $(a/a_0)^2$ plotted versus amplifier gain G for the conditions when the suppressor plate is set at a radius r such that (1) $r = 2.0 \pm 0.1$ cm and (2) $r = 1.6 \pm 0.1$ cm.

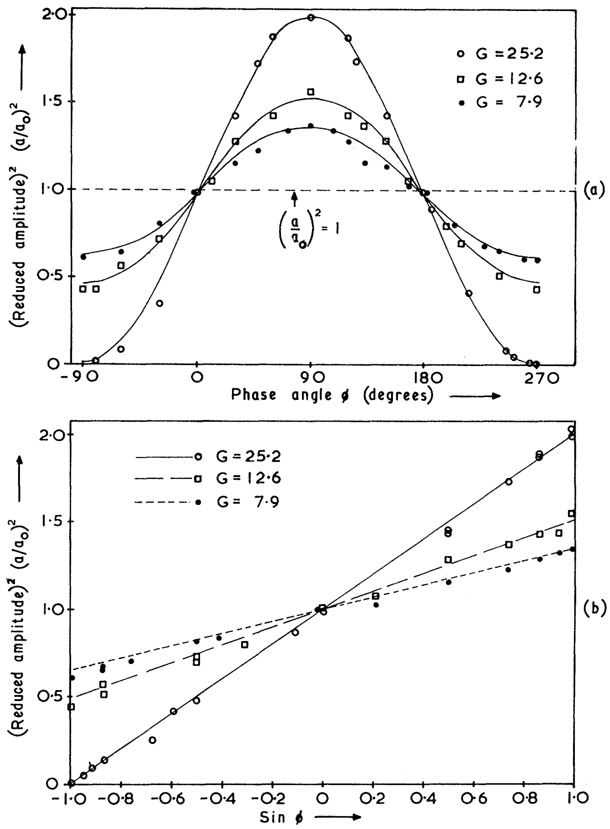

FIG. 2. A plot of the square of the reduced amplitude $(a/a_0)^2$ against (a) phase angle φ and (b) $\sin\varphi$.

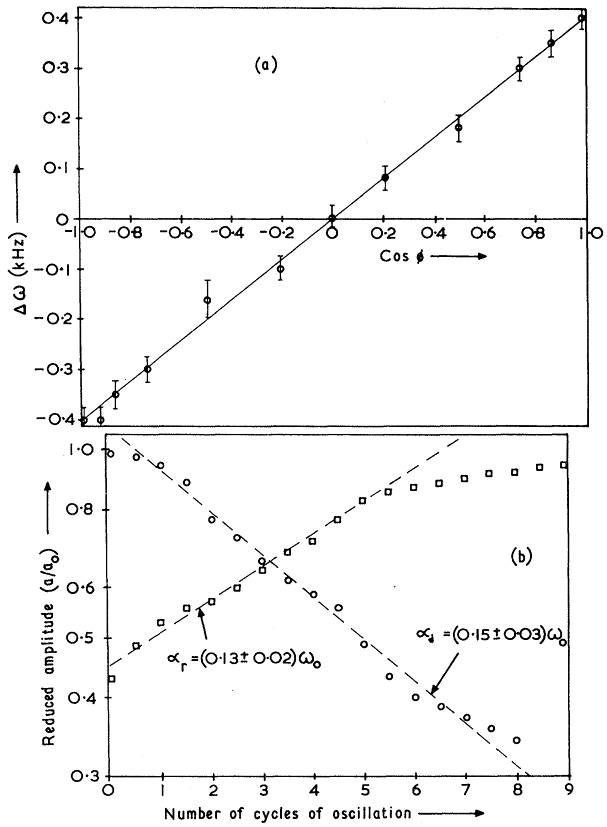

FIG. 3. (a) The change in the instability frequency $\Delta\omega$ (kHz) plotted versus the cosine of the phase angle ($\cos\varphi$). (b) The logarithm of the reduced amplitude $\ln(a/a_0)$ plotted against the number of periods of the wave for (i) the rise time and (ii) the decay time of the instability.

Eq. (10) shows that $2\Delta\omega/\omega_0 = g\cos\varphi$. The frequency shift $\Delta\omega$ was measured as a function of $\cos\varphi$ and this is shown plotted in Fig. 3(a); a good linear relationship was obtained. The slope of this line is proportional to $g = \alpha/\omega_0 = 0.12 \pm 0.02$. This allows the absolute gain g to be calibrated in terms of the amplifier gain G, and thus $g = \gamma G$ ($\gamma = 4.5\times 10^{-2}$). Consequently, the resulting theoretical variation of $(a/a_0)^2$ as a function of phase angle φ for each gain value was calculated using Eq. (9), and this variation is shown as the continuous lines in Fig. 2(a). Finally, a further check on the theory was made by measuring directly the linear growth rate α and comparing its value with the result obtained indirectly from $g = \alpha/\omega_0 = 0.12 \pm 0.02$. This was effected by using a tone-burst generator in the return loop, which gated the feedback signal at periodic intervals. The resulting instability signal was measured, and the rise and decay times were analyzed as in Fig. 3(b), to obtain a value for α. This resulted in a value from the rise-time data of $\alpha_r = (0.13 \pm 0.02)\omega_0$, and from the decay-time data of $\alpha_d = (0.15 \pm 0.03)\omega_0$, which shows remarkably good agreement with the value obtained above from theoretical considerations.

Concluding, it is seen that by adopting the phenomenological approach to the problem, in which a nonlinear equation of the Van der Pol type is used to explain the instability saturation conditions in a plasma, relationships can be obtained between the amplitude a and frequency shift $\Delta\omega$ as a function of gain g and phase shift φ in the feedback loop. The resulting measurements show the predicted variations and a consistent value for the growth rate α is obtained within experimental error. Therefore, remarkably good agreement between theory and experiment is achieved.

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MAGNETOPHONON RESONANCES IN ACOUSTOELECTRIC GAIN IN n-InSb *

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Novel resonances in the acoustoelectric gain have been observed in pure n-InSb at longitudinal magnetic fields corresponding to the well-known Gurevich-Firsov magnetophonon resonances. The magnetoacoustoelectric resonances were obtained at 77 and at 4.2°K, with acoustic phonons internally amplified from the thermal background. The resonant peaks in the gain are attributed to resonant cooling of hot carriers by the resonant enhancement of optical-phonon induced transitions between Landau levels.

We report here a new magnetoacoustoelectric (MAE) resonant phenomenon, consisting of the introduction of the magnetophonon resonances into the acoustoelectric interaction in n-InSb. Magnetophonon resonances, first proposed by Gurevich and Firsov,[1] are well known in the magnetoresistance.[2] They correspond to the resonant inelastic scattering of electrons between Landau levels which are separated by just the longitudinal-optical-phonon energy $\hbar\omega_0$. In the parabolic-band approximation, the resonances occur at values of magnetic field B determined by the condition

$$nBe/m^* = \omega_0, \tag{1}$$

where n is an integer and m^* is the electron effective mass. The novel[3] MAE resonances consist of strong positive peaks in the acoustoelectric gain at the resonant values of B. A comparison with the magnetoresistance (MR) variation shows that the MAE resonances are not merely a secondary manifestation of the MR resonances.

The present work was restricted to the longitudinal magnetic field configuration with B along the [110] length of the sample, parallel to both the electron-drift velocity v_d, and the phonon-propagation direction. The acoustic flux is produced internally, by acoustoelectric amplification of phonons from the thermal equilibrium background.[4-6] For sufficient net gain in pure n-InSb, this requires application of current pulses making v_d much greater than the piezoelectrically active shear-wave velocity $v_s = 2.3\times10^5$ cm/sec. The amplification is selective,[5,6] producing a beam of phonons in a narrow cone along the [110] direction, with a narrow band of frequencies centered around 1.7 GHz. The latter is the theoretical[7] frequency of maximum gain (ω_{max}), at 77°K, for n-InSb with carrier concentration $N \sim 4\times10^{13}/\mathrm{cm}^3$ and electron mobility $\mu \sim 6\times10^5$ cm^2/V sec. The important parameter ql (phonon wave vector × electron mean free path) is ~9.

Both measurement and analysis of the MAE are greatly simplified when the gain is achieved with very constant current pulses,[4-6] which maintain the current-dependent gain factor $v_d/v_s - 1$ constant. The amplified acoustic flux is detected through the delayed increase in the voltage across the sample, as illustrated by the recorded pulses in Fig. 1(a). During the delay of ~4 μsec, the acoustic flux propagating towards the anode grows exponentially by many orders of magnitude. Eventually it becomes detectable in the electrical resistance, producing the voltage rise ΔV_{ae}, which is determined by the rate of loss of electron drift momentum to the amplified phonon beam. The acoustoelectric signal ΔV_{ae}, is given by $\alpha\Phi/Nev_s$, where Φ is the amplified acoustic energy density integrated over the length of the sample, $\alpha = \alpha_0(v_d/v_s - 1)$ is the acoustoelectric power gain, and α_0 is a material and magnetic-field-dependent interaction coef-