~ 250 nsec in duration.

[8]This cross section is based on the recent analysis by F. T. Avignone of all available data on the beta spectrum from fission. We are grateful to Dr. Avignone for communication of his results prior to publication. See also, F. T. Avignone, III, S. M. Blakenship, and C. W. Darden, III, Phys. Rev. 170, 931 (1968).

[9]The $\bar{\nu}_e + p$ reaction in the liquid anticoincidence detector can give a signal in the plastic by gammas from capture of the product neutron in the NaI which surrounds the plastic. An improved experiment now under way will effectively eliminate this source of background by adding a thin Cd sheet external to the 3.8-cm Pb shield which absorbs the neutron-capture gammas prior to their entry into the NaI and the plastic detector. Consideration of the signal from $\bar{\nu}_e + p$ in the plastic target itself gives hope of adequately discriminating against this source of background as well via the two 0.51-MeV annihilation gammas and the associated neutron.

HADRON-NUCLEUS TOTAL CROSS SECTIONS

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Total cross sections for collisions between high-energy hadrons and nuclei are calculated using both Woods-Saxon and Gaussian density distributions for the nuclei. Comparisons with measurements are made. Errors incurred by truncating multiple-scattering series are investigated.

Many analyses of high-energy hadron-nucleus collisions have been based upon a diffraction approximation due to Glauber.[1] This approximation is most accurate for small-angle collisions. Consequently it is not unreasonable to expect that high-energy hadron-nucleus total cross sections, which depend only upon the corresponding hadron-nucleus forward elastic scattering amplitudes, could be calculated quite reliably for a given nuclear model. Such calculations have been carried out for a simple model in which nuclei are described by Gaussian density distributions.[2-4] Such a model, although quite unrealistic, is extremely useful because it leads to an analytic expression for the total cross sections which exhibits qualitative features that are likely to reappear in more realistic calculations.

We have performed analyses of total cross sections with a model in which the nuclei have Gaussian density distributions and with a model in which the nuclei have Woods-Saxon shapes for their density distributions. The quantitative results obtained with these two models differ by as much as 15%. Although such differences may seem relatively small, they are significant since the measurements have uncertainties considerably smaller than 15%. Nevertheless the qualitative results are clearly not very sensitive to the nuclear model. Consequently our predictions could serve as a rather severe test of the basic theory. Alternatively, if we have confidence in the theory, our predictions could serve as a test of the reliability of total cross-section measurements. Since a number of high-energy hadron-nucleus total cross-section measurements have been made in recent years,[5] we have calculated these cross sections and compared them with the data. In addition, we have investigated the speed with which the multiple-scattering series "converges" by finding the number of terms of the series that must be retained in order to obtain a numerically accurate result. Lastly, we have calculated the ratios of the real to imaginary parts of hadron-nucleus forward elastic scattering amplitudes, quantities which are of importance in analyses of the contribution of the interference between Coulomb and nuclear elastic amplitudes to the measured total cross sections. A more detailed presentation of our investigations and results will be given elsewhere.

The amplitude for collisions in which the nucleus makes a transition from an initial state i to a final state f and has momentum $\hbar q$ imparted to it may be written[1,6]

$$F_{fi}(\vec{q}) = (ik/2\pi)\int e^{i\vec{q}\cdot\vec{b}}\langle f|[1-e^{i\chi_{\rm tot}(\vec{b},\vec{r}_1,\cdots,\vec{r}_A)}]|i\rangle d^2b, \tag{1}$$

where $\hbar k$ is the momentum of the incident hadron, $\vec{r}_1, \cdots, \vec{r}_A$ are the coordinates of the target nucleons, and $\chi_{\rm tot}$ is a phase-shift function. The integration is over the plane of impact-parameter vectors $\vec{b}$

perpendicular to the direction of the incident beam. If we assume that χ_{tot} may be expressed as a sum of the individual hadron-nucleon phase-shift functions, which in turn may be expressed in terms of Fourier transforms of the corresponding hadron-nucleon elastic-scattering amplitudes $f_j(\vec{q})$,[1,6] we find

$$F_{fi}(\vec{q})=\frac{ik}{2\pi}\int e^{i\vec{q}\cdot\vec{b}}\left\langle f\left|\left\{1-\prod_{j=1}^{A}\left[1-\frac{1}{2\pi ik}\int e^{-i\vec{q}'\cdot(\vec{b}-\vec{s}_j)}f_j(\vec{q}')d^2q'\right]\right\}\right|i\right\rangle d^2b. \quad (2)$$

The forward elastic scattering amplitude may then be written as

$$F_{ii}(0)=\frac{ik}{2\pi}\int\psi_i^*(\vec{r}_1,\cdots,\vec{r}_A)\left\{1-\prod_{j=1}^{A}\left[1-\frac{1}{2\pi ik}\int e^{-i\vec{q}\cdot(\vec{b}-\vec{s}_j)}f_j(\vec{q})d^2q\right]\right\}\psi_i(\vec{r}_1,\cdots,\vec{r}_A)\prod_{j=1}^{A}d\vec{r}_j d^2b, \quad (3)$$

where $\psi_i(\vec{r}_1,\cdots,\vec{r}_A)$ is the ground-state wave function of the nucleus. As it stands, Eq. (3) may be regarded as an integral of dimension $3A+2$ (with two dimensions being contributed by the d^2b integration), whose integrand consists of terms involving a two-dimensional Fourier transform of $f_j(\vec{q})$.

Since we shall be dealing only with the forward elastic scattering amplitude, we shall ignore the constraint upon the center of mass of the nucleus. In general, the center-of-mass correction is significant only at angles away from the forward direction. To effect some simplification in the multidimensional integral (3), we let the ground-state wave function be a simple product wave function whose single-particle densities $\rho_j(\vec{r}_j)$ are all equal, so that

$$|\psi_i(\vec{r}_1,\cdots,\vec{r}_A)|^2=\prod_{j=1}^{A}\rho(\vec{r}_j). \quad (4)$$

Furthermore we let the hadron-proton and hadron-neutron elastic-scattering amplitudes be equal, so that $f_j(\vec{q})=f(\vec{q})$. If we assume spherical symmetry for the total density distributions of nuclei so that $\rho(\vec{r})=\rho(r)$, Eq. (3) may be written as

$$F_{ii}(0)=ik\int_0^\infty\{1-[1-(ik)^{-1}\int_0^\infty J_0(qb)f(q)S(q)qdq]^A\}\,bdb, \quad (5)$$

where $AS(q)$ is the nuclear form factor given by

$$S(q)=(4\pi/q)\int_0^\infty r\sin qr\rho(r)dr. \quad (6)$$

Equation (5) involves only three separate one-dimensional integrations and is amenable to numerical evaluation.

At high energies hadron-nucleon elastic-scattering amplitudes at small angles may be described by the form

$$f(q)=[(i+\alpha)k\sigma_N/4\pi]\exp(-\tfrac{1}{2}\beta q^2), \quad (7)$$

where σ_N is the hadron-nucleon total cross section, α is the ratio of the real to imaginary part of the forward elastic scattering amplitude, and β is the slope of the hadron-nucleon elastic-scattering intensity near the forward direction. If we use Eqs. (5) and (7) and the optical theorem, we find the hadron-nucleus total cross sections are given by

$$\sigma=4\pi\,\mathrm{Re}\int_0^\infty\{1-[1-(1-i\alpha)\sigma_N/4\pi\int_0^\infty J_0(qb)e^{-\beta q^2/2}S(q)qdq]^A\}\,bdb. \quad (8)$$

We have calculated hadron-nucleus total cross sections for a wide variety of hadrons, including protons, neutrons, pions, and K mesons. The analyses and the data extend over an incident-momentum range of 1-27 GeV/c. Our calculations contain no adjustable parameters. The parameters of the Woods-Saxon density distributions which we use in our analyses were taken from Elton,[7] and the parameters of Eq. (7) were taken from hadron-nucleon measurements. Our predictions are in excellent agreement with 1.1-GeV/c pion data,[8,9] 1.7-GeV/c proton data,[10,11] 19.3-GeV/c proton data,[12] 2.14-GeV/c neutron data,[11,13] 3.5-GeV/c neutron data,[11,14] 10-GeV/c neutron data,[5] and 7-GeV/c K-meson data.[15,16] Our predictions are not in agreement with 27-GeV/c neutron data.[17] Those measurements fall below the predicted curve to the extent of approximately 5% for deuterium and approximately 15% for heavy nuclei.

The individual cases will be discussed in detail elsewhere. As typical examples we shall present neutron data and calculations at 2.14 and 10 GeV/c. In Fig. 1 the 2.14-GeV/c neutron-nucleus data of Coor et al.[12] and Bugg et al.[11]

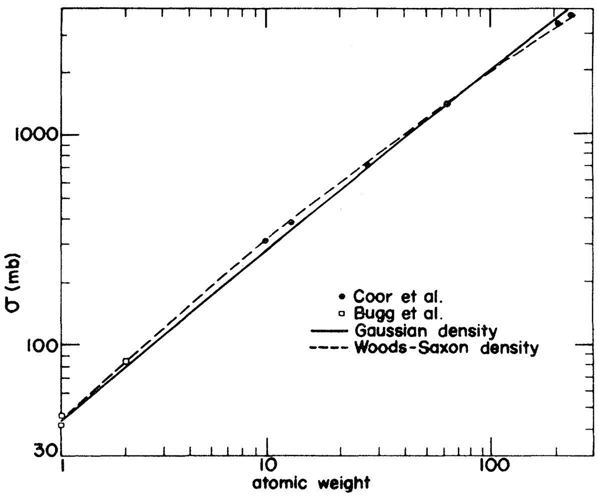

FIG. 1. Neutron-nucleus total cross sections calculated with a Woods-Saxon density distribution are compared with those calculated with a Gaussian density distribution and with the measurements of Bugg et al. (Ref. 11) and Coor et al. (Ref. 13). Both curves are obtained from Eq. (8).

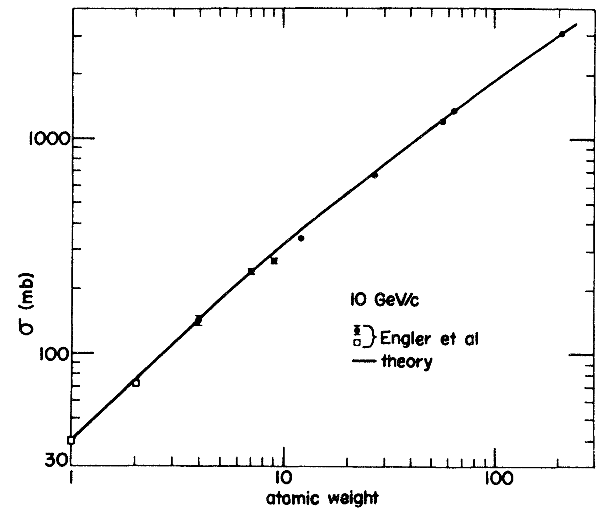

FIG. 2. Neutron-nucleus total cross sections calculated with a Woods-Saxon density distribution are compared with the measurements of Engler et al. (Ref. 5). The curve is obtained from Eq. (8).

are shown. The two curves are the predictions obtained with a Gaussian density distribution and a Woods-Saxon density distribution. We note that the variation of σ with A is not correctly given by the Gaussian distribution, and no acceptable variation of the nuclear radius with A will improve the results significantly. However, the Woods-Saxon distribution gives results which are in agreement with the measurements. In Fig. 2 the 10 GeV/c neutron-nucleus data of Engler et al.[5] are shown. The curve is the prediction obtained with a Woods-Saxon density distribution and is in good agreement with the measurements.

The density distribution $\rho(r)$ does not possess a sharp cutoff. Furthermore if we let the integrand in Eq. (8) be denoted by $b\Gamma(b)$, a study of the properties of $\Gamma(b)$ shows that $\mathrm{Re}\,\Gamma$ decreases less rapidly from its value at $b=0$ than does $\rho(r)$ from its value at $r=0$. Consequently the σ-A curves in Figs. 1 and 2 must have slopes greater than $\frac{2}{3}$ (i.e., $\sigma > \sigma_1 A^{2/3}$). This is indeed also observed experimentally. However, for heavy nuclei $\Gamma(b) \approx 1$ over a large range of values of b, and the bulk of the contribution to the cross section comes from these values of b.[18] Consequently the slope of the σ-A curve should be closer to $\frac{2}{3}$ in the region corresponding to the heavier nuclei than in the region of the lighter nuclei, and the second derivative of the σ-A curves should be negative. This is readily observed in Figs. 1 and 2 in the calculations using the Woods-Saxon distribution, and is even detectable in the calculation in Fig. 1 for which the more rapidly decreasing Gaussian distribution was used.

Equation (8) for σ may be expanded in powers of σ_N and the resulting series will contain A terms. Each term represents the effect of a particular order of multiple scattering. For heavy nuclei there will be many terms in the series. There are several geometrical arguments which lead to estimates of the number of significant terms in the series. These arguments all yield approximately the same result. We shall present one of them here. Let us imagine the nucleus to consist of A nucleons represented by spheres of radius r and let them be distributed uniformly throughout a sphere of radius $R=r_0A^{1/3}$. The number of nucleons per unit volume is then $A/\frac{4}{3}\pi R^3 = 3/4\pi r_0^3$. Now at high energies most of the hadron-nucleon collisions within the nucleus will be small-angle collisions, particularly if there is no net deflection. Consequently it is reasonable to expect that in the great majority of cases a hadron would undergo at most that number of collisions it would have experienced had it traversed the nucleus along its diameter. That number is the product of the number of nucleons per unit volume and the effective volume swept out by the hadron as it travels along a nuclear diameter. The effective volume is the product of the diameter $2R$ and the effective cross-sectional area of the moving hadron. If, for simplicity, we assume the radius of the hadron and the radius of nucleons are equal, then the effective cross-sectional area of the incident hadron is $\pi(2r)^2$ or $4\pi r^2$. Therefore, the number of collisions encountered by the hadron is $(r/r_0)^2 6A^{1/3}$. If we use the generally accepted values $r_0 = 1.1$ fm and $r \approx 0.8$ fm we obtain a result of approximately $3A^{1/3}$ as the maximum number of significant terms in the multiple-scattering series.

If we write the total cross section (8) as the sum

$$\sigma = \sum_{l=1}^{A} g_l(A), \qquad (9)$$

where $g_l(A)$ is the term in the expansion of Eq. (8) containing the factor $\sigma_N^{\,l}$, and if we define partial cross sections

$$\sigma(j, A) = \sum_{l=1}^{j} g_l(A), \qquad (10)$$

we may obtain an estimate of the speed of "convergence" of the multiple-scattering series by investigating $\sigma(j, A)$ as a function of j. This is done in Fig. 3 for $A = 180$, where we have used a Gaussian density distribution. We see that excellent numerical accuracy is obtained for j slightly smaller than $3A^{1/3}$. Although the arguments leading to our estimate of $3A^{1/3}$ assume $A \gg 1$, we find that excellent numerical accuracy is obtained for j slightly smaller than $3A^{1/3}$ for all nuclei.[19] Furthermore we note from Fig. 3

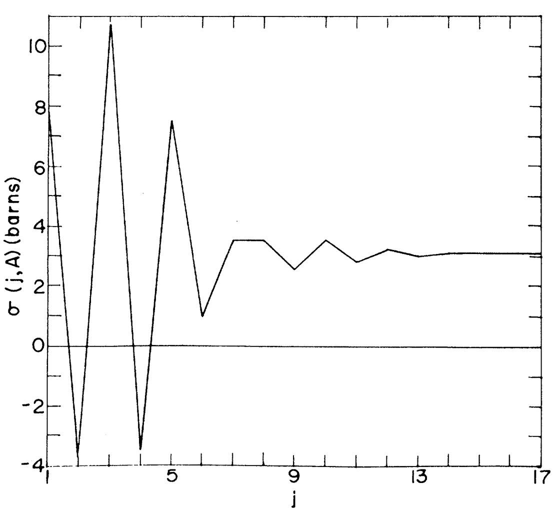

FIG. 3. Partial cross sections, given by Eq. (10), for an $A = 180$ nucleus. The single-scattering cross section is $\sigma(1,180)$. The total cross section is $\sigma(180, 180)$. The ordinates for successive integral values of j are connected by straight lines. Note that $\sigma(2,180)$ and $\sigma(4,180)$ are negative.

that a premature truncation of the series could lead to grossly erroneous (and occasionally even negative) values for the calculated cross section, since the truncation makes the corresponding S-matrix nonunitary.

We conclude by noting that high-energy interactions are likely to be more absorptive for heavier target nuclei than for lighter nuclei and, therefore, we expect the ratio $|\mathrm{Re}F_{ii}(0)|/\mathrm{Im}F_{ii}(0)$ to decrease with increasing A. Calculations we have done show this to be the case. The ratio decreases by ~50% when we vary A from unity to ~200.

We are grateful to Dr. Charles Critchfield for the hospitality extended us during a visit to the theoretical division of the Los Alamos Scientific Laboratory where this work was begun and where some of the numerical calculations were performed.

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[5]See, for example, J. Engler *et al.*, Phys. Letters 27B, 599 (1968), and 28B, 64 (1968).

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[15]W. L. Iakin *et al.*, Stanford University Report No. HEPL (A)123, 1969 (to be published).

[16]W. Galbraith *et al.*, Phys. Rev. 138, B913 (1965).

[17]L. W. Jones *et al.*, Phys. Letters 27B, 328 (1968).

[18]This property of Γ implies a relative insensitivity of the total cross section to any fine details of the density distribution.

[19]Technically we may claim this is true even for deuterium if we are willing to admit that 2 is slightly smaller than $3(2)^{1/3} \approx 3.8$. Since, however, the Glauber approximation stops with double scattering in the case of deuterium, discussion of the speed of "convergence" of the series (9) for $A = 2$ is not relevant here.