

for the existence of the two-phonon octupole vibration. The population of the two-phonon octupole band should also be possible in radioactive decay for other nuclei in the heavy-mass deformed region.

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## STRONG-CUT REGGEIZED ABSORPTION MODEL FOR BACKWARD SCATTERING\*

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The combined data for the high-energy backward reactions  $\pi N \rightarrow N\pi$ ,  $\pi N \rightarrow N\rho$ , and  $\gamma N \rightarrow N\pi$  are analyzed in terms of the strong-cut Reggeized absorption model proposed by Henyey, Kane, Pumplin, and Ross. Only the  $N$  and  $\Delta$  trajectories and the associated Regge cuts are necessary to describe the data, including the dip structure. The analysis lends strong support to the model, in particular to the feature that the Regge pole amplitudes do not have zeros at nonsense wrong-signature points.

In the past few years there have been proposed several models of high-energy two-body scattering processes which combine the physics of Regge-pole exchange and the absorptive properties of hadrons.<sup>1-4</sup> All of these models are essentially in agreement on the nature of the relation between absorption and Regge cuts and on the form of the leading Regge cut to be associated with each Regge pole. There are, however, significant differences in the form of the Regge-pole amplitude and in the expected size of the Regge cuts associated with inelastic intermediate states. In the strong-cut Reggeized absorption model (SCRAM) of Henyey, Kane, Pumplin, and Ross (HKPR) there are no nonsense wrong-signature zeros (NWSZ) in the Regge-pole amplitudes (it has been shown by several authors that there is no theoretical necessity for NWSZ; HKPR ref-

erence these arguments and discuss in detail the physical motivation for the omission of NWSZ in SCRAM), and there are significant contributions from Regge cuts associated with diffractively produced inelastic intermediate states. Both of these features enhance the strength of the net cut contribution and lead to dips in  $s$ -channel helicity amplitudes arising from pole-cut interference. We use the words "strong-cut" in reference to this model in order to emphasize these features.

Data.—We list here with references the backward reactions that will be discussed in this Letter.<sup>5</sup> We have limited our attention to reactions involving nonstrange baryon exchange at lab momenta  $\geq 5$  BeV/c. Only cross-section data are discussed; unfortunately there exist no measurements of polarizations or density matrices at these energies.<sup>6</sup> The reactions are as follows:

$\pi^- p \rightarrow p\pi^-$ ,  $^{7,9} \pi^+ p \rightarrow p\pi^+$ ,  $^{8,9} \pi^- p \rightarrow n\pi^0$ ,  $^{10,11} \pi^- p \rightarrow n\rho^0$ ,  $^{12} \pi^- p \rightarrow p\rho^-$ ,  $^{13} \gamma p \rightarrow n\pi^+$ ,  $^{14} \gamma p \rightarrow p\pi^0$ ,  $^{15} pp \rightarrow d\pi^+$ ,  $^{16} pp \rightarrow d\rho^+$ .

**Regge-pole models.**—An examination of the data on the above reactions indicates that no simple model having Regge-pole amplitudes with NWSZ at  $\alpha_N = -\frac{1}{2}$  can describe all of these data simultaneously. There are several aspects to this argument:

(A) The only trajectories that can reasonably be considered sufficiently high lying to describe high-energy data are  $N$ ,  $\Delta$ , and  $N_\gamma$ .

(B) In a NWSZ model any reaction in which nucleon exchange is allowed will exhibit a deep cross-section dip around  $u = -0.15$  if nucleon exchange dominates.<sup>17</sup> Such a dip is seen only in  $\pi^+ p \rightarrow p\pi^+$ . Therefore NWSZ models must have important  $\Delta$  and/or  $N_\gamma$  exchange contributions.

(C) The size of the  $\Delta$  contribution in photoproduction can be inferred from  $\rho^-$  production using vector dominance. The cross section for  $\pi^- p \rightarrow p\rho^-$  at 8 BeV/c and  $u = -0.15$  is about  $3.5 \mu\text{b}$ . Using vector dominance (with  $\gamma_\rho^2/4\pi = 0.5$ ) this gives an upper limit (corresponding to purely

transverse  $\rho^-$  production) of 1.4 nb (2.8 nb) for the  $\gamma p \rightarrow n\pi^+$  ( $p\pi^0$ ) cross section arising from  $\Delta$  exchange alone. The actual cross section is about 6 nb (4.5 nb). Thus, at least in charged photoproduction,  $\Delta$  exchange cannot account for a large percentage of the cross section at  $u = -0.15$  without grossly violating the hypothesis of vector dominance. If this argument is applied to the integrated backward cross sections it is still stronger because the backward peak in  $\rho^-$  production is considerably steeper than in photoproduction.

(D) If the photoproduction cross sections are primarily due to  $N_\gamma$  exchange at  $u = -0.15$ , as in the models of Barger and Weiler<sup>18</sup> and Beaupre and Paschos,<sup>19</sup> then factorization requires the  $N_\gamma$  also to fill in the dip in  $\pi^+ p \rightarrow p\pi^+$ . This is so because the  $pp \rightarrow d\pi^+$  and  $pp \rightarrow d\rho^+$  cross sections are approximately equal, and therefore the  $\pi NN_\gamma$  and  $\rho NN_\gamma$  vertices must be approximately equal at  $u = -0.15$  ( $\Delta$  exchange is forbidden in these reactions). A simple factorization and vector-dominance argument which neglects spin,  $\Delta$  exchange in photoproduction, and the isoscalar component of the photon leads to the following relations:

$$\sigma^{N_\gamma}(\pi^+ p \rightarrow p\pi^+) = \frac{4\gamma_\rho^2}{\pi\alpha} \frac{\sigma(\gamma p \rightarrow p\pi^0)\sigma(pp \rightarrow d\pi^+)}{\sigma(pp \rightarrow d\rho^+)} = \frac{2\gamma_\rho^2}{\pi\alpha} \frac{\sigma(\gamma p \rightarrow n\pi^+)\sigma(pp \rightarrow d\pi^+)}{\sigma(pp \rightarrow d\rho^+)}, \quad (1)$$

where  $\sigma$  denotes  $d\sigma/du$  at  $u = -0.15$  and  $\sigma^{N_\gamma}$  denotes the cross section coming from  $N_\gamma$  exchange alone. The observed value of  $\sigma(pp \rightarrow d\pi^+)/\sigma(pp \rightarrow d\rho^+)$  at 21 BeV/c is about 1. Using the same value at 10 BeV/c, the second and third members of Eq. (1) are 2.8 and 1.9  $\mu\text{b}$ , respectively. The observed  $\pi^+ p \rightarrow p\pi^+$  cross section at  $u = -0.15$  and 10 BeV/c is 0.15  $\mu\text{b}$ .

The approximations made in obtaining this estimate (except neglect of spin) are fairly well satisfied in the Regge-pole models<sup>18,19</sup> that have been used to describe the photoproduction data. We feel that the above estimate exhibits a possible order of magnitude discrepancy in spite of the rather crude way in which the numbers were obtained.

(E) A NWSZ model for  $\pi N \rightarrow N\pi$  which fits the  $\pi^- p \rightarrow p\pi^-$  and  $\pi^+ p \rightarrow p\pi^+$  data will have a deep dip in the  $\pi^- p \rightarrow n\pi^0$  cross section near the position of the  $\pi^+ p \rightarrow p\pi^+$  dip. However, the data show a shallow dip or break somewhat beyond  $u = -0.2$  and it therefore appears likely that no NWSZ model can quantitatively describe these processes. A fit to  $\pi^- p \rightarrow p\pi^-$  and  $\pi^+ p \rightarrow p\pi^+$  using SCRAM produces a  $\pi^- p \rightarrow n\pi^0$  cross section with a break or dip whose depth is quite parameter dependent, but whose position is rather insensitive to the parameters and agrees with the existing data.<sup>19a</sup>

(F) All of the above remarks apply with little change to "weak-cut" models in which the Regge pole amplitudes have NWSZ and there are no cuts arising from inelastic intermediate states.<sup>20</sup>

**SCRAM.**<sup>21</sup>—We consider processes of the type  $a + b \rightarrow c + d$  where  $a$  is a pion,  $b$  and  $c$  are nucleons, and  $d$  is a meson, isovector photon ( $\gamma^v$ ), or isoscalar photon ( $\gamma^s$ ). For convenience of presentation we use  $\pi N \rightarrow N\gamma$  amplitudes to calculate photoproduction cross sections; the full photoproduction amplitude is the sum of the  $\pi N \rightarrow N\gamma^v$  and the  $\pi N \rightarrow N\gamma^s$  amplitudes.

Let the particles have helicities  $\lambda_a = \lambda$ ,  $\lambda_b = \mu$ ,  $\lambda_c = \lambda'$ , and  $\lambda_d = \mu'$ , and define

$$n = |(\lambda' - \mu') - (\lambda - \mu)|, \quad n' = |(\lambda' - \mu') + (\lambda - \mu)|, \quad x = |\lambda' - \lambda| + |\mu' - \mu| - n - 1. \quad (2)$$

The Regge-pole  $s$ -channel helicity amplitudes for reaction  $R \equiv a + b \rightarrow c + d$  are

$$M_{\lambda'\mu'; \lambda\mu}^{P,R} = \sum_{I=N,\Delta} M_{\lambda'\mu'; \lambda\mu}^{P,R,I},$$

Table I. Vertex functions, isospin vector-dominance coefficients, and parameter values.

DEFINITIONS OF VERTEX FUNCTIONS <sup>a</sup>	ISOSPIN - VECTOR DOMINANCE COEFFICIENTS, $C_I^R$			NUMERICAL VALUES OF PARAMETERS <sup>b</sup>	
	R	I=N	I=Δ		
$\gamma_{\frac{1}{2}0}^{\pi N} = \gamma_{0\frac{1}{2}}^{\pi N} = -i\gamma_{0-\frac{1}{2}}^{\pi N} = i\gamma_{-\frac{1}{2}0}^{\pi N} = \beta^N$				$\alpha^N = -.54+.01\sqrt{u}+1.16u$	$\alpha^\Delta = .14-.23\sqrt{u}+1.08u$
$\gamma_{\frac{1}{2}0}^{\pi\Delta} = -\gamma_{0\frac{1}{2}}^{\pi\Delta} = -i\gamma_{0-\frac{1}{2}}^{\pi\Delta} = -i\gamma_{-\frac{1}{2}0}^{\pi\Delta} = \beta^\Delta$				$\alpha^N(M_N) \equiv 1/2$	$s_N^0 = .51$
$\gamma_{1\frac{1}{2}}^{\pi\Delta} = -i\gamma_{-1-\frac{1}{2}}^{\pi\Delta} = a_d^N$				$\lambda_{\Delta}^{\pi N \rightarrow N\pi} \equiv \lambda_{\Delta}^{\pi N \rightarrow N\omega} = 1.96$	$\alpha^\Delta(M_\Delta) \equiv 3/2$
$\gamma_{1-\frac{1}{2}}^{\pi\Delta} = i\gamma_{-1\frac{1}{2}}^{\pi\Delta} = b_d^N$				$\lambda_{\Delta}^{\pi N \rightarrow N\pi} \equiv \lambda_{\Delta}^{\pi N \rightarrow N\omega} = 1.77$	$s_\Delta^0 = 1.15$
$\gamma_{0-\frac{1}{2}}^{\pi\Delta} = -i\gamma_{0\frac{1}{2}}^{\pi\Delta} = c_d^N$				$\lambda_{\Delta}^{\pi N \rightarrow N\pi} = 3.17$	$\beta^\Delta = 2.48+3.65\sqrt{u}$
$\gamma_{1\frac{1}{2}}^{\rho\Delta} = i\gamma_{-1-\frac{1}{2}}^{\rho\Delta} = a^\Delta$				$\beta^N = 13.1-4.57\sqrt{u}$	$a^\Delta = -.416+4.29\sqrt{u}$
$\gamma_{1-\frac{1}{2}}^{\rho\Delta} = -i\gamma_{-1\frac{1}{2}}^{\rho\Delta} = b^\Delta$				$a_\rho^N = -7.31-11.8\sqrt{u}$	$b^\Delta = -4.67+4.40\sqrt{u}$
$\gamma_{0-\frac{1}{2}}^{\rho\Delta} = i\gamma_{0\frac{1}{2}}^{\rho\Delta} = c^\Delta$				$b_\rho^N = 17.8-6.90\sqrt{u}$	$c^\Delta = -5.44+5.02\sqrt{u}$
	$\pi^- p \rightarrow p \pi^-$	0	1	$c_\rho^N \equiv M_\rho b_\rho^N / \sqrt{2} M_N$	$\gamma_\rho^2 / 4\pi = .37$
	$\pi^+ p \rightarrow p \pi^+$	2/3	1/3	$a_\omega^N = -3.73-17.9\sqrt{u}$	$\gamma_\omega^2 \equiv 9\gamma_\rho^2$
	$\pi^- p \rightarrow n \pi^0$	$\sqrt{2}/3$	$-\sqrt{2}/3$	$b_\omega^N = .655+.223\sqrt{u}$	
	$\pi^- p \rightarrow p \rho^-$	0	1		
	$\pi^- p \rightarrow n \rho^0$	$\sqrt{2}/3$	$-\sqrt{2}/3$		
	$\pi^+ n \rightarrow p \gamma^v$	$-\sqrt{2}G_\rho/3$	$\sqrt{2}G_\rho/3$		
	$\pi^0 p \rightarrow p \gamma^v$	$G_\rho/3$	$2G_\rho/3$		
	$\pi^+ n \rightarrow p \gamma^s$	$\sqrt{2}G_\omega/3$	0		
	$\pi^0 p \rightarrow p \gamma^s$	$G_\omega/3$	0		

<sup>a</sup> $\beta^N, \beta^\Delta, a_d^N$ , etc. are real analytic functions of  $\sqrt{u}$ .

<sup>b</sup>Identity signs,  $\equiv$ , denote "hard" constraints that were imposed on the parameters during data fitting.

where

$$M_{\lambda'\mu'; \lambda\mu}^{P,R,I} = (M_N)^{1-n-x}(i)^{1-x}[(s)^{1/2} \cos \frac{1}{2}\theta]^n (\sin \frac{1}{2}\theta)^{n'} \times C_I^R \xi_I \left[ u^{x/2} \frac{\gamma_{\lambda'\lambda}^{aI}(\sqrt{u}) \gamma_{\mu'\mu}^{d'I}(\sqrt{u})}{u^{1/2} - M_I} \left( \frac{e^{-i\pi/2 S_I}}{s_0^I} \right)^{\alpha_I(\sqrt{u})} + (\sqrt{u} - \sqrt{-u}) \right]. \quad (3)$$

The vertex functions  $\gamma$  and the coefficients  $C_I^R$  are given in Table I;  $\xi_I = \exp(\frac{1}{2}i\pi S_I)$ , where  $S_I = \alpha_I(M_I)$  is the spin of particle  $I$ . The photoproduction amplitudes are related to  $\rho$ - and  $\omega$ -production amplitudes by vector dominance; thus the  $d'$  occurring in the vertex function  $\gamma_{\mu'\mu}^{d'I}$  is  $\rho$  ( $\omega$ ) if  $d$  is  $\gamma^v$  ( $\gamma^s$ ); otherwise  $d' = d$ .

This parametrization satisfies factorization, MacDowell symmetry, nonsense decoupling,<sup>22</sup> absence of parity doublets, and real analyticity conditions. Only the  $N$  and  $\Delta$  particle poles are retained; Regge recurrences are neglected since they are far from the scattering region. We have assumed that the  $N_\gamma$  trajectory is sufficiently low lying so that its contribution can be ignored.

The Regge-cut amplitudes for reaction  $R$  are (see HKPR appendix)

$$M_{\lambda'\mu'; \lambda\mu}^{C,R} = \sum_{I=N,\Delta} M_{\lambda'\mu'; \lambda\mu}^{C,R,I},$$

where

$$M_{\lambda'\mu'; \lambda\mu}^{C,R,I} = -(8\pi)^{-1} \lambda_I^{R'} \sigma_T (1-i\rho) e^{Av/2} \int_{-\infty}^0 dv' e^{Av'/2} I_n(A(vv')^{1/2}) M_{\lambda'\mu'; \lambda\mu}^{P,R,I}(u'), \quad (4)$$

and  $R' = a + b - c + d'$ ,  $v = u - u_{\max}$ , and  $v' = u' - u_{\max}$ . Here  $\sigma_T$ ,  $\rho$ , and  $A$  are, respectively, the total cross section, real-to-imaginary ratio of the forward amplitude, and slope of  $d\sigma/dt$  for  $\pi N$  elastic scattering. Their values are taken from experiment:  $\sigma_T = 27$  mb,  $\rho = -0.15$ ,  $A = 7$  BeV<sup>-2</sup>. The constant factor  $\lambda_I^{R'}$  is meant to take approximate account of cuts arising from inelastic intermediate states; it is treated as an adjustable parameter

but is expected to be in the range  $1.5 \lesssim \lambda \lesssim 2$  (see HKPR).

The full amplitudes for reaction  $R$  are  $M_{\lambda'\mu'; \lambda\mu}^{C,R} = M_{\lambda'\mu'; \lambda\mu}^{P,R} + M_{\lambda'\mu'; \lambda\mu}^{C,R}$  and the cross section is

$$d\sigma_R/du = (64\pi q^2 s)^{-1} \sum |M_{\lambda'\mu'; \lambda\mu}^{R}|^2, \quad (5)$$

where  $\sum$  means average over initial helicities and sum over final helicities. For photoproduc-

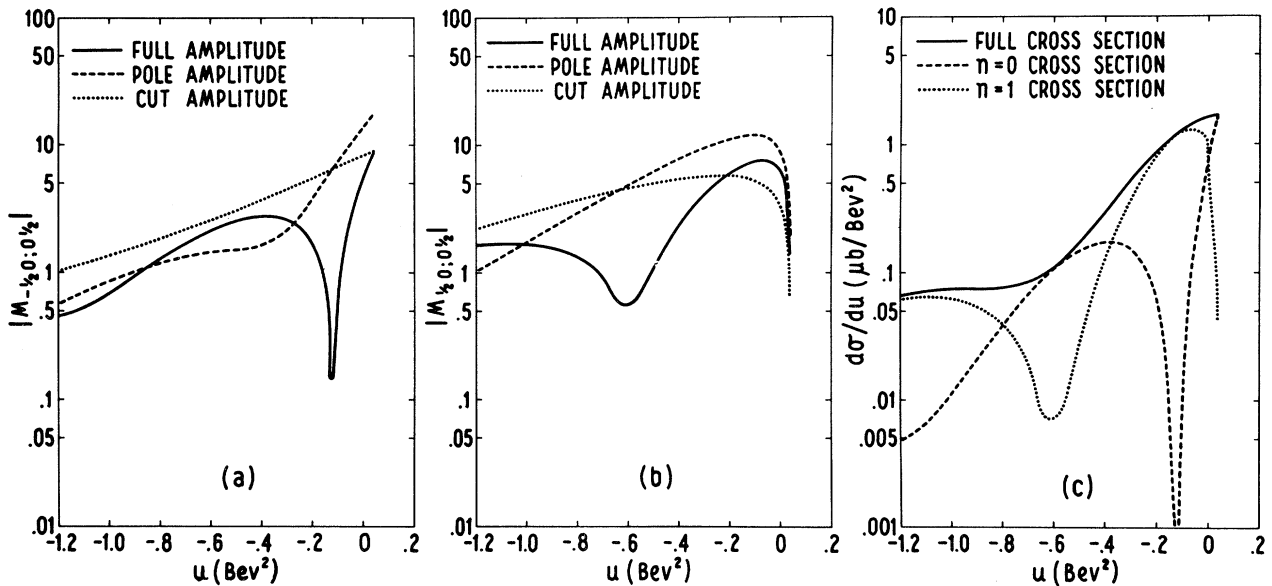


FIG. 1. All curves refer to  $\pi^- p \rightarrow p \pi^-$  amplitudes calculated with the parameters in Table I at 9.85 BeV/c. (a) Magnitude of pole, cut, and full  $n=0$  amplitudes. (b) Magnitude of pole, cut, and full  $n=1$  amplitudes. (c)  $n=0$ ,  $n=1$ , and full cross section.

tion the amplitudes in Eq. (5) should be replaced by the sum of the  $\gamma^v$  and  $\gamma^s$  parts.

The structure of some typical amplitudes is illustrated in Fig. 1. In general, helicity amplitudes must vanish at  $u = u_{\max}$  as  $(u_{\max} - u)^{n/2}$ ; so only  $n=0$  amplitudes can contribute in the backward direction. The magnitude of the cut term is usually less than that of the pole term near  $u=0$ , but the cut term has a smaller slope and eventually crosses the pole term. The pole and cut terms are about  $180^\circ$  out of phase [see Eq. (4)]; so there is a dip in the total amplitude near the point where they cross. For an  $n=0$  amplitude [Fig. 1(a)] this dip is usually in the range  $0.1 \lesssim -u \lesssim 0.25$  and is rather deep. In an  $n=1$  amplitude [Fig. 1(b)] the size of the cut term relative to the pole term is smaller than in an  $n=0$  amplitude because the integrand in Eq. (4) vanishes at  $u' = u_{\max}$  if  $n > 0$ . Thus the dip in an  $n=1$  amplitude is usually in the range  $0.5 \lesssim -u \lesssim 0.7$ . This dip is shallower than an  $n=0$  dip because the relative phase of the pole and cut moves away from  $180^\circ$  as  $-u$  increases.

Figure 1(c) shows the decomposition of the  $\pi^- p \rightarrow p \pi^-$  cross section into its  $n=0$  and  $n=1$  components. This illustrates how the full cross section can be smooth even though there are dips in the individual helicity amplitudes. Because of this "filling-in" effect we expect that there will hardly ever be deep dips in cross sections for reactions involving high-spin particles. The presence

of dips in the helicity amplitudes can be tested by measurements of spin density matrix elements. In particular, Fig. 1(c) indicates that the  $\pi^- p \rightarrow p \pi^-$  polarization is very small at  $u = -0.15$ . Measurement of this polarization would provide an important test of SCRAM. The presence of dips could also be tested by using finite-energy sum rules (FESR) for helicity amplitudes. In particular, we would predict a dip in the left-hand side of an FESR for the  $n=0$ ,  $\pi^- p \rightarrow p \pi^-$  amplitude at about  $u = -0.15$ , but no such dip in an FESR for the  $n=1$ ,  $\pi^+ p \rightarrow p \pi^+$  amplitude. NWSZ models would predict exactly the opposite.

**Fits to data.**—Our current best fit to the data on seven backward reactions is shown in Fig. 2 along with predictions at 50 BeV/c.<sup>23</sup> The  $\chi^2$  for this fit, including all data shown, is 1030 for 291 points. The worst part of the fit occurs in the  $\pi N - N\pi$  reactions. This is partially accounted for by apparent inconsistencies in the relative normalization of data from different experimental groups. For example, most of the 8-BeV/c  $\pi^- p \rightarrow p \pi^-$  data lie slightly above the 6.9-BeV/c data and this alone accounts for over 10% of  $\chi^2$ .

One of the more interesting features of the fit is the predicted structure of  $\pi^- p \rightarrow p \pi^-$  near  $u = -0.5$ . Although the precise shape of the theoretical curves should not be taken too seriously, we do believe that there will be a break in the slope of the cross section around  $u = -0.5$  due to the secondary maxima in the  $n=0$  amplitudes.

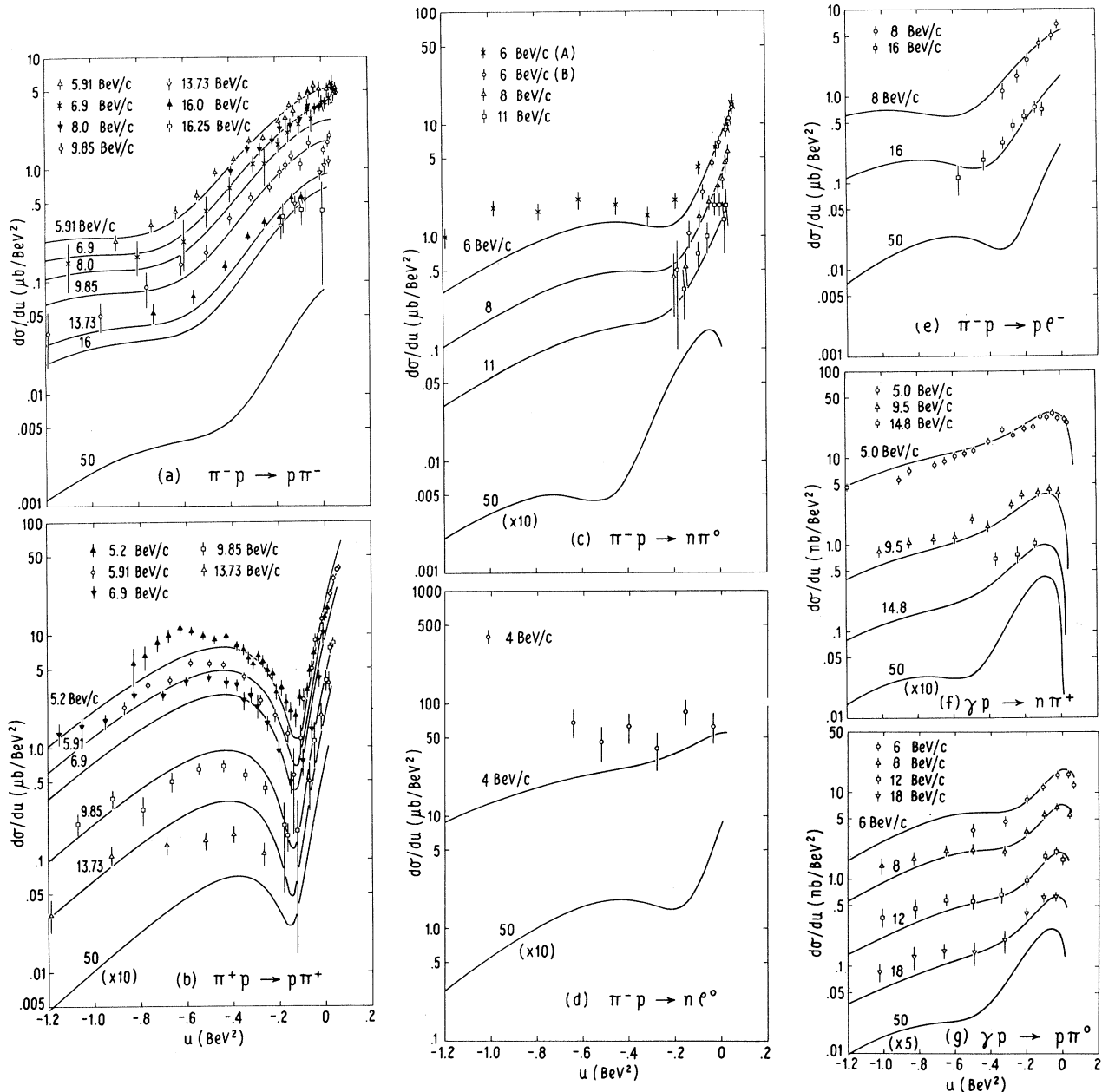


FIG. 2. Fits to data. The data shown were obtained from the following sources: (a) 5.91, 9.85, 13.73, and 16.25, Ref. 9; 6.9, Ref. 8; 8 and 16, Ref. 7. (b) 5.2 and 6.9, Ref. 8; 5.91, 9.85, and 13.73, Ref. 9. (c) 6(A), Ref. 10; 6(B), 8, and 11, Ref. 11. (d) Ref. 12. (e) Ref. 13. (f) Ref. 14. (g) Ref. 15.

The parameters used in the fit are shown in Table I. There is a total of 28 independent partially adjustable parameters for the seven reactions, or four per reaction. There were 17 "soft" constraints imposed on these parameters. These were that each trajectory should pass close to its first recurrence (2 constraints),  $1.5 \lesssim \lambda \lesssim 2$  (3 constraints),  $0.5 \lesssim s_0 \lesssim 1.5$  (2 constraints),  $\gamma_\rho^2/4\pi \approx 0.5$  (1 constraint), and that each vertex function

should extrapolate to the correct value at the  $N$  or  $\Delta$  pole (in terms of known coupling constants for  $\pi NN$ ,  $\pi N\Delta$ ,  $\rho NN$ ,  $\rho N\Delta$ , and  $\omega NN$ ) within about a factor of 2 (9 constraints). All but one of these constraints are satisfied by the parameters in Table I;  $\lambda_N^{\pi N \rightarrow N\rho}$  had to be taken anomalously large. This may be due to a strong  $\Delta + \rho$  exchange cut in  $\pi N \rightarrow N\rho$  since this cut would have about the same energy dependence as the  $N$  exchange am-

plitudes. Some support for this argument comes from a similar result for pion exchange in forward charged photoproduction.<sup>5</sup>

The quality of the fit can be improved by either relaxing some of the constraints or omitting some of the data. However, we believe that a good fit to all the data with physically meaningful parameters is more useful than a perfect fit to part of the data with unphysical parameters.

Concerning the pole extrapolations we mention in particular that we have a  $\Delta$  width of 90 MeV. The  $\Delta$  width has invariably come out too small in Regge-pole fits to  $\pi^-p \rightarrow p\pi^-$ . We can get a large value and still fit the data because the cancellation between pole and cut allows the pole amplitudes to be larger than in the absence of the cut. Still larger values can be obtained if one fits the  $\pi^-p \rightarrow p\pi^-$  data alone.

A number of topics not discussed here will be covered in a paper now in preparation. These include analysis of  $N\bar{N} \rightarrow \pi\pi$  and  $\pi N \rightarrow N\omega$ , and more detailed treatments of  $\pi N \rightarrow N\pi$  polarization and  $\pi N \rightarrow N\rho$ .

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<sup>6</sup>It is interesting that lower energy polarizations calculated with the parameters in Table I happen to agree

rather well with recently obtained data on  $\pi^+p \rightarrow p\pi^+$  polarization at 2.75 and 3.25 BeV/c: D. J. Sheriden, N. E. Booth, G. Conforto, R. J. Esterling, J. H. Parry, J. Scheid, and A. Yokasawa, to be published.

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<sup>19</sup>J. V. Beaupre and E. A. Paschos, Stanford Linear Accelerator Center Report No. SLAC-PUB-655, 1969 (to be published). In this model only charged photoproduction is dominated by  $N\gamma$  exchange at  $u = -0.15$ .

<sup>19a</sup>After this Letter was submitted we received a Cornell preprint with new data on  $\pi^-p \rightarrow n\pi^0$  at 6 BeV/c. These data exhibit a  $u = 0$  cross-section and a dip position consistent with the Minnesota data (Ref. 10) but have a deeper dip depth. J. P. Boright *et al.*, Phys. Rev. Letters **24**, 964 (1970).

<sup>20</sup>C. B. Chiu and J. Finkelstein, Nuovo Cimento **59A**, 92 (1969).

<sup>21</sup>The formalism in this section is necessary and sufficient for the interested reader to reproduce our results.

<sup>22</sup>Nonsense decoupling at the nucleon pole requires that  $b_\rho^N = M_\rho c_\rho^N / \sqrt{2} M_N$  at  $\sqrt{u} = M_N$ . In data fitting we have imposed this condition at all values of  $\sqrt{u}$ . A similar condition holds with  $\rho$  replaced by  $\omega$ , but  $c_\omega^N$  does not enter into the seven reactions fitted here.

<sup>23</sup>Experiments at momenta like 50 BeV/c may yield the first observation of a higher lying Regge trajectory ( $\Delta$ ) taking over dominance from a lower lying but more strongly coupled trajectory ( $N$ ) which dominates at lower energies. This may first become evident in  $\pi^-p \rightarrow n\pi^0$  which has a fairly strong  $\Delta$  contribution even at lower energies.