TRANSVERSE KELVIN-HELMHOLTZ INSTABILITY IN A Q-MACHINE PLASMA*

D. L. Jassby and F. W. Perkins
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08540
(Received 22 December 1969)

The Q-machine "edge oscillation" is identified as a Kelvin-Helmholtz instability by a comparison of theory with experiment. Specifically, measurements of frequency and of radial variation in wave phase and amplitude agree with numerical integrations of the radial wave equation, including finite parallel wavelength. The computations show that the instability is caused principally by velocity shear.

The "edge oscillation"[1,2] of Q-machine plasmas has been conjectured to be a transverse Kelvin-Helmholtz instability[2,3] because of the strong shear in the equilibrium $\vec{E}\times\vec{B}$ rotation that occurs at the edge of these devices. The pioneering attempt to substantiate this conjecture is the recent work of Kent, Jen, and Chen,[2] who showed that the frequency and the radial dependence of wave intensity agreed well with a rough theoretical model. The present work reports important extensions to both the theoretical model and the experimental measurements that support, in detail, the model of a Kelvin-Helmholtz instability.

From a general point of view, any confined plasma in which nonambipolar processes lead to $\vec{E}\times\vec{B}$ velocities that exceed the average diamagnetic velocity may be susceptible to a velocity-shear instability (whose nonlinear state is a convective cell).[4] Indeed, convective cells occur in multipoles.[5] A Q-machine plasma, which is not strictly confined but where the radial electric field is, to some degree, under the control of the experimenter,[6] provides an opportunity to study this instability and compare it with theory.

In this work we compare measurements of the edge oscillation with theoretical eigenfunctions and frequencies obtained from numerical integration of the complete radial wave equation, using measured values for the equilibrium radial electric field and density profiles, and also including the effects of finite parallel wavelength and ion collisional viscosity. Furthermore, by deleting terms in the radial wave equation, we can turn off certain physical processes (e.g., centrifugal force) and assess their contribution to the growth rate.

The theoretical model is a cylindrical, low-β, isothermal plasma column immersed in an axially uniform magnetic field B. The equilibrium profiles of plasma density n and radial electric field E are described by the parameters

$$\omega_D(r) = -\frac{c}{B}\frac{kT}{e}\frac{1}{r}\frac{1}{n}\frac{dn}{dr}$$

and

$$\omega_E(r) = (c/B)\,(E/r), \tag{1}$$

respectively.

The linearized radial wave equation for a normal mode with a $\psi(r)\exp(im\theta+i\omega t)$ dependence is[7]

$$\frac{d}{dr}\,\mathcal{T}\frac{d\psi}{dr}+\frac{1-m^2}{r^2}\mathcal{T}\,\psi+\omega^2 r^2\frac{dn}{dr}\psi+2i\frac{\Sigma n r^3}{a^2}\frac{(m\omega_D+\omega-m\omega_E)(\omega-m\omega_E)}{\omega-m\omega_E-i\Sigma}\psi + i\nu_{\parallel}\tfrac{1}{4}na^2r^2(\omega-m\omega_E-m\omega_D)\nabla_\perp{}^4 r\psi=0, \tag{2}$$

where

$$\mathcal{T}=nr^3(\omega-m\omega_E)(\omega-m\omega_E-m\omega_D), \tag{3}$$

$$\Sigma=k_\parallel{}^2\frac{kT}{m_e}\frac{1}{\nu_e},\quad a^2=\frac{2kT}{M}\frac{1}{\omega_{ci}{}^2}, \tag{4}$$

$$\nabla_\perp{}^2=\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}-\frac{m^2}{r^2},$$

and m is the azimuthal mode number. The dependent variable

$$\psi=-\frac{c}{B}\frac{m}{r}\frac{\tilde{\varphi}}{\omega-m\omega_E}$$

is the radial displacement of a guiding center; $\tilde{\varphi}$ is the perturbed potential.

The numerical integration of Eq. (2) was simplified by neglecting the last term, which describes the effects of ion collisional viscosity. The boundary conditions were: (1) $\psi\to r^{m-1}$ as $r\to 0$, and (2) $|\psi|$ decreases monotonically as $r\to\infty$. These conditions determined the complex eigenvalue resulting from the last term, under the assumption that the eigenfunction varies rapidly compared with the equilibrium.[8]

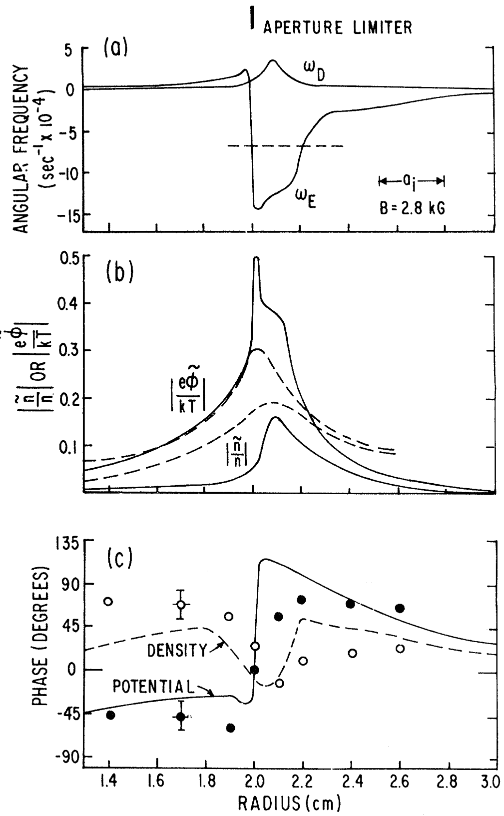

FIG. 1. (a) The plasma equilibrium is given by the radial variations of ω_E and ω_D. The quantity a_i shows the size of the ion Larmor radius. The dashed line shows the measured oscillation frequency for $m=2$. (b) Comparison of measured rms amplitudes of density and potential fluctuations (dashed lines) with theoretical eigenfunctions (solid lines) for the $m=2$ mode with $\lambda_{\parallel}=330$ cm. The theoretical results have been normalized so that the potential fluctuation agrees with the experimental value at $r=1.6$ cm. In these measurements, the small admixture of $m=1$ has been filtered out. (c) Comparison of measured radial phase variation of the potential and density fluctuations with theoretical pahses (smooth curves). The open circles are the experimental density phases; the closed circles are the experimental potential phases. The potential phases were set equal to zero at $r=2.0$ cm. The vertical lines represent the experimental uncertainty; the horizontal lines show the length of the probe (0.4 mm).

The fact that in most experiments the velocity shear layer is thin compared with the azimuthal wavelengths leads, theoretically, to an important observation: The radial displacement ψ is approximately constant throughout the velocity shear layer. This can be seen either from numerical calculations or by extending the arguments of Drazin and Howard on the hydrodynamic Kelvin-Helmholtz instability[9] to the plasma case.

The potential and density perturbations (as derived from the electron-fluid equations) are

$$\tilde{\varphi}=(B/c)(r/m)(\omega-m\omega_E)\psi, \tag{5}$$

$$\frac{\tilde{n}}{n}=-\frac{1}{n}\frac{dn}{dr}\psi\frac{(m\omega_D+i\Sigma)(\omega-m\omega_E)}{m\omega_D(\omega-m\omega_E-i\Sigma)}. \tag{6}$$

These relations are useful in identifying the Kelvin-Helmholtz instability experimentally. Specifically, (1) if ψ is constant, Eq. (5) predicts a large change in the phase of $\tilde{\varphi}$ where $\mathrm{Re}(\omega)=m\omega_E$; and (2) if Σ is small and ψ is constant, Eq. (6) predicts little radial variation in the phase of $\tilde{n}$. In the regions free of velocity shear, the phase between $\tilde{\varphi}$ and $\tilde{n}$ depends on the direction of the radial electric field in the shear layer (because this determines ω), and the amplitudes are often such that $|\tilde{n}/n| \ll |e\tilde{\varphi}/kT|$.

The experiments were performed in a cesium plasma in the Princeton Q-3 device[10] in double-ended operation (both end plates hot). The column length was 110 cm, and the maximum density was 4×10 cm^{-3}. The aperture limiter set the diameter of the main plasma column at 4 cm. The end-plate temperature of 2200°K produced ion sheaths resulting in small density gradients and gave no drift waves in the main plasma column for $B<5.5$ kG. Figure 1(a) shows the steady-state density and potential profiles at $B=2.8$ kG in terms of ω_D and ω_E [see Eq. (1)]. The $m=2$ mode was dominant under these conditions, although a small admixture of $m=1$ (20% in power) was present. Measurements of the parallel wavelength gave $\lambda_{\parallel}=330$ cm, with a 30% uncertainty.

Figures 1(b) and 1(c) compare theoretical and experimental determinations of the radial dependence of the amplitudes and phases of the potential and density perturbations. The agreement shows that the oscillation is indeed described by Eqs. (2), (5), and (6). In particular, Eq. (6) shows that the radial variation of phase of the density perturbation $\tilde{n}$ results from a nonzero parallel wave number. The agreement between theory and experiment here gives added confidence to the parallel wavelength measurement. Theoretically, the frequency also depends significantly on the parallel wave number (see Fig. 2), and again there is agreement. Figure 2 does show that any attempt to accurately predict the frequency of the edge oscillation requires a good knowledge of the parallel wave number.

Next we turn to the question of what physical effect is responsible for the instability. Is it velocity shear, centrifugal force, or electron dis-

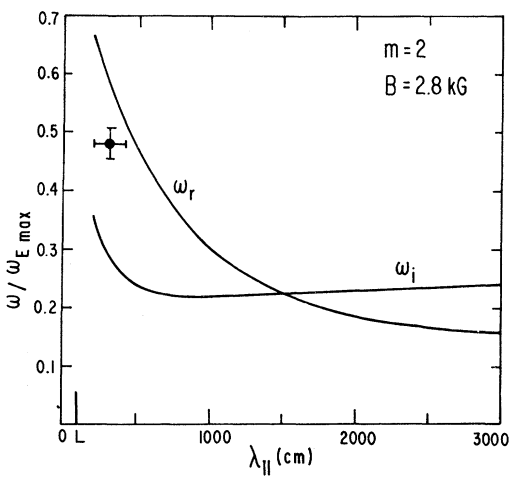

FIG. 2. Dependence of the frequency on parallel wavelength. The experimental oscillation frequency is shown at the measured parallel wavelength. The column length is indicated by L.

sipation (which leads to drift instability)? By numerically integrating Eq. (2) with certain terms deleted, a theoretical answer to this question is provided. In particular, deleting the third term removes the centrifugal instability (CF in Table I), deleting ω_D in the expression for $\mathcal{T}$ [Eq. (3)] removes ion finite-Larmor-radius (FLR) stabilization, and deleting ω_D in the fourth term removes the destabilizing portion of the electron dissipation (ED). In all of these integrations the first two terms of Eq. (2), which represent velocity shear (VS) were present. Table I gives the eigenfrequencies found by the integrations. It is evident that velocity shear makes the largest contribution to the growth rate; so we may call the edge oscillation a Kelvin-Helmholtz instability. This confirms the conclusion of Kent, Jen, and Chen.[2]

Table I. The effect of various physical processes on the instability frequency.

Processes[a]	$\omega_r / \omega_{E\,max}$ [b]	$\omega_i / \omega_{E\,max}$ [b]
VS	0.64	0.19
VS + CF	0.51	0.24
VS + ED	0.72	0.24
VS + CF + ED	0.57	0.285
VS + CF + ED + FLR	0.58	0.28

[a] VS stands for velocity shear, CR for centrifugal force, ED for electron dissipation, and FLR for finite Larmor radius. See the text for details.

[b] In all cases, $\lambda_{\|} = 330$ cm and the effect of ion-collisional viscosity was less than 5%. The uncertainty in the determination of the eigenvalue is ±0.01.

Finally, we should remind the reader that the theoretical work is based on the finite-Larmor-radius fluid equations, which require that the scale sizes be large compared with the ion gyroradius and the frequency small compared with the ion gyrofrequency ($\omega_{ci} = 2.0 \times 10^5$ sec^{-1} for these measurements). Since neither approximation is well satisfied, the agreement between theory and experiment is surprisingly good.

The authors are grateful to Dr. R. W. Motley for valuable discussions and to Dr. H. W. Hendel and Dr. C. Oberman for reading the manuscript.

---

*Work performed under the auspices of the U. S. Atomic Energy Commission, Contract No. AT(30-1)-1238. Use made of computer facilities supported in part by National Science Foundation Grant No. NSF-GP 579.

[1]H. W. Hendel, T. K. Chu, and P. A. Politzer, Phys. Fluids 11, 2426 (1968). These authors point out that most of the early Q-machine experiments on "drift waves" were in fact concerned with the edge oscillation.

[2]G. I. Kent, N. C. Jen, and F. F. Chen, Phys. Fluids 12, 2140 (1969).

[3]S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability (Clarendon Press, Oxford, England, 1961); C. C. Lin, The Theory of Hydrodynamic Stability (Cambridge U. P., Cambridge, England, 1966).

[4]J. A. Byers, Phys. Fluids 9, 1038 (1966).

[5]D. M. Meade, A. W. Molvik, J. W. Rudmin, and J. A. Schmidt, Phys. Rev. Letters 22, 1354 (1969); R. Freeman, M. Okabayashi, H. Pacher, G. Pacher, and S. Yoshikawa, Phys. Rev. Letters 23, 756 (1969).

[6]Instabilities in Q-machine plasmas produced by manipulating the radial electric field have been reported by L. Enriques, A. M. Levine, and G. B. Righetti, in Proceedings of a Conference on Plasma Physics and Controlled Nuclear Fusion Research, Novosibirsk, U. S. S. R., 1968 (International Atomic Energy Agency, Vienna, Austria, 1969), Vol. I, p. 641, and by C. W. Hartman and R. H. Munger, in Proceedings of the Conference on Physics of Quiescent Plasmas, Frascati, Italy, 1967 [Laboratori Gas Ionizzati (Association Euratom Comitato Nazionale per l'Energie Nucleare, Frascati, Italy), 1967], Pt. I, p. 49.

[7]M. N. Rosenbluth and A. Simon, Phys. Fluids 8, 1300 (1965), derived the first three terms of this equation. The other terms are obtained by following well-known methods; e.g., T. K. Chu, B. Coppi, H. W. Hendel, and F. W. Perkins, Phys. Fluids 12, 203 (1969).

[8]For the edge oscillation the eigenfunction varies on the same scale as the equilibrium. However, since the density is low, collisional viscosity has a very small effect on the eigenvalue; so this approximation is not crucial.

[9]P. G. Drazin and L. N. Howard, J. Fluid Mech. 14, 257 (1963).

[10]N. Rynn and N. D'Angelo, Rev. Sci. Instr. 31, 1326 (1960).