# PROTON POLARIZATION IN $\Sigma^{+} \rightarrow p \pi^{0} \dagger^{*}$ 

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#### Abstract

The polarization of protons from the decay of polarized $\Sigma^{+}$hyperons has been measured by scattering the protons in a carbon-plate spark chamber. A sample of 1335 useful scatters gave $\alpha_{0}=-0.98 \pm 0.05$ and $\varphi_{0}=22^{\circ} \pm 90^{\circ}$, where $\tan \varphi_{0}=\beta_{0} / \gamma_{0}$. Using the data on $\Sigma^{+} \rightarrow n \pi^{+}$and $\Sigma^{-} \rightarrow n \pi^{-}$and fitting to the $|\Delta \overrightarrow{\mathrm{I}}|=\frac{1}{2}$ rule gave $\chi^{2}=0.3$ for 2 degrees of freedom.


The test of the $|\Delta \overrightarrow{\mathrm{I}}|=\frac{1}{2}$ selection rule for nonleptonic decays of $\Sigma^{ \pm}$hyperons has been limited by experimental uncertainty in the asymmetry parameter $\alpha_{0}$ for the decay $\Sigma^{+} \rightarrow p \pi^{0}$. Two measurements of $\alpha_{0}$ have been reported. The first was performed by Beall et al., ${ }^{1}$ who measured the decay proton helicity by scattering the protons in carbon and obtained $\alpha_{0}=-\overline{0.80} \pm 0.18$. The second was performed by Bangerter et al., ${ }^{2}$ who observed a proton asymmetry of the form ( $1+\alpha_{0} P_{\Sigma} \cos \omega$ ) relative to the hyperon spin direction, and then deduced $\alpha_{0}$ from a phase-shift analysis of the $1520-\mathrm{MeV} Y_{0}{ }^{*}$ which predicted $P_{\Sigma}$. Their result was $\alpha_{0}=-0.986 \pm 0.072$, in good agreement with the $|\Delta \overrightarrow{\mathrm{I}}|=\frac{1}{2}$ rule, which requires $\alpha_{0} \approx-1$. It seemed desirable to repeat with greater statistical accuracy a direct measurement of the proton spin following the technique of Ref. 1 in order to avoid possible uncertainties in the $Y_{0} *$ phaseshift analysis, and to measure the decay parameter $\gamma_{0}$, observable if the $\Sigma^{+}$hyperons are highly polarized in production.

The objective of this experiment was the measurement of the spin vector $\langle\vec{\sigma}\rangle$ for protons from the decay of polarized $\Sigma^{+}$hyperons. This spin vector is given in terms of $\alpha_{0}, \beta_{0}, \gamma_{0}$ by

$$
\begin{equation*}
\langle\vec{\sigma}\rangle=\left(1+\alpha_{0} \overrightarrow{\mathrm{P}}_{\Sigma} \cdot \hat{p}\right)^{-1}\left[\left(\alpha_{0}+\overrightarrow{\mathrm{P}}_{\Sigma} \cdot \hat{p}\right) \hat{p}+\beta_{0} \overrightarrow{\mathrm{P}}_{\Sigma} \times \hat{p}+\gamma_{0} \hat{p} \times\left(\overrightarrow{\mathrm{P}}_{\Sigma} \times \hat{p}\right)\right], \tag{1}
\end{equation*}
$$

where $\overrightarrow{\mathrm{P}}_{\Sigma}$ is the $\Sigma$ polarization vector, $\left|\overrightarrow{\mathrm{P}}_{\Sigma}\right| \equiv \bar{P}$, and $\hat{p}$ is the proton-momentum unit vector in the hyperon rest frame. The spin parameters $\alpha_{0}, \beta_{0}, \gamma_{0}$ are not all independent, but satisfy the constraint $\alpha_{0}{ }^{2}+\beta_{0}{ }^{2}+\gamma_{0}{ }^{2}=1$. The parameter $\beta_{0}$ vanishes if time reversal invariance is valid and final-state $\pi^{0} p$ interactions are ignored. ${ }^{3}$ The constraint can be expressed by defining $\beta_{0}=\left(1-\alpha_{0}{ }^{2}\right)^{1 / 2} \sin \varphi_{0}$, and $\gamma_{0}=(1$ $\left.-\alpha_{0}{ }^{2}\right)^{1 / 2} \cos \varphi_{0}$. Equation (1) then has three unknowns, $\alpha_{0}, \varphi_{0}$, and $\bar{P}$. The product $\alpha_{0} \bar{P}$ can be measured independently by observing the asymmetry distribution of protons relative to the hyperon spin direction $N(\omega)=\left(1+\alpha_{0} \bar{P} \cos \omega\right) \equiv\left(1+\alpha_{0} \overrightarrow{\mathrm{P}}_{\Sigma} \cdot \hat{p}\right)$. The spin vector $\langle\vec{\sigma}\rangle$ can be measured by scattering the protons in the laboratory off carbon nuclei. If $\hat{k}_{i}$ and $\hat{k}_{f}$ are the initial and final laboratory momentum unit vectors of the proton scattered by carbon, and $\hat{n}=\hat{k}_{i} \times \hat{k}_{f} /\left|\hat{k}_{i} \times \hat{k}_{f}\right|$, then the likelihood function

$$
\begin{equation*}
L\left(\alpha_{0} \varphi_{0}\right)=\prod_{j=1}^{n}\left[1+A_{j}\left(\theta_{j}, E_{j}\right)\left\langle\vec{\sigma}_{j}\left(\alpha_{0}, \varphi_{0}\right)\right\rangle \cdot \hat{n}_{j}\right] \tag{2}
\end{equation*}
$$

can be formed for the $j=1 \rightarrow n$ events of the sample. Here the coefficient $A_{j}\left(\theta_{j}, E_{j}\right)$ is the carbon-analyzing power for a $p$-carbon scatter at polar angle $\theta$ and energy $E .^{4}$. The unknown parameters $\alpha_{0}, \varphi_{0}$ can be calculated by maximizing $L\left(\alpha_{0} \varphi_{0}\right)$.

Positive pions at $1.12 \mathrm{GeV} / c$ from the Princeton-Pennsylvania accelerator produced $\Sigma^{+}$hyperons by the reaction $\pi^{+} p \rightarrow \Sigma^{+} K^{+}$in a liquid-hydrogen target. The experimental arrangement is shown in Fig. 1. The beam contained protons in the ratio $p / \pi^{+}=3 / 1$; these protons were easily eliminated by time of flight using the time-bunching feature of the accelerator. Velocity and range were used to identify the $K^{+}$mesons electronically. The time between the $K^{+}$stop and the decay $\mu^{+}$was recorded on film for each event. The $K^{+}$track was recorded in a foil spark chamber. Protons from $\Sigma^{+}$decay entered a carbon-plate spark chamber with 32 plates each $2.2 \mathrm{gm} / \mathrm{cm}^{2}$ thick. A scatter from carbon was not required in the trigger. The trigger rate was about $15 / \mathrm{min}$. One quarter of the triggers was associated production and the remainder was background. Of a total data sample of 400000 pictures, about $5 \%$ had proton scatters which appeared satisfactory on the film. The film was scanned for events with


FIG. 1. Plan view of the apparatus. A positive-pion beam of $10^{5} / \mathrm{sec}$ entered from the right at $1.12 \mathrm{GeV} / \mathrm{c}$ mean momentum and $\pm 3 \%$ bite. The first two counters were timed relative to a master time signal to identify pions by time of flight $(\pi) . K^{+\prime}$ s produced to the right of the hydrogen target satisfied $K_{1} K_{2} \overline{\mathrm{H}_{2} \mathrm{O}}$ and stopped in the large water counter. Ten wrap counters $W$ surrounding the large water counter used to count the decay $\mu^{+}$. Decay protons were counted in $P_{1} P_{2}$ and entered the carbon-plate spark chamber. The trigger was ( $\left.\pi K_{1} K_{2} \overline{\mathrm{H}_{2} \mathrm{O}} P_{1} P_{2}\right) \times\left(\mathrm{H}_{2} \mathrm{OW}\right)$, the second $\mathrm{H}_{2} \mathrm{O}$ in parentheses being the large water tank.
a single $K^{+}$track and a single decay-proton track which scattered in the carbon and stopped in the chamber volume. The $K^{+}$direction, the initial and final proton directions, and both the total proton range and the residual range after the scatter were measured. The data contained about $40 \%$ background at this stage. Since elastic $\pi p$ scattering could not satisfy the counter geometry, this background was caused by multiple pion production in the hydrogen or in the target walls. The background was reduced by requiring a vertex in the liquid hydrogen and by requiring the $K^{+}$and proton angles to be consistent with associated production and $\Sigma^{+}$decay kinematics. A total of 8550 events remained in the sample.

Figure 2 shows the distribution in delay time between the stopping $K^{+}$and the decay $\mu^{+}$for these 8550 events. An excess of 2600 events at prompt delay times is apparent in this curve. To eliminate this prompt background only those events with delay times $t \geqslant 0.6 \tau_{K}$ were accepted for further analysis. For these 3304 events, two laboratory proton energies could be calculated assuming the sequence $\pi^{+} p \rightarrow \Sigma^{+} K^{+}, \Sigma^{+} \rightarrow p \pi^{0}$. The lower proton energy was usually insufficient


FIG. 2. Plot of the time difference between $K_{2}$ and $W$ for 8550 events. The dashed line is the curve expected for a pure $K^{+}$decay sample, normalized to $t>0.6 \tau_{K^{+}}$. The 2600 events in the shaded area were "prompt" events, not caused by $K^{+} \Sigma^{+}$production. The data to the right of the vertical line at $t=0.6 \tau_{K^{+}}$were selected to be free of the prompt background. Because of the sharp bunching of the Princeton-Pennsylvania Accelerator beam ( $1-\mathrm{nsec}$ pulse every 66 nsec ) there was no continuous accidental background under this curve.
to give a useful carbon scatter. The upper energy could be compared with the energy inferred from the observed proton range in carbon. Figure 3 shows the result of this comparison; $\delta \equiv$ ob-served-minus-predicted range in sparks. One spark corresponded to 8 MeV for a typical event. A satisfactory fit to this histogram was obtained by combining the spark-chamber resolution with the proton energy spread due to geometrical uncertainties. The data sample at this stage was consistent with a pure $\Sigma^{+} \rightarrow p \pi^{0}$ signal, but to eliminate possible remaining background, the observed range was required to agree with the predicted range to within $\pm 3$ sparks, corresponding to the full width at half-maximum of the curve in Fig. 3. There were 2000 events in this peak.

Each of these 2000 events had a proton scatter in carbon with no visible recoil tracks at the scatter vertex. To avoid obvious geometrical bias each scatter was required to satisfy a "cone test." 86 events were eliminated because the scattered proton track could be forced to leave the spark-chamber fiducial volume by rotating it


FIG. 3. Observed-minus-predicted range curve ( $\delta$ curve) for the 3300 events with $t \geqslant 0.6 \tau_{K^{+}}$in Fig. 2. The Monte Carlo fit to this histogram is consistent with no background.
about the incident-proton track. No other geometrical distortions were found despite the fact that the carbon-plate spark chamber was not symmetrical with respect to the incident protons. The spatial asymmetry for the decays was calculated using $\underset{\vec{P}}{N}(\omega)=\left(1+\alpha_{0} \bar{P} \cos \omega\right)$ and defining $P$ parallel to $\overrightarrow{\mathrm{P}}_{\pi_{i n}} \times \overrightarrow{\mathrm{P}}_{K_{\text {out }}}$ as positive. The result averaged over center-of-mass production angles $-0.6 \leqslant \cos \theta^{*} \leqslant+0.3$ was $\alpha_{0} \bar{P}=0.59 \pm 0.04$ for the 1914 events in the sample. Requiring the $p$-carbon analyzing power to be greater than 0.1 eliminated 579 events.
In summary, the final data sample was subjected to the kinematic and vertex requirements, the decay-time requirement $t \geqslant 0.6 \tau_{K^{+}}$, and the range-agreement requirement $|\delta| \leqslant 3$ sparks. The scatters had to pass the cone test, and had to have an analyzing power greater than $10 \%$. The average carbon analyzing power for these events was 0.41 . The center-of-mass $\cos \theta^{*}$ region was divided into three equal bins and an $\alpha \bar{P}$ was determined for each bin for use in the likelihood function. The likelihood function defined in Eq. (2) was calculated in terms of $\alpha_{0}$ and $\varphi_{0}$. Solutions for the maximum value of $L$ were

$$
\begin{equation*}
\alpha_{0}=-0.98_{-0.02}^{+0.04}, \quad \varphi_{0}=22^{\circ} \pm 78^{\circ} . \tag{3}
\end{equation*}
$$

The relativistic spin transformation from center of mass to laboratory has been included in the


FIG. 4. Likelihood curves for the final data sample. Likelihood contours in $\alpha_{0}, \varphi_{0}$ space show that the two parameters are essentially uncorrelated, although $\varphi_{0}$ is poorly determined because $\alpha_{0}$ is very close to -1 .
analysis. The errors were variations in $\alpha_{0}, \varphi_{0}$ which changed $\ln (L)$ by $\frac{1}{2}$. These likelihood curves are shown in Fig. 4. Variations in the $|\delta|$ cut produced no change in the result. Relaxing the delay time requirement $t \geqslant 0.6 \tau_{K^{+}}$and admitting all delay times increased the data sample to 2300 events, but also introduced $20 \%$ prompt backg round. For this enlarged sample the results were $\alpha_{0} \bar{P}=0.54 \pm 0.03, \alpha_{0}=-0.86$ $\pm 0.04$, and $\varphi_{0}=25^{\circ} \pm 20^{\circ},{ }^{5}$ consistent with Monte Carlo calculations of the expected effects of an unpolarized background on the experimental result.

The sensitivity of the results to various systematic effects has been investigated. $\mathrm{A} \pm 2 \%$ uncertainty in $\alpha_{0}$ has been ascribed to possible remnant unpolarized background in the final data sample. The dependence of $\alpha_{0}$ on the $p$-carbon analyzing power was slight. ${ }^{4}$ Nonlinear distortions in the carbon-plate-chamber optics were studied with grid pictures and straight beam tracks. Uncertainties in these corrections led to an additional error of $\pm 2 \%$ in $\alpha_{0}$ and $\pm 45^{\circ}$ in $\varphi_{0}$, giving the final results

$$
\begin{equation*}
\alpha_{0}=-0.98_{-0.02}^{+0.05}, \quad \varphi_{0}=22^{\circ} \pm 90^{\circ} . \tag{4}
\end{equation*}
$$

These values can be converted into values for $\alpha_{0}, \beta_{0}$, and $\gamma_{0}$, giving

$$
\begin{align*}
& \alpha_{0}=-0.98_{-0.02}^{+0.05}, \quad \beta_{0}=+0.08_{-0.28}^{+0.16}, \\
& \gamma_{0}=+0.19_{-0.35}^{+0.23} . \tag{5}
\end{align*}
$$

Final-state $\pi^{0} p$ interactions predict $\beta_{0}=-0.03$ if the $|\Delta \vec{I}|=\frac{1}{2}$ rule is assumed. Using the latest data on $\Sigma^{ \pm}$lifetimes, ${ }^{6}$ the $\alpha_{ \pm}$data compiled by
N. Barash-Schmidt et al., ${ }^{3}$ and Eq. (4), a $\chi^{2}=0.3$ for two degrees of freedom was obtained for the hypothesis $|\Delta \overrightarrow{\mathrm{I}}|=\frac{1}{2}$ with no violation of time-reversal invariance. Possible $|\Delta \overrightarrow{\mathrm{I}}|=\frac{3}{2}$ amplitudes were computed from the formula

$$
\begin{equation*}
\sqrt{2} A_{0}+A_{+}-A_{-}=-3\left(\frac{2}{5}\right)^{1 / 2} B_{3} \tag{6}
\end{equation*}
$$

where $B_{3}$ is the $|\Delta \overrightarrow{\mathrm{I}}|=\frac{3}{2}$ term. Assuming all amplitudes to be real, $B_{3}$ was found to have $S$-wave and $P$-wave components

$$
\begin{align*}
& S_{3} / S_{-}=-0.04 \pm 0.05, \\
& P_{3} / P_{+}=-0.04 \pm 0.05 \tag{7}
\end{align*}
$$

Here $S_{-} \approx A\left(\Sigma^{-} \rightarrow n \pi^{-}\right), P_{+} \approx A\left(\Sigma^{+} \rightarrow n \pi^{+}\right)$, and $S_{-}$ $\approx P_{+}$in magnitude.
This experiment is consistent with the $|\Delta \overrightarrow{\mathrm{I}}|=\frac{1}{2}$ rule, with time-reversal invariance, and confirms the validity of the $Y_{0}{ }^{*}$ phase-shift analysis used in Ref. 2.
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*Work performed at the Princeton-Pennsylvania Accelerator.
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# DOES THE SLOPE OF THE HIGH-ENERGY ELASTIC PROTON-PROTON SCATTERING CROSS SECTION INCREASE AT SMALL MOMENTUM TRANSFER?* 

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#### Abstract

Experimental information relating to the slope of the elastic proton-proton scattering cross section in the region of $-t=0.15(\mathrm{BeV} / \mathrm{c})^{2}$ is reviewed. For proton energies greater than 18 BeV , most of the available data indicate that the slope changes from less than $9.0(\mathrm{BeV} / c)^{-2}$ for $-t>0.2(\mathrm{BeV} / c)^{2}$ to a value greater than $10.0(\mathrm{BeV} / c)^{-2}$ for $-t<0.15$ $(\mathrm{BeV} / c)^{2}$.


Over the past several years a number of experiments have shown that proton-proton elastic scattering has several distinct regions of momen-tum-transfer dependence. The experiments of Akerlof et al. ${ }^{1}$ and Allaby et al. ${ }^{2}$ exhibit a change in the character of the slope of the cross section near $-t=6.0(\mathrm{BeV} / c)^{2}$. A distinct break in the cross section at $-t=1.2(\mathrm{BeV} / c)^{2}$ appears in measurements taken in a Brookhaven isobar run ${ }^{3}$ and the experiment of Allaby et al. ${ }^{2}$

Krisch ${ }^{4}$ has emphasized this structure by separating the cross section into three exponential
regions. There are a number of theoretical models which can explain the qualitative features of a three-region structure. In particular, some optical models ${ }^{5}$ predict a cross section in which there should be an even number of breaks ${ }^{6}$ and consequently an odd number of regions. Reggepole models ${ }^{7}$ and hybrid models ${ }^{8}$ do not have this constraint. In this note it will be shown that there is experimental evidence indicating the existence of a fourth region below $-t=0.15(\mathrm{BeV} /$ $c)^{2}$.

It is useful to discuss cross-section parametri-

