ISOLATION OF POMERANCHUK CONTRIBUTION TO $\bar{K}N$, $\bar{p}N$, AND πN TOTAL CROSS SECTIONS*

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A procedure for isolating the Pomeranchuk contribution to total cross sections is suggested. Application to the K^-N and $\bar{p}N$ systems yields the same value within errors as for the K^+N and pN systems, respectively, in agreement with the Pomeranchuk theorem. For πN, an asymptotic cross section of $18.1_{-1.5}^{+1.2}$ mb is obtained.

There has for a long time been interest in the asymptotic behavior of total cross sections for various hadron-hadron collisions. The goal of testing the Pomeranchuk theorem[1] has motivated experiments of increasing precision at the highest energies available in existing accelerators. Data from such experiments suggest that both the K^+p, K^+n and the pp, pn systems come close to their asymptotic limits at presently available beam energies whereas the K^-p, K^-n, the $\bar{p}p, \bar{p}n$, and the $\pi^\pm p$ total cross sections are still significantly dropping.[2]

In this paper we suggest a procedure, based on plausible assumptions but otherwise independent of any particular model, to determine from data at presently reachable energies the asymptotic limits of K^-N, $\bar{p}N$, and πN total cross sections. This procedure, applied to K^-N and $\bar{p}N$, gives limits of $17.0_{-1.6}^{+1.3}$ mb and 44.6 ± 5.8 mb, respectively, in good agreement with the near-asymptotic limits of 17.2 mb for K^+N (Ref. 2) and 38.9 mb for pN (Foley et al.[3]). Application of the same procedure to the $\pi^\pm p$ data gives an asymptotic cross section of $18.1_{-1.5}^{+1.2}$ mb, substantially lower than most estimates based on Regge fits and remarkably close to the K-nucleon limit.

We present our method of analysis by considering specifically the K^--nucleon system. We assume that (a) the K^-p and K^-n systems have a common asymptotic limit, a supposition experimentally well supported by the rapid decrease with energy of the forward K^- charge-exchange amplitude ($K^-p \to \bar{K}^0 n$); (b) the high-energy behavior of the K^-p and K^-n total cross sections can be represented by relations of the form

$$\sigma_1(s) = \sigma(\infty) + a_1 F(s), \tag{1a}$$

$$\sigma_2(s) = \sigma(\infty) + a_2 F(s), \tag{1b}$$

where s is the c.m. energy squared, the subscripts 1, 2 refer to K^-p and K^-n, respectively, a_1, a_2 are constants, and $F(s)$ is a monotonic function which vanishes at infinite energy. Arguments for the plausibility of (1a), (1b) and the smoothness of $F(s)$ can be made as follows:

(i) Above a few GeV/c, the K^-p and K^-n total cross sections do appear to drop smoothly; furthermore, the K^-n cross section, which is the smaller one, is dropping more slowly than the K^-p cross section.[2] A similar situation is known known, with much higher accuracy, to hold for the $\pi^\pm p$ system where the smaller π^+p cross section drops more slowly than the larger π^-p cross section.[4]

(ii) From the point of view of Regge poles, the forms (1a), (1b) follow from the assumption of degeneracy for the P', ω, ρ, and A_2 trajectories. In fact, the degeneracy of (P', ω) and (ρ, A_2) combinations has been invoked to explain the flatness of the K^+p and K^+n total cross sections as a function of energy.[5] It has been shown by Lipkin[6] that the postulated quadruple degeneracy is required if other systems with "exotic quantum numbers" such as $\pi^+\pi^+$ and $K^+\pi^+$ are to exhibit flat total cross-section behavior as do the K^+N and pN systems. It must be emphasized here, however, that we do not assume the Regge power dependence for $F(s)$, and indeed, as will be shown elsewhere, the data give only a poor fit to such power-law representation.

(iii) The forms (1a), (1b) have the esthetic feature that any linear combination of σ_1 and σ_2 corresponding, for example, to pure isovector or isoscalar t-channel exchanges also has a smooth monotonic behavior. In a sense, we are making the simplest possible assumption compatible with the experimental observations, namely that the smoothness observed at a few GeV continues to infinite energy.

The consequence of our assumptions is that there exists a linear combination of σ_1 and σ_2, namely $a_2\sigma_1 - a_1\sigma_2$, which cancels out the energy-dependent terms leaving a constant cross section essentially equal to the asymptotic limit. Our actual procedure is to construct the linear combination

$$\sigma_\lambda(s) = \sigma_1(s) - \lambda[\sigma_1(s) - \sigma_2(s)] \tag{2}$$

and choose the parameter λ, using the measured values of σ_1 and σ_2, to make $\sigma_\lambda(s)$ as flat as possible as a function of s. The value of σ_λ, when this condition is fulfilled, is then a measure of the asymptotic cross section $\sigma(\infty)$. In effect we are isolating the Pomeranchuk contribution to the total K^-N cross section.

Similarly, to isolate the Pomeranchuk contributions in $\bar{p}$ and pion interactions, we again construct a form like (2), where in the first case the subscripts 1 and 2 refer to $\bar{p}p$ and $\bar{p}n$ cross sections and in the second case, these same subscripts refer to π^+p and π^-p cross sections. We now consider these analyses in detail.

(1) K^--nucleon.—To apply Eq. (2) and construct a σ_λ of maximum flatness, it is necessary to have as precise values of $\sigma(K^-p)$ and $\sigma(K^-n)$ as possible. Measurements of $\sigma(K^-p)$ to ± 0.2 mb at energies up to 18 GeV have been carried out by Galbraith et al.[2] These authors have also measured $\sigma(K^-n)$, but because this measurement requires a subtraction of $\sigma(K^-p)$ from deuterium cross-section measurements of only ± 0.4-mb accuracy, as well as the calculation of Glauber corrections, the precision is not sufficiently good to obtain a useful result for the asymptotic cross section. To get around this difficulty we have used determinations of the forward cross section of the K^-p charge-exchange reaction by Astbury et al.,[7] coupled with the optical theorem to determine directly the difference $\sigma(K^-p) - \sigma(K^-n)$. This calculation assumes that the forward amplitude of the K^-p charge exchange is largely imaginary, a result already known to be essentially correct and explained on the basis of Regge theory.[8] Actually even as much as a 30% real component of the forward amplitude would introduce negligible error. In Table I, for momenta where good charge-exchange data exist, we give interpolated K^-p cross sections, $K^-p - K^-n$ cross-section differences obtained from the charge-exchange data, calculated K^-n cross sections obtained from combining the K^-p and charge-exchange data, and finally interpolated K^-n cross sections from the deuterium data.[2,9] It is clear that the last two columns are in good agreement as expected, but the K^-n cross sections derived from the charge-exchange data are more precise than those obtained from the K^-d measurements.

A least-squares fit requiring maximum constancy for σ_λ gives $\lambda = 2.9 \pm 0.8$ and $\sigma_\lambda = 17.0^{+1.3}_{-1.6}$ mb, with a χ^2 of 0.5 for two degrees of freedom. This asymptotic value of σ_λ is in excellent agreement with the value of 17.2 mb for the K^+p and K^+n cross sections. This is all the more remarkable in that at the highest momentum used in the analysis, 12.3 GeV/c, the K^-p cross section is still 4.5 mb above the asymptotic value. It is worth noting that the K^-p, K^-n asymptotic limit could be substantially improved with more precise and higher energy charge-exchange data.

(2) $\bar{p}$-nucleon.—Since $\bar{p}n$ measurements suffer from the same problems as K^-n, we have followed a similar procedure in that we have used

Table I. $\bar{K}N$ and $\bar{p}N$ cross sections.

K^- momentum (GeV/c)	$\sigma(K^-p)$ (mb)	$\sigma(K^-p) - \sigma(K^-n)$ (from c.e.) (mb)	$\sigma(K^-n)$ (from c.e.) (mb)	$\sigma(K^-n)$ (from K^-d) (mb)
5.0	25.0±0.6	2.91±0.33	22.1±0.7	--
7.1	23.8±0.2	2.21±0.18	21.6±0.3	21.2±0.4
9.5	22.6±0.2	1.98±0.20	20.6±0.3	20.6±0.4
12.3	21.7±0.2	1.60±0.12	20.1±0.3	20.2±0.4
$\bar{p}$ momentum (GeV/c)	$\sigma(\bar{p}p)$ (mb)	$\sigma(\bar{p}p) - \sigma(\bar{p}n)$ (from c.e.) (mb)	$\sigma(\bar{p}n)$ (from c.e.) (mb)	$\sigma(\bar{p}n)$ (from $\bar{p}d$) (mb)
5.0	62.0±1.5	7.25±0.6	54.75±1.6	--
6.0	59.3±1.1	7.20±0.6	52.1 ±1.2	59.5±4.0
7.0	57.8±1.1	5.71±0.45	52.1 ±1.2	58.4±4.0
9.0	55.5±1.1	4.75±0.4	50.75±1.2	56.4±3.9

$\bar{p}p \to \bar{n}n$ charge-exchange data, plus the optical theorem to determine $\sigma(\bar{p}p)-\sigma(\bar{p}n)$, again assuming that the forward amplitude is largely imaginary. In lower part of Table I we have given the cross sections $\sigma(\bar{p}p)$, $\sigma(\bar{p}p)-\sigma(\bar{p}n)$ from charge exchange, $\sigma(\bar{p}n)$ from combining the previous results, and finally $\sigma(\bar{p}n)$ obtained directly from the deuterium data.[2,10] Unlike the K^-n situation, the $\bar{p}n$ cross sections determined from deuterium are nearly equal to $\bar{p}p$ cross sections and do not agree well with those obtained from the optical theorem and the charge-exchange results. It must however be pointed out that a very large Glauber correction is required to obtain $\sigma(\bar{p}n)$ from deuterium. Furthermore the value of $\langle r^{-2}\rangle$ for deuterium used by Galbraith et al.[2] to obtain $\sigma(\bar{p}n)$ is about $1\frac{1}{2}$ times as large as the values found appropriate by Baker et al.,[11] Carter et al.,[12] and Abrams et al.[13] A reduction in $\langle r^{-2}\rangle$ by a factor of 1.5-2 would remove the discrepancy between $\sigma(\bar{p}n)$ from charge exchange and $\sigma(\bar{p}n)$ from deuterium. At the same time, it would lead to better agreement between np cross sections from the pd experiment of Galbraith et al. and directly measured np cross sections,[14] while, because of the smallness of the K^- cross sections, it would not significantly affect the good agreement between the two ways of obtaining $\sigma(K^-n)$ previously mentioned.

The analysis in the $\bar{p}$-nucleon case is perhaps more dubious than for the other situations considered, largely because of the low energies of availability of data on the $\bar{p}p$ charge exchange. The $\bar{p}p$ cross section is still varying significantly although very slowly above the highest energy used in the fit. In any case our analysis gives a limiting value of $\sigma_\lambda = 44.6 \pm 5.8$ mb for $\lambda = 2.2 \pm 1.0$. The χ^2 value of the fit is 1.1 for two degrees of freedom. Within the rather large errors this result is compatible with the near-asymptotic value of 38.9 mb obtained for pp at 26 GeV.[3]

(3) Pion-nucleon.—Using the precise cross-section data of Foley et al. between 8 and 22 GeV/c,[4] we obtain a best-fit pion-nucleon asymptotic cross section of $18.1^{+1.2}_{-1.5}$ mb. This value is remarkably close to the limiting cross section for the kaon-nucleon system. The χ^2 is 4.9 for six degrees of freedom, and the corresponding value of $\lambda = 3.6^{+0.9}_{-0.7}$. A plot of σ_λ for this value of λ for pions is shown in Fig. 1. We have plotted not only the 8- to 22-GeV/c data from which the fit was made, but also other data extending all the way down to 1.1 GeV/c.[12,15] Inspection of Fig. 1 shows the following features:

(i) For momenta above 4.5 GeV/c, the cross section σ_λ with the above choice of λ is completely flat within the errors. The fit for λ was done only for the Foley data above 7 GeV/c, and the fact that the flatness persists in the lower energy data of Citron et al.[15] is a satisfactory consistency check. The discontinuity in the actual values of σ_λ between the two sets of data reflects discontinuities in the measured values of the π^+ and π^- cross sections presumably arising from systematic errors. If the systematic error is actually in the experiment of Foley et al. rather than that of Citron et al., the asymptotic limit may be more like 17 than 18 mb.

(ii) As one goes below 4.5 GeV/c, the behavior of σ_λ exhibits wiggles of increasing amplitude due to resonances. These oscillations are strikingly regular and reminiscent in shape of damped sinusoidal behavior. The maxima and minima are roughly 0.60 GeV/c apart and decrease in amplitude by a constant factor of 3.1. One aspect of this regularity, well established before and connected with the uniform spacing of the wiggles, is the interleaving of the Δ and N_γ resonances on linear parallel Chew-Frautschi plots. The regularity of Fig. 1 also implies, however, smooth momentum dependence of widths and

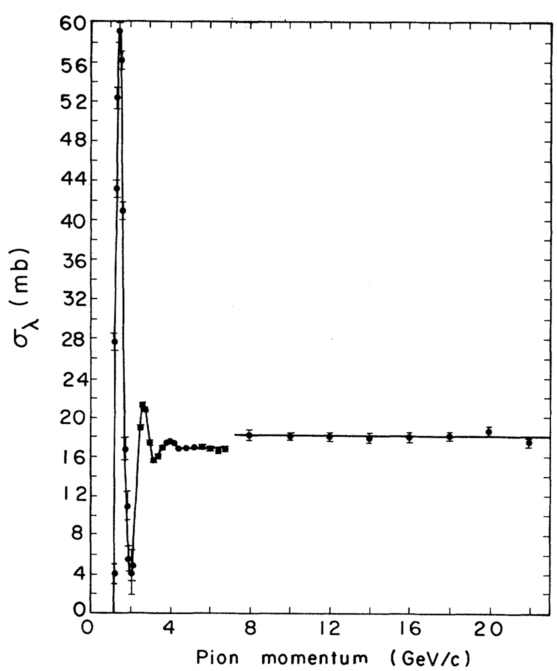

FIG. 1. Plot of σ_λ (see text) with $\lambda = 3.6$ for $\pi^\pm p$ collisions as a function of the incident momentum of the pion. The curves are as follows: above 7 GeV/c, constant cross section of 18.1 mb; below 7 GeV/c, approximate fit to the experimental points.

elasticities. These regularities are not nearly so evident in the energy dependences of the individual π^+ and π^- total cross sections. The absence of oscillations above 4.5 GeV/c arises from the fact that the oscillation amplitudes have decayed to less than 0.15 mb and are therefore within the errors of measurement. In effect, the resonances presumably continue to higher energies, but are so inelastic as to be undetectable in the total cross section. It is perhaps worth noting that the presence of resonance oscillations in what becomes a pure Pomeranchuk amplitude at high energy in no way invalidates Harari's[16] proposal that the Pomeranchuk contribution is built up from background only. Since σ_λ for pions contains positive contributions for $T=\frac{3}{2}$ and negative contributions for $T=\frac{1}{2}$ states, it is perfectly possible for the resonances to give a vanishing average contribution. In this sense our σ_λ is different from other pure Pomeranchuk cross sections like $\sigma(K^+p)$ or $\sigma(pp)$ in which resonance contributions are strictly positive and Harari's proposal therefore implies the total absence of resonant behavior at low energy.

We conclude by noting that the values of λ permit us to determine the ratio between t-channel isovector and isoscalar contributions to the imaginary part of the forward amplitude, exclusive of the Pomeranchuk contribution. This ratio is just $(2\lambda+1)^{-1}$, and amounts to 15% for K^-N, 14% for $\bar{p}N$, and 12% for $\pi^\pm p$.

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