LIMIT OF CROSS SECTIONS AT INFINITE ENERGY*

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At infinite energy, we predict: (1) σ_{tot} approaches infinity; (2) the ratio of the real part to the imaginary part of the forward elastic amplitude approaches zero; (3) σ_{el}/σ_{tot} approaches $\frac{1}{2}$; (4) the width of diffraction peak approaches zero; its product with σ_{tot} is a constant. We give theoretical evidence based on massive quantum electrodynamics as well as experimental evidence in support of these predictions, and a physical picture for high-energy scattering.

With the construction of larger and larger accelerators, it is hoped that the mysteries of high-energy scattering will unfold in the near future. It is therefore particularly urgent that theoretical progress will match the experimental progress in stride. On the basis of our results in quantum electrodynamics,[1-4] we shall make in this Letter a number of predictions on some of the fundamental questions in high-energy physics.[5] While these predictions cannot be rigorously proven in a mathematical way, the reasoning that leads to them is based on physical insights learned from field theory, and are sufficiently convincing to warrant attention. For clarity of presentation, we shall first list these predictions. Theoretical evidence in support of them will follow next. Experimental evidence as well as a physical picture for high-energy scattering will be given lastly.

For any hadron-hadron scattering process at extremely high energies, we make the following predictions:

Forward direction.—Let us denote the forward elastic scattering amplitude by $\mathfrak{M}(s,0)$, where s is the square of the c.m. energy of the system. We predict:

$$\frac{\mathrm{Re}\mathfrak{M}(s,0)}{\mathrm{Im}\mathfrak{M}(s,0)}=\frac{\pi}{\ln|\mathcal{S}|}+O((\ln|\mathcal{S}|)^{-2}), \tag{1}$$

and

$$\sigma_{tot}=2\pi R^2+O(\ln|\mathcal{S}|), \tag{2}$$

where

$$\mathcal{S}=\left\{-\frac{(-s)^a}{[\ln(-s)]^2}+\frac{(-u)^a}{[\ln(-u)]^2}\right\}^{1/a}, \tag{3}$$

with u the third Mandelstam variable and a a positive constant, and

$$R=R_0\ln|\mathcal{S}|, \tag{4}$$

with R_0 a constant independent of the energy.

Nonforward directions.—The usual diffraction peak is expected for the elastic-scattering amplitude around the forward direction. Furthermore, let Γ be the value of $-t$ at which the first dip occurs, where t is the negative of the momentum-transfer squared. We predict:

$$\Gamma\sigma_{tot}=2\pi^3\beta_1^{\,2}+O((\ln|\mathcal{S}|)^{-1}), \tag{5}$$

independent of the processes,

$$\sigma_{el}=\pi R^2+O(\ln|\mathcal{S}|), \tag{6}$$

and

$$\sigma_{el}/\sigma_{tot}=\tfrac{1}{2}+O((\ln|\mathcal{S}|)^{-1}). \tag{7}$$

In (5), $\beta_1=1.2197$ is the first zero of $J_1(\beta\pi)$. Thus

$$2\pi^3\beta_1^{\,2}=92.254=35.92\text{ mb }(\mathrm{BeV}/c)^2. \tag{8}$$

Predictions (2), (5), (6), and (7) are model independent and are believed to be firm, while (1) is a specific result of quantum electrodynamics, and may be modified in strong interactions.

The above predictions mark a drastic departure from current concepts of high-energy scattering. This is why we were slow in recognizing its full significance when the relevant formula was first found.[2] After more than a year of deliberation, we now realize that the above predictions constitute the only answer supported by all theoretical evidences from field theory. We shall next write down the basic formula and list all the evidence.

At high energies, the fermion-fermion elastic-scattering amplitude was found to be[1,2]

$$\tfrac{1}{2}im^{-2}s\,\delta_{12}\delta_{1'2'}\int d\vec{x}_\perp e^{i\vec{\Delta}\cdot\vec{x}_\perp}(1-e^{-A}). \tag{9}$$

In (9), m is the mass of the fermion, δ_{12} and $\delta_{1'2'}$ are Kronecker deltas of the spins, $\vec{\Delta}$ is

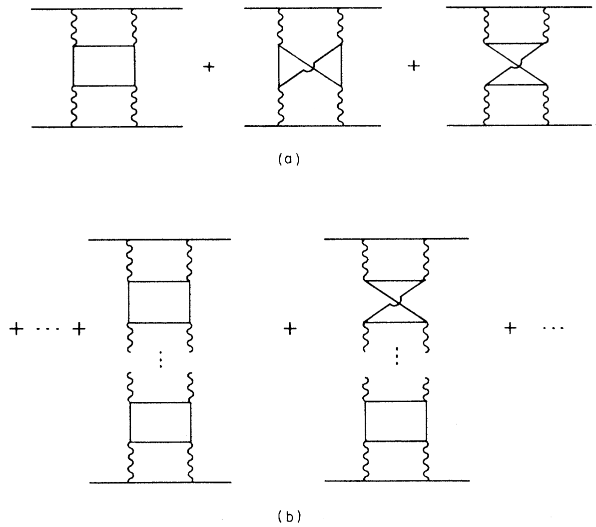

FIG. 1. The one-tower diagrams.

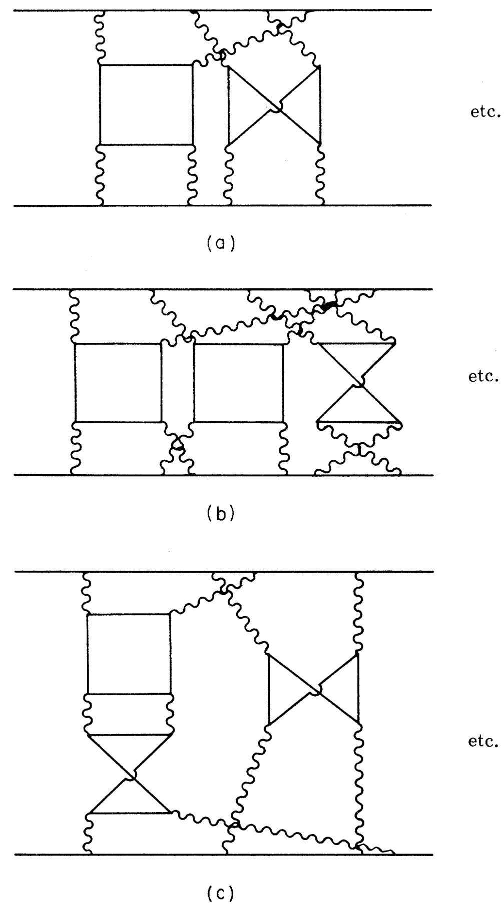

FIG. 2. Examples of the multitower diagrams.

the momentum transfer, and A is related to the Fourier transform of the asymptotic amplitude due to the sum of one-tower diagrams illustrated in Fig. 1. Specifically, if we expand $(1-e^{-A})$ in (9) into a Taylor series in A, the term in (9) proportional to A is the high-energy amplitude due to the sum of the one-tower diagrams. In fact, the A^N term in (9) is the high-energy amplitude for the sum of the N-tower diagrams. [Two of the two-tower diagrams are illustrated in Figs. 2(a) and 2(c), and a three-tower diagram is illustrated in Fig. 2(b).]

Equation (9) was obtained by summing up the leading terms in all multitower diagrams. Although this method has no mathematical justification, we believe that (9) is correct by the following steps of reasoning:

(1) The one-tower diagrams with n loops are the lowest-order diagrams which yield terms of the order of $s(\ln s)^n$.[6] By the optical theorem, the corresponding cross section for the production of n pairs is proportional to $(\ln s)^n$ for large s. Summing up these cross sections over n, we get a total cross section of the order of $s^a(\ln s)^{-2}$, where $a=11\alpha^2\pi/32$.[7-9] The fact that this result is larger than any power of $\ln s$ cannot be blamed on the process of summing only the leading terms. This is because the cross section for n-pair production is always positive. By choosing n sufficiently large, the corresponding n-pair production cross section is already larger than any given power of $\ln s$. Thus the sum must also be larger than any power of $\ln s$.

(2) To understand this large result $[s^a(\ln s)^{-2}]$, we must first recognize that the appearance of the logarithmic factors is due to the creation of low-energy pairs in the c.m. system.[3] It therefore bears a strong resemblance to the familiar infrared divergence in quantum electrodynamics. We recall that a charged particle in an external field can emit soft photons, and the number of logarithmic divergences is equal to the number of soft photons emitted. The scattering cross section is thus infinite, and experimental resolutions must be introduced to obtain finite answers. High-energy scattering processes closely parallel this situation in the following way: If the kinetic energy of the charged particle is T and the mass of the photon is λ, then the maximum number of photons that the charged particle can emit is kinematically limited to T/λ. Only in the physical case of massless photons can an infinite number of photons be emitted. In a high-energy scattering process, the maximum number of pairs that are kinematically allowed to be created in the c.m. system is ω/m, where ω is the

c.m. energy of one of the incident particles. In the limit of infinite energy, this number is also infinite. Thus the limit of $\omega \to \infty$ parallels that of $\lambda \to 0$. Since it is well known that the cross section for a charged particle scattered by an external field is more divergent than any power of $\ln\lambda$ due to the emission of an arbitrary number of soft photons, the cross section for high-energy scattering must be larger than any power of $\ln\omega$ due to the creation of slow particles in the c.m. system, i.e., due to pionization.[3] Thus we believe that the previously obtained answer of the order of $s^a/(\ln s)^2$ is correct. In strong interactions, the numerical value of a may be of the order of 1.

(3) This result $s^a/(\ln s)^2$ is not to be interpreted as a violation of the Froissart bound.[10] Rather, it ought to be regarded as a realization of the strongly absorptive "potential" with a coupling constant increasing with energy, as conceived in Froissart's original paper.[10] Thus, in a two-particle scattering process, if the interaction takes place at a sufficiently close transverse distance from the center of the target, the incident particle creates slow particles in the c.m. system and is lost to the beam. If the transverse distance involved is large, the incident particle does not necessarily create pionization products and may survive. Mathematically, this is achieved by including not only the one-tower diagrams but also all of the multitower diagrams as well. The resulting amplitude is (9). In the limit $s \to \infty$ and $|\vec{x}_\perp| = O(\ln s)$, we have[11]

$$A \sim b i^{-1} s^a e^{-\mu|\vec{x}_\perp|}, \tag{10}$$

where b and μ are real constants. In fact, $\mu < 2\lambda$, where λ is the mass of the vector meson.

(4) The inclusion of the multitower diagrams is not arbitrary, but has a physical basis. Since (9) is in the form of the impact-parameter representation, the quantity $1-e^{-A}$ is the opacity at the distance and energy at which A is evaluated. For a large fixed s, we may choose $|\vec{x}_\perp|$ sufficiently large so that A is very small. Then $1-e^{-A} \sim A$. Thus at sufficiently large transverse distances, A is proportional to the Fourier transform of the high-energy scattering amplitude. Since very little scattering occurs at this distance, the contribution of any diagram to A is always small. In fact, the diagrams with higher thresholds in the t channel are in general expected to contribute less to A for large $|\vec{x}_\perp|$. Thus the important diagrams are those with two-vector-meson cuts in the t channel. The one-tower diagrams are the leading diagrams of this kind, and the inclusion of other diagrams of this kind merely modifies the kernel $\mathcal{K}$ in Ref. 8. We therefore conclude that (10) is correct when $|\vec{x}_\perp|$ is large, although the constants a, b, and μ should be modified if the coupling is strong. If we now fix $|\vec{x}_\perp|$ and increase s, the right-hand side of (10) increases. This means that the interaction extends into larger and larger transverse distances as the energy increases. When the energy is sufficiently high or the transverse distance is not large enough, A becomes appreciable, and we may not approximate $1-e^{-A}$ by A. This means that, no matter what happens to an incident particle, it is, at most, taken from the beam and cannot yield a large cross section that increases as some power of the energy. In this way the Froissart bound is obeyed. As shown in Ref. 2, this exponentiation is achieved by the exchange of two, three, or more towers. It is interesting to note that, while the amplitude from the sum of one-tower diagrams is so large that unitarity is violated, the amplitude from the sum of two-tower diagrams is even larger. Fortunately, these two amplitudes are of opposite sign. In this way unitarity is restored.

(5) Finally, we emphasize that (9) has (i) all leading terms, (ii) unitarity, (iii) analyticity, and (iv) crossing symmetry.

The above considerations lead to a very dramatic physical picture. This picture is perfectly general and holds for all hadron-hadron scattering processes. At extremely high energies, a particle acts like a Lorentz-contracted pancake which can be roughly separated into the following two regions: (i) a black core (completely absorptive) with a radius R given by (4) which expands with energy [this core contributes the leading terms in Eqs. (1), (2), (5), (6), and (7)]; (ii) a gray fringe (partially absorptive) which extends further out and contributes to the next-order terms in these equations.

It is perhaps instructive to compare the present physical picture with those from the Regge-pole model[12-16] and the droplet model.[17-19] In each case, a particle is pictured either as a sphere or a disk, depending on the coordinate system used. In the Regge-pole model, the disk becomes larger and more transparent as energy increases; in the droplet model, the properties of the disk are independent of the energy at high energies; in the present physical picture, the disk becomes larger and more absorptive as energy increases. In particular, both in the Regge-pole model and in the present physical picture, the diffraction peak shrinks in width as energy increases. However, the present picture differs from both the Regge-pole model and the droplet model in the high-energy behavior of σ_{tot}, the ratio of σ_{el} to σ_{tot}, and also the position of the dip in elastic scattering.

While much about the hard core is model independent, it is not so for the gray fringe. For the purpose of comparing with experiment, we must remember that, for proton-proton scattering at 30 BeV, the quantity $\ln[s/(\ln s)^2]$ is only about 1.3 if the energy scale is taken to be due to the rest mass of the proton. Thus the contribution of the gray fringe is comparable with that of the black core in this energy range; indeed the black core may not be black yet. Without a precise knowledge of the gray fringe, our predictions cannot be tested against the existing experimental data. For example, in view of our results from quantum electrodynamics, we can take A to be of the form (10). With this assumption, the contribution from the gray fringe can then be obtained precisely. Instead of dwelling on the possible assumptions, we shall briefly comment on some of the important aspects in the trends of high-energy experiments.

Total cross sections.—The total cross sections of $\pi^- $-$p$, π^--n, K^--p, and K^--n have all become energy independent in the region from 30 to 65 BeV.[20] No upward trend is noted yet, although the experimental data may be in agreement with slowly rising cross sections.[20]

The ratio $\mathrm{Re}\mathfrak{M}(s,0)/\mathrm{Im}\mathfrak{M}(s,0)$.—This ratio is still negative at 25 BeV for both $\pi^\pm$-p[21] and p-p[22] scattering. For both the π^+-p and π^--p cases, there is a definite trend for this ratio to rise above zero. The trend for the p-p case is not well established.

The product $\Gamma\sigma_{tot}$.—For proton-proton scattering at 19.2 BeV, this product is 39.5 mb $(\mathrm{BeV}/c)^2$,[23,24] which agrees quite well with (8). Since the diffraction peak continues to shrink[25] at 70 BeV, the agreement ought to improve at this energy.

The ratio σ_{el}/σ_{tot}.—Experimentally, in the region 8-20 BeV, this ratio is approximately 17% for both π^+-p and π^--p scattering, and 26% for p-p scattering.[26] By using the model of quantum electrodynamics, the terms of the orders of $\ln|s|$ and 1 in both (2) and (6) can be obtained explicitly. These terms are positive for σ_{tot} and negative for σ_{el}. As a result, at intermediate energies this ratio σ_{el}/σ_{tot} is much smaller than $\frac{1}{2}$, its value at infinite energy. At 20 BeV, this ratio is estimated to be very roughly in the neighborhood of 20%.

In conclusion, we emphasize the uniqueness of the present physical picture from our knowledge of high-energy behavior in field theory. Confrontation with the experimental data in the next few years will be most exciting.

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IMPLICATIONS OF LOCAL DUALITY IN MESON-DECUPLET SCATTERING*

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The consequences of the assumption that the imaginary parts of the direct-channel helicity amplitudes vanish in the forward (backward) scattering if the crossed t channel (u channel) is exotic are examined in the case of meson-decuplet scattering. Confining ourselves to a limited energy region above the threshold, we show that the solutions of the set of equations provided by different external helicities predict the existence of certain patterns of octet and decuplet baryons degenerate in mass and having spin and parity assignments $\frac{5}{2}^{\pm}$, $\frac{3}{2}^{\pm}$, $\frac{1}{2}^{\pm}$.

Recently several authors[1,2] have attempted to predict "exchange degeneracy" patterns for hadronic trajectories on the basis of crossing, SU(3) symmetry, and duality. The scattering amplitude is assumed to consist of two parts: (1) a primarily diffractive background, and (2) a resonant part, the imaginary part of which is expressible at high energies in terms of Regge trajectories in the crossed channels. Hence if a particular scattering amplitude is characterized by internal quantum numbers for which no resonances exist, the Regge trajectories in the crossed channels must exhibit exchange degeneracy so that the corresponding imaginary parts cancel.

In the present note, we investigate the consequences of duality in a local energy region in the case of of the reactions $P+D\rightarrow P+D$, where P and D represent, respectively, the octet of pseudoscalar mesons and the decuplet of $J^P=\frac{3}{2}^+$ baryons. Following Mandula, Weyers, and Zweig,[3] we assume that there is a region of s and small t on the one hand, and a region of s and small u on the other, within which the imaginary part can be calculated in two alternate ways, in terms of direct-channel resonances or in terms of Regge trajectories of the crossed channels. Therefore, if we select the s-channel reactions so that their t or u channels are characterized by exotic quantum numbers (viz., 27 in the t channel, 27 and 35 in the u channel), the imaginary parts due to s-channel resonances must add up to zero. We consider a set of direct-channel resonances degenerate in mass but with different spins and parities and examine the constraints on their coupling constants implied by the above requirement in the forward and the backward scattering. We show by including considerations of spin that the particles on leading trajectories must be accompanied by daughters with prescribed ratios of coupling constants between the parent and the daughter particles. We discuss the general pattern of the nontrivial solutions and compare the results with experiments in the case of nonstrange baryons.

The imaginary parts of the six independent s-channel helicity amplitudes free of kinematic singularities[4] can be written as

$$\mathrm{Im}\overline{G}^{R}_{0\mu;0\lambda}(s,\theta_s)=K(s)\sum_{JLL'R'}X^{RR'}(2L'+1)^{1/2}(2L+1)^{1/2}C(L'\tfrac{3}{2}J;0\mu)C(L\tfrac{3}{2}J;0\lambda)\chi_{R'}{}^{J}(L')\chi_{R'}{}^{J}(L)d_{\lambda\mu}{}^{J}(\theta_s), \quad (1)$$

where R characterizes the irreducible SU(3) representation of the crossed t or u channel and $X_{R'}{}^{R}$ are the elements of the corresponding crossing matrix. L,λ (L',μ) denote the orbital angular momentum and helicity of the initial (final) state, and $K(s)$ is a kinematic factor. The χ^J's are related to the partial widths through

$$\chi^J(L)=(\Gamma_{PD}{}^{J}\Gamma^J)^{1/2}\{[(W^*-W_{\mathrm{c.m.}})/\tfrac{1}{2}\Gamma]^2+1\}^{-1/2}, \quad (2)$$

where Γ^J is the total width and $\Gamma_{PD}{}^{J}$ the partial width of a resonance of angular momentum J and mass