lasers, see C. A. Klein, in Physics of Quantum Electronics, edited by P. L. Kelley, B. Lax, and P. E. Tannenwald (McGraw-Hill, New York, 1965), p. 424.

[2]J. D. Kingsley and G. W. Ludwig, Bull. Am. Phys. Soc. 13, 421 (1968).

[3]D. Pines, Elementary Excitations in Solids (Benjamin, New York, 1964).

[4]K. Colbow, Phys. Rev. 141, 742 (1966).

[5]Type "B" CdS grown by Eagle-Picher, with a liquid-nitrogen temperature resistivity of ~300 Ω cm. These crystals exhibit the phenomenon of "conductivity storage" as described by B. A. Kulp, J. Appl. Phys. 36, 553 (1965).

[6]One-phonon line at 5230 Å, as reported by C. Z. van Doorn, J. Phys. Chem. Solids 29, 601 (1968).

[7]R. K. Swank, Phys. Rev. 153, 844 (1967).

[8]Data from Ref. 7 were used to determine the Fermi level and conduction-band energy in CdS. No change in band bending was observed upon cooling to ~80°K, in contrast to observations in III-V semiconductors; see T. E. Fischer, Phys. Rev. Letters 21, 31 (1968), and A. Y. Cho and J. R. Arthur, Phys. Rev. Letters 22, 1180 (1969).

[9]D. L. Dexter, in Proceedings of the International Conference on Semiconductor Physics, Prague, 1960 (Academic, New York, 1961), p. 122.

[10]K. Sawamoto, J. Phys. Soc. Japan 18, 1224 (1963).

[11]T. Bergstresser and M. Cohen, Phys. Rev. 164, 1069 (1967).

[12]C. B. Duke and C. W. Tucker, Jr., Surface Sci. 15, 231 (1969).

[13]M. Gershenzon and R. M. Mikulyak, Appl. Phys. Letters 8, 245 (1966); this paper shows the influence of surface recombination on the luminescent efficiency of GaP.

[14]W. Shockley, Czech. J. Phys. B11, 85 (1961).

[15]V. S. Vavilov, J. Phys. Chem. Solids 8, 223 (1959).

[16]W. E. Spicer, J. Phys. Chem. Solids 22, 365 (1961).

INTERACTION OF DISLOCATIONS WITH ELECTRONS AND WITH PHONONS*

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The interaction parameter of moving dislocations with electrons and with phonons in aluminum was determined from measurements of ultrasonic attenuation changes with stress. The results indicate that the interaction with electrons is temperature independent and that the interaction with phonons increases with increasing temperature. These results are consistent with theoretical predictions.

In this note we report a new approach to measuring, by means of ultrasonic methods, the interaction of dislocations with electrons and with phonons in solids. (Specifically, we study the resistive force on a moving dislocation.) This method does not depend on a knowledge of the dislocation density and of other inaccurately known or difficult to determine features of the dislocation network. The main difficulties in obtaining reliable values for this interaction are thus eliminated. By using this method, the magnitude and temperature dependence of the interaction parameter (hereafter called B) were obtained in aluminum, in the temperature range $10°K \lesssim T \lesssim 250°K$.

Previous attempts to measure B by ultrasonic methods[1-3] required an independent determination of the dislocation density, thus involving a large uncertainty in the results. Other attempts were also made to obtain B from average dislocation velocity determined in mechanical tests.[4-6] A comparison of the values of B obtained with those derived from previous ultrasonic experiments was given by Fanti et al.[3] In the present approach, small changes in ultrasonic attenuation $\Delta\alpha$ are measured as a function of frequency. These changes result from the application of a small bias stress to the specimens during the experiment. From an analysis of this incremental attenuation it is possible to extract the value of B without knowing the dislocation density. In order for this analysis to be applicable the bias stress must be large enough to cause unpinning of dislocations from weak pinning points, but smaller than the stress required to generate new dislocations. Bias stresses in this range do not affect measurably the attenuation due to mechanisms other than dislocation vibrations. The measured $\Delta\alpha$ are, therefore, due entirely to the unpinning of dislocations with the resulting increase in the average loop length.

The analysis of the relation between $\Delta\alpha$ and B proceeds as follows. For the extensible string model of a vibrating dislocation, the total attenuation due to dislocations is given by[7]

$$\alpha(L_c) = \frac{4R\Lambda G b^2}{\pi^2 A} \frac{\omega^2 d}{(\omega_0^2 - \omega^2) + (\omega d)^2}, \qquad (1)$$

where R is an orientation factor, Λ is the total length of dislocations per unit volume, L_c is the dislocation loop length between pinning points, A is the mass per unit length, C is the dislocation line tension, ω is the driving frequency, $\omega_0 = (\pi/L_c)(C/A)^{1/2}$, and $d = B/A$.

When a distribution of loop lengths is considered, the total attenuation due to dislocations may be written in the form

$$\alpha_D = \int_0^\infty \alpha(l) l N(l) dl ,$$

where $N(l)dl$ is the number of dislocations whose loop length lies between l and $l+dl$. The distribution function can be approximated by the following expression:

$$N(l)dl \cong (\Lambda/L_c^2) e^{-l/L_c} dl, \tag{2}$$

where L_c is the average loop length. The application of a small bias stress affects this distribution by causing a slight increase δL_c in L_c, and the resulting attenuation change $\Delta\alpha_D$ can be expressed as follows:

$$\Delta\alpha_D = (\partial/\partial L_c)\int_0^\infty \alpha(l) l N(l) dl (\delta L_c) = (\Lambda/L_c^3)(\delta L_c)\int_0^\infty \alpha(l) l (l/L_c - 2) e^{-l/L_c} dl . \tag{3}$$

The main feature of this expression is that the dislocation density Λ and the increment δL_c of the average loop length are outside of the integral.[8] It follows that the characteristics of the $\Delta\alpha_D(\omega)$ relation are completely determined by the quantities ω_0 and B. The effect of a given bias stress on unpinning of dislocations, i.e., on δL_c, may differ for different temperatures. This does not affect the above analysis, however.

The experiments were carried out on an aluminum (99.99% pure) single crystal with the waves propagating in the [100] direction. A "dynamic" bias stress was used in the form of a large-amplitude wave at a frequency of 5 or 10 MHz, propagating in the [010] direction, i.e., at right angles to the "measuring" wave. The bias-wave pulse was synchronized with the measuring-wave pulse and the duration of the former was longer than the time separation between consecutive echoes of the latter. The change in ultrasonic attenuation $\Delta\alpha_D$ resulting from application of the bias wave was recorded for different measuring-wave frequencies.[9] The frequency range covered was 10 to 90 MHz at 10°K and it increased with increasing temperature. In the temperature range $50°K \lesssim T \lesssim 250°K$ the frequency range covered was from 10 to 315 MHz. (At the lowest temperatures it was not possible to obtain reliable results for the higher frequencies because of the large attenuation due to conduction electrons.)

Typical results for $\Delta\alpha_D$ as a function of frequency, obtained by the above method at several temperatures, are shown in Fig. 1. The points represent experimental values. In order to compare these results with the predictions of Eq. (3) it is necessary to obtain numerical solutions of this equation. The curves in Fig. 1 represent a computer fit of the solutions of Eq. (3) to the experimental points, using a least-squares method.

As can be seen, the curves of $\Delta\alpha$ as a function of frequency have a maximum which shifts to higher frequencies as the temperature decreases. This feature is a qualitative indication that B decreases as the temperature decreases. It is also noteworthy that this maximum is not of the thermal relaxation type. The damping parameter B obtained from the curve-fitting procedure mentioned is plotted as a function of temperature in Fig. 2. B is essentially temperature independent from the lowest temperature explored to about 50°K and increases with increasing temperature thereafter.

Two contributions to B are considered here, one from the interaction of dislocations with conduction electrons, B_e, the other from interactions with phonons, B_{ph}. Calculations by Holstein[10] and by Kravchenko[11] yield an expression for B_e which is independent of temperature. The results of previous measurements[12-14] as well as

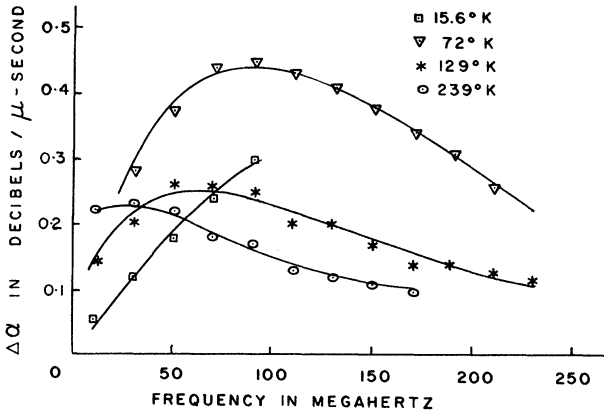

FIG. 1. The incremental attenuation $\Delta\alpha_D$ as a function of frequency for four representative temperatures. The points represent experimental data; the continuous curves represent computer fits of Eq. (3) to these points.

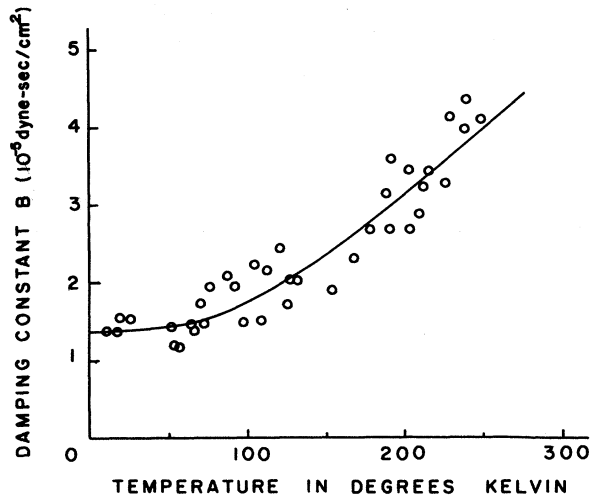

FIG. 2. The damping parameter B as a function of temperature. Open circles represent the values of B obtained from the curves of $\Delta\alpha_D(\omega)$. The continuous curve represents a computer fit of Eq. (6) to these points.

the present results are in agreement with these predictions. On the other hand, the calculations of Mason[15] give an expression for B_e which is inversely proportional to the electrical resistivity and is thus a strongly decreasing function of temperature, at low temperatures. The present results are clearly not in agreement with these predictions.

The temperature-dependent part of B, observed above 50°K, is attributed to dislocation interactions with phonons. Leibfried[16] derived the following expression for the force (per unit length) due to thermal phonons interacting with a moving dislocation:

$$K = -\sigma(v/c)\bar{\epsilon}/10. \quad (4)$$

From this, the phonon part of B, B_{ph}, is easily extracted:

$$B_{ph} = (\sigma/c)\bar{\epsilon}/10. \quad (5)$$

σ is the scattering width a dislocation presents to a phonon (assumed independent of phonon frequency and wave vector), v the velocity of the dislocation, c the velocity of sound, and $\bar{\epsilon}$ the thermal-energy density. The total B can then be expressed as

$$B = B_e + \sigma(1/c)\bar{\epsilon}/10. \quad (6)$$

The solid line through the points representing B in Fig. 2 is a computer fit of expression (6) to the data. In the calculation of $\bar{\epsilon}$ a Debye approximation is used for the phonon distribution, and B_e and σ are treated as fitting parameters. The best fit is obtained with $B_e = 1.37\times 10^{-5}$ dyn sec/cm^2, whereas the value obtained for edge dislocations using Holstein's[10] expression for B_e, with free-electron parameters for aluminum, is 1.5×10^{-5} dyn sec/cm^2. {Note that this value includes a correction by a factor $(2\pi)^{-1}$ that was left out of Holstein's formula due to a misprint, and a factor $[(1-2\nu)/(1-\nu)]^2$ associated with the dilatation at an edge dislocation; ν is Poisson's ratio taken to be $\frac{1}{3}$.} With an average phonon velocity $c = 3\times 10^5$ cm/sec, the value of σ yielding the best fit is $\sigma = 3.5\times 10^{-8}$ cm. Leibfried[16] suggested that the scattering width should be of the order of the Burgers' vector, which is 2.9×10^{-8} cm in aluminum.

The temperature dependence of B_{ph} found here is about 3 times larger, and the numerical value of B about two orders of magnitude smaller than the values predicted by Mason's theory[17] based on the phonon viscosity model. There is also a qualitative difference between the present results and those obtained on lead by Parameswaran and Weertman.[6]

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[1]G. A. Alers and D. O. Thompson, J. Appl. Phys. 32, 283 (1961).

[2]T. Suzuki, A. Ikushima, and M. Aoki, Acta Met. 12, 1231 (1964).

[3]F. Fanti, J. Holder, and A. V. Granato, J. Acoust. Soc. Am. 45, 1356 (1969).

[4]J. A. Gorman, S. D. Wood, and T. Vreeland, Jr., J. Appl. Phys. 40, 833 (1969).

[5]W. G. Ferguson, A. Kumar, and J. E. Dorn, J. Appl. Phys. 38, 1863 (1967).

[6]V. R. Parameswaran and J. Weertman, Scripta Met. 3, 477 (1969).

[7]A. V. Granato and K. Lücke, J. Appl. Phys. 27, 583 (1956).

[8]For a more detailed discussion of this distribution function and of its physical meaning and of experimental details, see A. Hikata, R. A. Johnson, and C. Elbaum, to be published.

[9]A separate experiment was performed on an alloy with a negligible dislocation contribution to α in order to ascertain whether $\Delta\alpha$ was associated with energy loss from the measuring wave due to (anharmonic) frequency mixing, and no evidence of such effects was found at the amplitudes used. It is concluded, therefore, that the measured $\Delta\alpha_D$ is due to dislocation unpinning, as postulated.

[10]Calculation due to T. Holstein, quoted by B. R. Tittman and H. E. Bömmel, Phys. Rev. 151, 178 (1966).

[11]V. Ya. Kravchenko, Fiz. Tverd. Tela 8, 927 (1966) [Soviet Phys. Solid State 8, 740 (1966)].

[12]A. Hikata and C. Elbaum, Phys. Rev. Letters 18, 750 (1967).
[13]C. Elbaum and A. Hikata, Phys. Rev. Letters 20, 264 (1968).
[14]A. Hikata and C. Elbaum, Suppl. Trans. Japan Inst. Metals 9, 46 (1968).
[15]W. P. Mason, Phys. Rev. 143, 229 (1966).
[16]G. Leibfried, Z. Physik 127, 344 (1950).
[17]W. P. Mason and A. Rosenberg, Phys. Rev. 151, 434 (1966).

ENERGY BANDS IN ONE-DIMENSIONAL APERIODIC POTENTIALS*

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By modifying the characteristic-momentum formalism of Williams and Matthews, we demonstrate the existence of two characteristic lengths of quite different magnitude. The larger, analogous to the mean free path for extended states in higher dimensions, is the distance of localization within energy bands. The smaller is that of states in the band tails. Finally, a formula for the conductivity in terms of the characteristic momentum is obtained.

Williams and Matthews[1] have made an analysis of electronic motion in one-dimensional aperiodic potentials based upon the introduction of a maximized average momentum, called the characteristic momentum. Their numerical results properly interpreted in the framework of a modified formalism confirm in the one-dimensional case the Mott and Cohen-Fritzsche-Ovshinsky (CFO) band model[2-4] for amorphous materials. We present here the modified Williams-Matthews formalism as well as some related mathematical results and physical ideas.

Consider a finite segment of the line, $a<x<b$ with $b-a=L$. The potential within the segment is aperiodic but can be imagined as derivable from a periodic potential by a disordering process. The Schrödinger equation, being a real, second-order, ordinary differential equation, has two linearly independent real solutions for each energy E, $\psi_1(x)$ and $\psi_2(x)$. Choose $\psi_1(x)$ so that

$$\psi_1(a)=\psi_1(b), \quad \int_a^b \psi_1^2(x)dx=1; \tag{1}$$

these conditions uniquely determine $\psi_1(x)$. Given $\psi_1(x)$, $\psi_2(x)$ is uniquely determined by

$$\int_a^b \psi_2(x)\psi_1(x)dx=0, \quad \int_a^b \psi_2^2(x)dx=1. \tag{2}$$

Williams and Matthews point out that the solutions of the present aperiodic case which most resemble the Bloch functions of the periodic case are those linear combinations φ of ψ_1 and ψ_2 which satisfy an extremal condition in relation to the average value of momentum $\langle\hat{p}\rangle$. We choose φ so that the real part of $\langle\hat{p}\rangle$ is an extremum. Without loss of generality, φ can be written as

$$\varphi=\psi_1+(\alpha+i\beta)\psi_2/[1+\alpha^2+\beta^2]^{1/2}, \tag{3}$$

where x and y are real. The average value of $\hat{p}$ is then

$$\langle\hat{p}\rangle=-i\hbar\int\varphi^*(d\varphi/dx)dx=\hbar\frac{\beta(\pi_{12}-\pi_{21})-i[\pi_{11}+(\alpha^2+\beta^2)\pi_{22}+\alpha(\pi_{12}+\pi_{21})]}{1+\alpha^2+\beta^2}, \tag{4}$$

where

$$\pi_{ij}=\int_a^b \psi_i d\psi_j, \quad i,j=1,2. \tag{5}$$

The extremum requirement on $\mathrm{Re}\langle\hat{p}\rangle$ gives

$$\alpha=0,\ \beta=1 \text{ or } \alpha=0,\ \beta=-1. \tag{6}$$

The above solutions leave us with a nonzero imaginary part of the expectation value of a dynamical variable. This situation is not allowed in quantum mechanics. Our approach to eliminating this difficulty differs from that of Williams and Matthews. We restrict ourselves to those energies for which

$$\psi_2(a)=\psi_2(b) \tag{7}$$

holds. We have thus imposed periodic boundary conditions on both ψ_1 and ψ_2. It can easily be