N. Barash-Schmidt et al.,[3] and Eq. (4), a $\chi^2=0.3$ for two degrees of freedom was obtained for the hypothesis $|\Delta\vec{I}|=\frac{1}{2}$ with no violation of time-reversal invariance. Possible $|\Delta\vec{I}|=\frac{3}{2}$ amplitudes were computed from the formula

$$\sqrt{2}A_0+A_+-A_-=-3(\tfrac{2}{5})^{1/2}B_3, \qquad (6)$$

where B_3 is the $|\Delta\vec{I}|=\frac{3}{2}$ term. Assuming all amplitudes to be real, B_3 was found to have S-wave and P-wave components

$$S_3/S_-=-0.04\pm0.05,$$

$$P_3/P_+=-0.04\pm0.05. \qquad (7)$$

Here $S_-\approx A(\Sigma^-\to n\pi^-)$, $P_+\approx A(\Sigma^+\to n\pi^+)$, and $S_-\approx P_+$ in magnitude.

This experiment is consistent with the $|\Delta\vec{I}|=\frac{1}{2}$ rule, with time-reversal invariance, and confirms the validity of the $Y_0{}^*$ phase-shift analysis used in Ref. 2.

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DOES THE SLOPE OF THE HIGH-ENERGY ELASTIC PROTON-PROTON SCATTERING CROSS SECTION INCREASE AT SMALL MOMENTUM TRANSFER?*

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Experimental information relating to the slope of the elastic proton-proton scattering cross section in the region of $-t=0.15$ (BeV/c)^2 is reviewed. For proton energies greater than 18 BeV, most of the available data indicate that the slope changes from less than 9.0 (BeV/c)$^{-2}$ for $-t>0.2$ (BeV/c)^2 to a value greater than 10.0 (BeV/c)$^{-2}$ for $-t<0.15$ (BeV/c)^2.

Over the past several years a number of experiments have shown that proton-proton elastic scattering has several distinct regions of momentum-transfer dependence. The experiments of Akerlof et al.[1] and Allaby et al.[2] exhibit a change in the character of the slope of the cross section near $-t=6.0$ (BeV/c)^2. A distinct break in the cross section at $-t=1.2$ (BeV/c)^2 appears in measurements taken in a Brookhaven isobar run[3] and the experiment of Allaby et al.[2]

Krisch[4] has emphasized this structure by separating the cross section into three exponential regions. There are a number of theoretical models which can explain the qualitative features of a three-region structure. In particular, some optical models[5] predict a cross section in which there should be an even number of breaks[6] and consequently an odd number of regions. Regge-pole models[7] and hybrid models[8] do not have this constraint. In this note it will be shown that there is experimental evidence indicating the existence of a fourth region below $-t=0.15$ (BeV/c)^2.

It is useful to discuss cross-section parametrizations before considering the evidence for a change of slope. In the diffraction region the parametrization frequently takes the form

$$\frac{d\sigma}{dt} = \exp(a + bt + ct^2).$$

Since the cross-section slope is $b + 2ct$, the c term is a minor contribution to the slope at low $|t|$ values. The existence of the c term is only weakly established in elastic pp scattering at high energy and small momentum transfer, so that the existing data can also be fitted without it. The presence of this term may reflect the onset of the break in the cross section near $|t| = 1.2$ $(\mathrm{BeV}/c)^2$. Typically at $|t| = 0.2$ $(\mathrm{BeV}/c)^2$ the contribution to the slope is -0.4 $(\mathrm{BeV}/c)^{-2}$ and at $|t| = 0.5$ $(\mathrm{BeV}/c)^2$ is -1.0 $(\mathrm{BeV}/c)^{-2}$. In the comments that follow, b is sometimes used as an indication of the slope. Inclusion of the c term averaged over a $|t|$ interval would give a slightly smaller value for the slope. These parametrizations are only weakly affected by the choice of the kinematical quantities used in the expansion. For instance, using $p_\perp^2$ rather than $|t|$ changes the slope by only -0.2 $(\mathrm{BeV}/c)^{-2}$ at $|t| = 0.8$ $(\mathrm{BeV}/c)^2$ and an incident momentum of 20.0 BeV/c. If the variable $(p_\perp\beta)^2$ is used to parametrize the data, a new slope constant must be used which corresponds approximately to b divided by $(1-4m^2/s)$. In that case the slope of the cross section will again change by only -0.2 $(\mathrm{BeV}/c)^{-2}$ at $|t| = 0.8$ $(\mathrm{BeV}/c)^2$.

The evidence for a larger slope at small $|t|$ is the following:

(a) The precision total cross-section measurements of Foley et al.[9] give a cross section near 39 mb at high energy. These and the real-part measurements of the same group can be used to determine a differential cross section at $t = 0$, $d\sigma(0)/dt$, which is 81.0 mb/$(\mathrm{BeV}/c)^2$ at 20.0 BeV/c and estimated to be 78.3 mb/$(\mathrm{BeV}/c)^2$ at 30 BeV/c. The same group[10] previously measured the cross section as a function of energy in the range 0.2 $(\mathrm{BeV}/c)^2 < -t < 0.8$ $(\mathrm{BeV}/c)^2$. There they find b values of approximately 8.5 ± 1.0 $(\mathrm{BeV}/c)^{-2}$. These results have been confirmed by other experiments.[3,11] Slopes for neutron-proton scattering in the same $-t$ region are in good agreement with the p-p slopes.[12]

When these p-p measurements are extrapolated into the optical point, they tend to give values for $d\sigma(0)/dt$ that are distinctly below the values determined from the total cross-section measurements. For instance, the results of Foley et al. extrapolate to $d\sigma(0)/dt = 60$ mb/$(\mathrm{BeV}/c)^2$ at 24.63 BeV/c incident momentum, while their total cross-section measurements would indicate $d\sigma(0)/dt = 80$ mb/$(\mathrm{BeV}/c)^2$. These two types of experiments can be reconciled if the slope of the cross section increases below $-t = 0.2$ $(\mathrm{BeV}/c)^2$. A slope of about 11.0 $(\mathrm{BeV}/c)^{-2}$ in the region below $-t = 0.2$ $(\mathrm{BeV}/c)^2$ is required to make them consistent.

(b) A slope for the cross section in a region beyond the Coulomb interference region but below $-t = 0.1$ $(\mathrm{BeV}/c)^2$ has been determined by Bellettini et al.[13] in their real-part measurements. They find that the slope is 10.0 ± 0.2 $(\mathrm{BeV}/c)^{-2}$ at 19.33 BeV/c and 10.2 ± 0.2 $(\mathrm{BeV}/c)^{-2}$ at 25.42 BeV/c. In addition, the extrapolated cross sections at $t = 0$ are higher than the corresponding optical points determined from total cross-section measurements. Foley et al.[9] have not reported cross section slopes for their real-part measurements.

(c) Serpukhov results[14] indicate a slope of 10.7 ± 0.1 $(\mathrm{BeV}/c)^2$ at 29.7 BeV, increasing to 11.4 ± 0.1 $(\mathrm{BeV}/c)^2$ at 69 BeV in the region of 0.008 $(\mathrm{BeV}/c)^2 < -t < 0.12$ $(\mathrm{BeV}/c)^2$. Absolute normalization information is not available for this experiment. [These slopes also have a systematic error of $\Delta b = 0.3$ $(\mathrm{BeV}/c)^{-2}$.]

Figure 1 illustrates the situation for incident momenta near 19.0 BeV/c. The measurements of Harting et al.,[11] Allaby et al.,[2] and Foley et al.[10] are in essential agreement in the region above $|t| = 0.2$ $(\mathrm{BeV}/c)^2$. In particular the data of both Harting et al. and Allaby et al. can be fitted with a constant slope over the region $0.2 < -t < 1.0$ $(\mathrm{BeV}/c)^2$. The solid line shows such a fit for the Allaby et al. cross section data with the form e^{a+bt} in the region $0.23 < -t < 1.0$ $(\mathrm{BeV}/c)^2$. The small $-t$ behavior is determined in part by the Bellettini et al.[13] real-part measurement. A solid line with the slope of the Serpukhov[14] data has been fitted through their data.

The Krisch parametrization is also plotted in Fig. 1. This form does reproduce part of the slope change that the experimental data exhibit. Over the interval $0 < |t| < 0.5$ $(\mathrm{BeV}/c)^2$ it predicts a total slope change of -0.9 $(\mathrm{BeV}/c)^{-2}$. The region $0 < |t| < 0.2$ $(\mathrm{BeV}/c)^2$ gives an average slope of 8.9 $(\mathrm{BeV}/c)^{-2}$, while the region $0.2 < |t| < 0.5$ $(\mathrm{BeV}/c)^2$ gives an average slope of 8.5 $(\mathrm{BeV}/c)^{-2}$ so that there is a slope change of -0.4 $(\mathrm{BeV}/c)^{-2}$ when the effects of averaging are included. The experimental slope change is -1.9 $(\mathrm{BeV}/c)^2$. Thus the Krisch form underestimates the actual

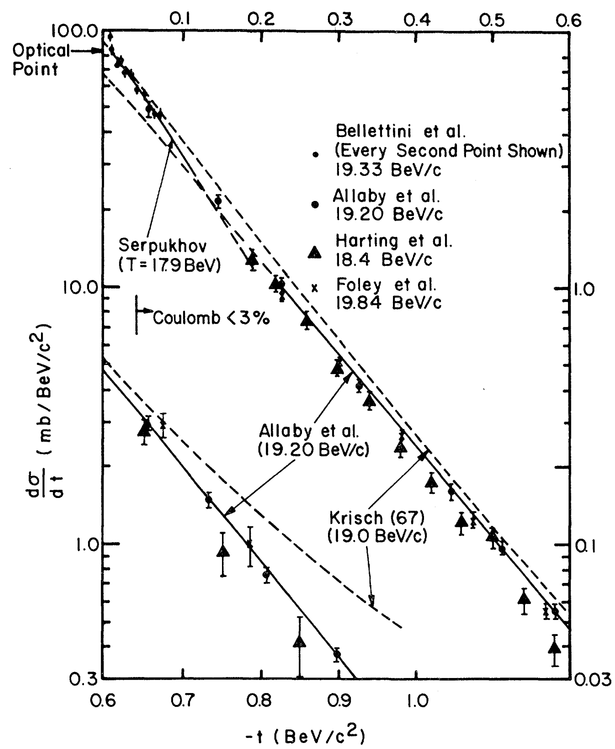

FIG. 1. pp elastic scattering for incident momenta near 19.0 BeV/c, illustrating the indication of a slope change near $|t| = 0.2$ BeV/c^2. The left and upper axes apply to the upper curves and points, while the right and lower axes apply to the lower curves and points.

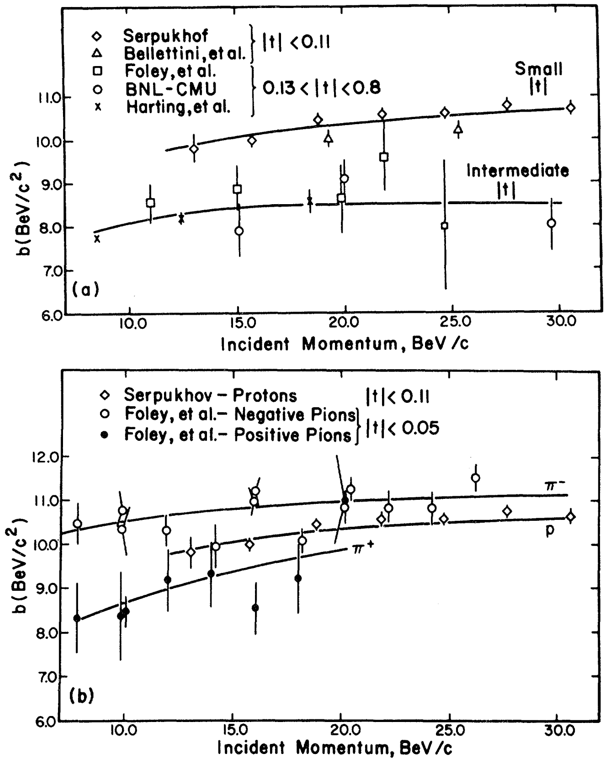

FIG. 2. (a) Slopes for elastic p-p scattering as a function of incident momentum for small and intermediate $|t|$ values. The curves only serve to link points from particular momentum-transfer regions. (b) Slopes for elastic p-p and $\pi^{\pm}$-p scattering for small $|t|$ values. In the cases of Foley et al. and BNL-CMU (Ref. 3) b is not the slope but the linear term in a cross-section parametrization $\exp(a+bt+ct^2)$.

slope change. One of the features of the Krisch parametrization is that it includes the effect of amplitudes that are important beyond $|t| = 1.2$ (BeV/c)^2. At this momentum the presence of these amplitudes in the Krisch formula fails by a factor of 2 to 4 to account for the magnitude of the slope change.

Figure 2(a) summarizes the available information on the slopes as a function of incident momentum. The curves shown are only intended to link points connected with particular momentum-transfer regions. The two sets of measurements below $-t < 0.15$ (BeV/c)^2 give slopes greater than 10 (BeV/c)$^{-2}$ while the three sets for $-t > 0.15$ BeV/c^2 give slopes less than 10.0 BeV/c^{-2}.

An insight into the character of the small $|t|$ scattering can be gained by considering that the cross section arises from two amplitudes with different slopes and no interference term. Then, roughly, the amplitude that makes a contribution only at small $-t$ can be found by subtracting the cross section extrapolated from intermediate t from the overall cross section. This gives a squared amplitude with a slope of 18 (BeV/c)$^{-2}$ and a total contribution to the cross section of 1 mb. Different assumptions about the relative phase of the amplitudes would strongly affect these values.

There is also evidence that the slope of pion-proton elastic scattering may increase at small momentum transfer. For the region 0.2 (BeV/c)^2 $< |t| < 0.8$ (BeV/c)^2 Foley et al.[10] find an average b that is about 9.0 (BeV/c)$^{-2}$ for π^- elastic scattering. For $|t| < 0.05$ they find[15] an average b that is 11.0 (BeV/c)$^{-2}$. Figure 2(b) shows their data for $\pi^{\pm}$ elastic scattering in the interval $|t| < 0.05$ (BeV/c)^2 plotted with the Serpukhov proton data. The different processes seem to have distinctly different slopes at low momenta but converge at higher momenta. Other scattering processes also exhibit steeper slopes at small momentum transfer. For example, neutron charge-exchange scattering[16] and π^+ photoproduction[17] have high-slope forward peaks. In these cases the effects are manifested at much lower incident energies.

Clearly further high-precision experiments are desirable which span the t region of the suggested slope change at the highest possible energy. If such a slope change exists it will complicate theoretical interpretations of the diffraction region. In particular the shrinkage observed at Serpukhov could not be directly associated with the major part of the diffraction peak, since the measurements only cover the small $|t|$ range where the slope change may occur.

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