DISCREPANCIES IN THE LOW-ENERGY PROTON-PROTON SCATTERING DATA*

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A strong disagreement for predictions near 10 MeV is found between two previous multienergy phase-shift analyses. The data also contain contradictions, but are found to definitely favor one of the analyses over the other. Unresolved normalization discrepancies make further experimental work in the energy range 1-10 MeV mandatory.

For most of the past nine years, the definitive proton-proton differential cross section in the vicinity of the strong low-energy peak in the $^1S_0$ phase shift has been the cross section at 9.69 MeV due to Johnston and Young[1] at Minnesota. However, last year Slobodrian, Conzett, Shield, and Tivol[2] at Berkeley published the results of new cross-section measurements at 6.141, 8.097, and 9.918 MeV. These authors noted that their 9.918 cross section was in disagreement with the 9.69 Minnesota cross section by more than could be accounted for by the small difference in energy. This latter result was confirmed by MacGregor, Arndt, and Wright (MAW),[3] who found that the two cross sections behaved rather differently in their most recent phase-shift analysis,[3,4] MAW-X.

MacGregor, Arndt, and Wright's phase-shift analysis MAW-X was of 1076 data in the energy range 1-450 MeV. Each phase shift $\delta_{lj}(E)$ was given a separate phenomenological parametrization in energy, and the parameters were adjusted to obtain a least-squares fit of these particular representations to the data. MAW used a total of 26 parameters and obtained a χ^2 value of 1126 for the 1076 data. Among the various data sets, the 9.918-MeV Berkeley one stood out as having an abnormally high χ^2 value of 41 for its 17 data: It appeared to be "tipped" with respect to the predection of the multienergy phase-shift solution. MAW concluded from their work that their form of the energy dependence of the phase shifts was clearly established and that the 9.918 MeV data must be in error.

In the course of trying to reproduce the MAW values of χ^2 from the MAW phase shifts, we found that the angles used for the Berkeley data in the MAW analysis were not correct.[5] When the angles were changed to their correct values, we found that the value of χ^2 using the MAW phases changed from 41 to the even worse value of 64 for the 17 data at 9.918 MeV.

There has been another recent multi-energy phase-shift analysis which included the data in question: that of Sher, Signell, and Heller[6] (SSH). Their analysis used 157 data in the energy range 0-27.5 MeV. By restricting the energy range, SSH could use an effective-range series for each of the $l=1$ phase shifts, $^3P_0$, $^3P_1$, and $^3P_2$. This representation had the advantage of clearly allowing sufficient freedom for the P waves: One need not worry about having an overly restrictive energy parametrization. For the $^1S_0$ state, SSH used a potential, and the higher angular momentum phases were taken to be essentially given[6] by one-pion exchange. SSH found[6] that their multienergy analysis gave values for the "tensor" P-wave phase-shift combination $^3\Delta_T$ which were close to one-pion exchange for $E<11$ MeV; and values for the "spin-orbit" P-wave phase-shift combination $^3\Delta_{LS}$ which were close to zero in that same energy range. These two results were in agreement with results from potential and one-boson–exchange models. SSH's values of the $^1S_0$ phase shift and uncertainties were in precise agreement with those of MAW's analysis. However, the SSH and MAW-X values of the "central" P-wave phase shift combination $^3\Delta_C$ were in substantial disagreement. SSH also made single-energy analyses[6] of each data set for $E<11$ MeV, holding Δ_T and Δ_{LS} fixed at various values near those of the multienergy-analysis solution. For each cross section, the $^1S_0$ and $^3\Delta_C$ values were then determined from the data: The $^1S_0$ phase shift so obtained was found to be slightly dependent upon the $^3\Delta_T$ assumed, but the value of $^3\Delta_C$ was found to be quite independent of it.

The SSH individual-data-set analysis $^1S_0$ and $^3\Delta_C$ values are plotted as points with error bars in Figs. 1 and 2, along with shaded areas representing the bands of uncertainty in the multiener-

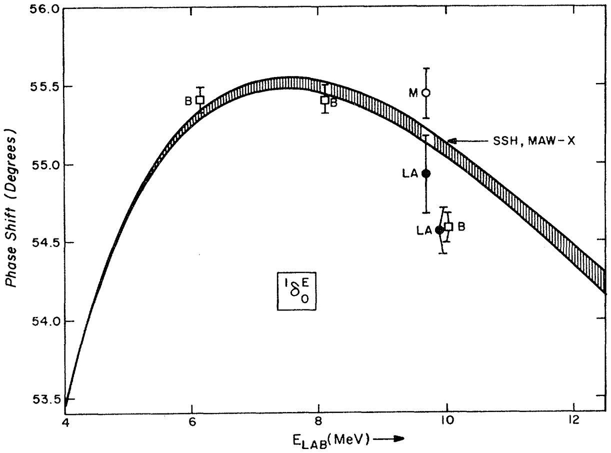

FIG. 1. Values of the $^1S_0$ electric-nuclear phase shift (Ref. 6). The multienergy-analysis band is that of Refs. 4 and 6, while the single-data-set error bars are from Ref. 6 and the present work. The data are from Berkeley (Ref. 2), Minnesota (Ref. 1), and Los Alamos (Ref. 7). Note that the uncertainty in the Berkeley $^1S_0$ value at 9.918 MeV is smaller than the uncertainty in the Los Alamos value, even though the absolute errors on the individual Berkeley differential cross-section data are larger than those for the Los Alamos data (Ref. 8) [see Fig. 1 of the preceding Letter (Ref. 7)].

gy-analysis values, analogous to the single-energy error bars. As stated above, the MAW and SSH phenomenological predictions are seen to be in agreement for the $^1S_0$ phase but are in strong disagreement for the $^3\Delta_C$. No error corridor is shown for the MAW $^3\Delta_C$ because this quantity was not computed by MAW. Presumably, MAW's $^3\Delta_C$ error corridor would be smaller than that for SSH because their errors on the $^3P_{0,1,2}$ phase shifts were smaller than were those of SSH. The SSH $^3\Delta_C$ curve is seen to be a considerably better fit to the data than is the MAW curve. This is presumably due to the MAW parametrization being overly restrictive for the ^3P phases. However, the SSH parametrization was of sufficient generality as to include[6] that of MAW as a particular region of its parameter space.

The disagreement of the Minnesota and Berkeley data near 10 MeV is striking. The low Berkeley $^1S_0$ phase correlates to some extent with the relatively low Berkeley cross-section normalization, while the relatively low Berkeley $^3\Delta_C$ correlates solely with the different shape of the cross section. These statements can be easily verified from equation 21 of Ref. 6. The departure of the data and the single-data-set solutions from the multienergy solution is shown in Fig. 3.

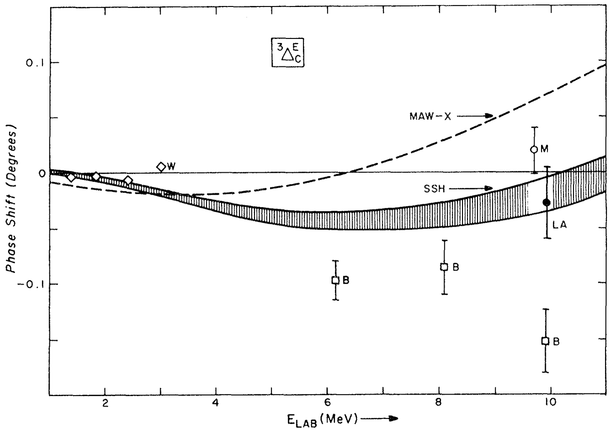

FIG. 2. Values of the "central" combination of $^3P_{0,1,2}$ phases (see text), similar to Fig. 1. The broken line is from Ref. 4, the band from Ref. 6. The data are from Wisconsin (Ref. 9), Berkeley (Ref. 2), Minnesota (Ref. 1), and Los Alamos (Ref. 7).

In order to resolve the discrepancies noted above for the data near 10 MeV, Jarmie, Brown, Hutson, and Detch[7] have recently made high-accuracy measurements at 5 angles at 9.69 MeV and at 11 angles at 9.918 MeV. We have made single-data-set analyses of each of these two data sets, adjusting the $^1S_0$ and $^3\Delta_C$ phases as described above for the older data. The most-forward-angle datum was that at 9.918 MeV, $\theta_{\rm lab} = 10°$. Its removal from the data set resulted in a drop in χ^2 of 5 for the single-data-set analysis, so this datum was discarded as inconsistent[10] with the other 10 data in the set. The resulting $^1S_0$ and $^3\Delta_C$ values for the new data are shown as sol-

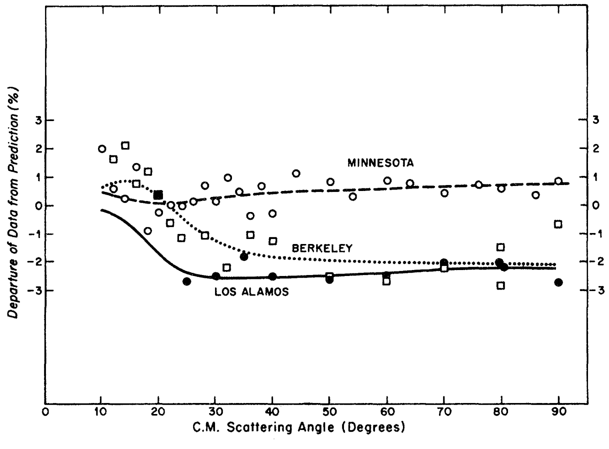

FIG. 3. Data values and single-data-set-analysis values, minus the corresponding multienergy-analysis values of Ref. 6, for the three data sets near 10 MeV. For sake of clarity, data errors are not shown. Note that the Berkeley and Los Alamos data coincide at 20°.

id circles in Figs. 1 and 2. The value of $^3\Delta_C$ for the new 9.69 MeV data was too poorly determined to be useful,[11] due to the restricted angular range of the new data at that energy.

It is seen in Fig. 2 that the value of $^3\Delta_C$ from the new data appears to be quite consistent with that from the data at lower and higher energies. The new value of the $^1S_0$-phase shift at 9.918 MeV is in agreement with the Berkeley-data value, but is considerably below the multienergy value (Fig. 1). From the work of Noyes,[12] the $^1S_0$-phase shift appears to be determined up to 10 MeV solely by its scattering length, effective range, and one-pion exchange. Assuming the Brolley, Seagrave, and Beery[9] relative data at 0.3 to 0.4 MeV as a low-energy anchor, confirmation of the new low $^1S_0$ values near 10 MeV would require re-examination of the normalization of the 1-3 MeV Wisconsin[13] data and would raise the previous value of the effective range parameter,[6] $r_0 = 2.83 \pm 0.02$ F, by 0.06 F.

One concludes that the $^3\Delta_C$ situation is moderately well settled, but that further experiments in the 1-10 MeV region are definitely necessary in order to clear up the $^1S_0$ discrepancies.

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[1] L. H. Johnston and D. E. Young, Phys. Rev. 116, 989 (1959).

[2] R. J. Slobodrian, H. E. Conzett, E. Shield, and W. F. Tivol, Phys. Rev. 174, 1122 (1968). We find that the values of χ^2 given in Table III of this reference do not correspond to the phase shifts in that table. The $^1S_0$ phase shifts listed in this reference are the *K* type of M. S. Sher, P. Signell, and L. Heller, Michigan State University Report No. COO-5061-4, 1969 (unpublished), and to be published (SSH) (R. J. Slobodrian, private communication).

[3] M. H. MacGregor, R. A. Arndt, and R. M. Wright, Phys. Rev. 179, 1624 (1969). The $^1S_0$-phase shifts listed in this reference are of the *K* type of SSH (R. A. Arndt, private communication).

[4] M. H. MacGregor, R. A. Arndt, and R. M. Wright, Phys. Rev. 182, 1714 (1969). Comparison with Ref. 3 and SSH shows that the $^1S_0$-phase shifts listed in this reference are of the electric-nuclear type of SSH.

[5] In the analyses published in Refs. 3 and 4, the c.m. angles were (erroneously) taken as twice the published lab angles of Ref. 2 (R. A. Arndt, private communication).

[6] Sher, Signell, and Heller, Ref. 2.

[7] N. Jarmie, R. E. Brown, R. L. Hutson, and J. L. Detch, Jr., preceding Letter [Phys. Rev. Letters 24, 240 (1970)].

[8] This is because the systematic errors in the Berkeley data are quoted as negligible and hence each angular point constitutes an independent measurement of the cross-section normalization (R. J. Slobodrian, private communication). Since there are 17 Berkeley angular data at 9.918 MeV, the effective normalization error is roughly $(17)^{-1/2}$ times a representative value for the absolute errors quoted at the individual angles. This effective normalization error for the Berkeley data is smaller than the normalization error quoted for the Los Alamos data. Of course in the actual analyses reported here the Berkeley data were treated as absolute while the Los Alamos data were treated as relative with a single normalization datum and accompanying normalization error. These treatments follow the interpretations of the errors placed on them by their authors.

[9] J. E. Brolley, J. P. Seagrave, and J. G. Berry, Phys. Rev. 135, B1119 (1964).

[10] See, for example, P. Signell, "The Nuclear Potential," in Advances in Nuclear Physics, edited by M. Baranger and E. Vogt (Plenum Press, Inc., New York, 1969), Vol. 2. With the forward datum left in, $^1\delta_0^E = 54.47° \pm 0.18°$, $^3\Delta_C = -0.058° \pm 0.035°$. See Ref. 7 for a discussion of the quoted error on the forward datum.

[11] The value was $^3\Delta_C = -0.03° \pm 0.09°$.

[12] H. P. Noyes, in Proceedings of the Second International Symposium on Polarization Phenomena of Nucleons, Karlsruhe, September, 1969, edited by P. Huber and H. Schupper (Birkhauser Verlag, Stuttgart, Germany, 1966), p. 238, and references contained therein. Noyes appears to have been the first to suggest the general approach to the problem taken here.

[13] D. J. Knecht, P. F. Dahl, and S. Messelt, Phys. Rev. 148, 1031 (1966).