Upon integration of Eq. (15), we get the usual expression

$$\langle n(x)n(y)\rangle = n_0^2 + f(\vec{R}, \sigma(\tau)), \qquad (16)$$

which result may be obtained by a direct evaluation of the correlation function.

The author wishes to thank Dr. Charlie Harper and Dr. Charles Ray Smith for helpful suggestions and comments.

*Work supported in part by the U. S. Atomic Energy Commission and in part by the Air Force Office of Scientific Research, under Grant No. AF-AFOSR-68-1616A.

[1]N. N. Bogolyubov, Jr., and B. I. Sadovnikov, Zh. Eksperim. i Teor. Fiz. 43, 677 (1962) [Soviet Phys. JETP 16, 482 (1963)].

[2]R. Kubo, J. Phys. Soc. Japan 12, 570 (1957).

[3]B. I. Sadovnikov, Dokl. Akad. Nauk SSSR 164, 785 (1965) [Soviet Phys. Doklady 10, 934 (1966)].

[4]B. I. Sadovnikov, Dokl. Akad. Nauk SSSR 164, 1024 (1965) [Soviet Phys. Doklady 10, 953 (1966)].

[5]B. I. Sadovnikov, Physica 32, 858 (1966).

[6]J. C. Herzel, J. Math. Phys. 8, 1650 (1967).

[7]N. N. Bogolyubov and S. V. Tyablikov, Dokl. Akad. Nauk SSSR 126, 53 (1959) [Soviet Phys. Doklady 4, 589 (1959)].

[8]D. N. Zubarev, Usp. Fiz. Nauk. 71, 71 (1960) [Soviet Phys. Usp. 3, 320 (1960)].

EVIDENCE FOR THE VALIDITY OF THE LANDAU THEORY OF LIQUID He^3

Allen Widom and James L. Sigel
Physics Department, Northeastern University, Boston, Massachusetts 02115
(Received 27 April 1970)

We observe that the Landau theory of liquid He^3 makes a definite prediction about the behavior of the liquid structure factor $S(k)$ when $\hbar k \ll P_F$. Using the two-parameter *Ansatz* $F(\chi) = F_0 + F_1 P_1(\cos\chi)$ for the spin-symmetric part of the quasiparticle interaction function with F_0 and F_1 determined by zero sound and compressibility, we obtain a qualitatively good fit to recent x-ray data for $S(k)$.

In a recent Letter[1] it was reported that the Feynman-Bijl relation $S(k) = \hbar k/2mc$ for the liquid structure factor of liquid He^4 was experimentally verified by x-ray diffraction methods. Similar measurements have been carried out by Achter and Meyer,[2] who also measured the liquid He^3 structure factor. The purpose of this Letter is to point out that the recent $S(k)$ data for liquid He^3 provide strong evidence for the validity of the Landau theory of zero-temperature density oscillations.

Pines and Nozières[3] have observed that the behavior of $S(k)$ for $\hbar k \ll P_F$ is in principle determined by the Landau theory. We begin by briefly describing how this prediction is made. Suppose that the Fermi liquid is in its ground state in the remote past and that a small (spin-independent) scalar potential

$$\varphi(\vec{r}t) = \mathrm{Re}[\varphi(\vec{k})e^{i(\vec{k}\cdot\vec{r}-\zeta t)}], \quad \mathrm{Im}\,\zeta > 0, \qquad (1)$$

is applied. The response in the density,

$$\delta\langle\rho(\vec{r}t)\rangle = \mathrm{Re}[R(\vec{k})e^{i(\vec{k}\cdot\vec{r}-\zeta t)}], \qquad (2)$$

is related to the dynamic liquid structure factor $S(k,\omega)$ via the usual rules of linear response theory,

$$R(\vec{k}) = \chi(k,\zeta)\varphi(\vec{k}), \qquad (3a)$$

$$\chi(k,\zeta) = \frac{2\rho}{\hbar}\int_0^\infty \frac{S(k,\omega)\omega d\omega}{\omega^2-\zeta^2}, \qquad (3b)$$

$$\mathrm{Im}\chi(k,\omega+i0^+) = (\rho\pi/\hbar)[S(k,\omega)-S(k,-\omega)]. \qquad (3c)$$

The density response $\delta\langle\rho(\vec{r}t)\rangle$ can be computed in the Landau theory via the change in the quasiparticle distribution $\delta n(\vec{r},\vec{p},t)$:

$$\delta\langle\rho(\vec{r}t)\rangle = 2\int[d^3p/(2\pi\hbar)^3]\,\delta n(\vec{r},\vec{p},t). \qquad (4)$$

From the zero-temperature Landau transport equation, it is not difficult to show that

$$\delta n = \mathrm{Re}[-\delta(\epsilon_0-\mu)\Psi(x,s)\varphi(\vec{k})e^{i(\vec{k}\cdot\vec{r}-\zeta t)}], \qquad (5)$$

where $x = \cos\theta = \vec{k}\cdot\vec{p}/kP_F$, $s = \zeta m^*/kP_F$, $\epsilon_0(\vec{p})$ is the single quasiparticle energy spectrum, and μ is the chemical potential. $\Psi(x,s)$ is determined by the integral equation

$$\Psi(x,s) = \frac{x}{s-x}\left[1 + \tfrac{1}{2}\int_{-1}^{+1} G(x,y)\Psi(y,s)dy\right], \qquad (6)$$

where

$$G(x, y)=\sum_{l=0}^{\infty} F_l P_l(x) P_l(y). \qquad (7)$$

The quantities F_l are the coefficients of the Legendre polynomials $P_l(\cos\chi)$ in the standard expansion[4]

$$F(\chi)=\sum_{l=0}^{\infty} F_l P_l(\cos\chi) \qquad (8)$$

of the spin-symmetric part of the quasiparticle interaction function on the Fermi surface. From Eqs. (2)-(5) it follows that the Landau theory predicts

$$\chi(k, \zeta)=-(m^* P_F/2\pi^2\hbar^3)\int_{-1}^{+1}\Psi(x, s)dx. \qquad (9)$$

The Landau prediction for $S(k, \omega)$ follows from Eqs. (6), (9), and (3c).

It is well known[5] that the dynamic liquid structure factor $S(k, \omega)$ obeys the following sum rules:

$$\lim_{k\to 0}\int_0^\infty S(k, \omega)\frac{d\omega}{\omega}=\frac{\hbar}{2mc^2}, \qquad (10a)$$

$$\int_0^\infty \omega S(k, \omega)d\omega=\hbar k^2/2m, \qquad (10b)$$

and

$$\int_0^\infty S(k, \omega)d\omega=S(k). \qquad (10c)$$

In the limit $k\to 0$, the Landau prediction for $S(k, \omega)$ [Eqs. (3c) and (9)] also obeys these sum rules since only long wavelengths $\hbar k \ll P_F$ and low frequencies $\hbar\omega \ll |\mu|$ are relevent to the integrals. A somewhat lengthy but direct calculation [using Eqs. (6), (9), and (3c)] shows that Eqs. (10) imply, respectively,

$$c^2=(1+F_0)P_F^2/3mm^*, \qquad (11a)$$

$$m^*/m=1+\tfrac{1}{3}F_1, \qquad (11b)$$

and

$$S(k)=\alpha(3\hbar k/4P_F) \text{ as } k\to 0, \qquad (11c)$$

where

$$\alpha=-(2/\pi)\int_{-1}^{+1}dx\int_0^\infty d\nu\, \mathrm{Im}\Psi(x, \nu+i0^+). \qquad (12)$$

Equations (11a) and (11b) have been previously derived by other methods. They show that the Landau prediction for $S(k, \omega)$ obeys the sum rules (10a) and (10b). The sum rule (10c) for $\hbar k \ll P_F$ must also be obeyed by the Landau prediction since any high-frequency error in the integrals would be larger for the known result (11b) than for the new result (11c) of this paper.

Equations (6), (11c), and (12) constitute a rigorous prediction of the Landau theory for the behavior of the liquid structure factor $S(k)$ as $k\to 0$. We have evaluated the coefficient α by assuming that $F_l=0$ for $l\geq 2$. From the data of Abel, Anderson, and Wheatley[6] on the velocity of first ($c=188$ m/sec) and zero ($c_0=194$ m/sec) sound, one obtains the values $F_0=10.8$ and $F_1=6.25$.

With the above assumption of only two nonzero parameters, the integral equation (6) can be solved exactly and yields the result

$$\int_{-1}^{+1}\Psi(x, s)dx = \frac{2Q_1(s)(1+\frac{1}{3}F_1)}{(1+\frac{1}{3}F_1)-[F_0(1+\frac{1}{3}F_1)+F_1 s^2]Q_1(s)}, \qquad (13)$$

where $Q_1(s)$ is the first-order Legendre function of the second kind, i.e.,

$$Q_1(s)=\tfrac{1}{2}s\ln[(s+1)/(s-1)]-1. \qquad (14)$$

One must, in doing the integral in Eq. (12), take into account the zero-sound pole at $s=3.60$; this yields a contribution

$$\alpha(\text{zero sound})=0.571. \qquad (15)$$

The remaining contribution to α is the quasiparticle-quasihole pair background which is found by simple numerical integration as

$$\alpha(\text{pairs})=0.004. \qquad (16)$$

Altogether,

$$\alpha=0.575. \qquad (17)$$

It is of interest to note that the continuous quasiparticle-quasihole background part of α is only about 1% as large as the zero-sound contribution. In other words, the zero-sound pole practically exhausts the sum rule (10c) in the limit $k\to 0$. Equations (11c) and (17) imply that $S(k)$ for liquid He^3 ($\hbar k \ll P_F$) is 57.5% of the result that one obtains for an ideal Fermi fluid at the same density (i.e., $\alpha_{\text{ideal}}=1$).

We have plotted our results for $S(k)$ in Fig. 1 along with the experimental data of Achter and Meyer[2] for the values $\hbar k/P_F \lesssim 1$. The agreement is qualitatively good. Note, in this connection, that the deviations at particularly small values of $\hbar k/P_F$ are to be expected since the experiment was carried out at finite temperature ($T\approx 0.56$°K). The limit of $[S(k)/k]$ as $k\to 0$ is finite only at absolute zero.

We have shown that a prediction for the He^3 structure factor $S(k)$, in semiquantitative agreement with recent x-ray data, can be obtained from the Landau theory in which F_0 and F_1 are

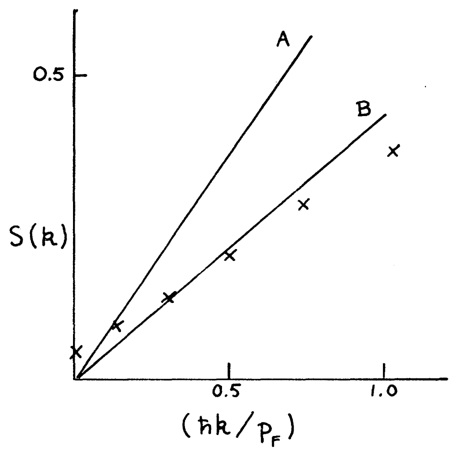

FIG. 1. Line $\boldsymbol{B}$ is the prediction of the Landau theory for the structure factor $S(k)$, i.e., $S(k) = \alpha\,(3\hbar k/4P_F$. $\boldsymbol{\alpha}$ is determined to be 0.575 for He^3. Line $\boldsymbol{A}$ indicates the structure factor for an ideal Fermi gas ($\alpha_{ideal} = 1$). The experimental points are from the data of Ref. 2.

determined from a knowledge of the compressibility and the speed of zero sound, and F_l is assumed to vanish for $l \geqslant 2$. We have used neither the theory nor the data for the specific heat (the theory being quite controversial) in our considerations. It seems to us that our result constitutes strong evidence for the validity of the Landau theory of density oscillations in Fermi liquids.

[1]R. B. Hallock, Phys. Rev. Letters 23, 830 (1969).

[2]F. K. Achter and L. Meyer, Phys. Rev. 188, 291 (1969).

[3]D. Pines and P. Nozières, *Theory of Quantum Liquids* (Benjamin, New York, 1966), Chap. 2.

[4]A. A. Abrikosov and I. M. Khalatnikov, Progr. Phys. 22, 329 (1959).

[5]See Pines and Nozières, Ref. 3.

[6]W. R. Abel, A. C. Anderson, and J. C. Wheatley, Phys. Rev. Letters 17, 74 (1966). See also W. E. Keller, *Helium-3 and Helium-4* (Plenum, New York, 1969), Chap. 6.

THICKNESS OF THE FLOWING SUPERFLUID FILM*

D. L. Goodstein† and P. G. Saffman
California Institute of Technology, Pasadena, California 91109
(Received 4 May 1970)

It has recently been shown experimentally that the thickness of a saturated helium film is independent of its state of superflow. We present a simple model in which the thickness of such a film is fixed by dynamical considerations at the gas-film interface, and the required balance of forces is then provided by relatively minor adjustments in the film surface tension.

In the two-fluid hydrodynamics of He II, steady-state flow is governed by the equation

$$\nabla(\mu + \tfrac{1}{2}u_s^2) = 0, \tag{1}$$

where μ is the chemical potential and u_s the superfluid velocity.[1] This is closely analogous to Bernoulli's equation in ordinary fluid mechanics. It means that, if u_s changes with position, some other quantity, such as the fluid pressure, must vary to change μ and balance Eq. (1). This is required since changes in u_s are accelerations in a frame moving with the superfluid, and need balancing forces.

Kontorovich[2] has predicted that in helium films the film thickness z would change in order to balance changes in u_s by means of the van der Waals force binding the film to the wall. Keller[3] has shown experimentally that the predicted changes in z do not occur. Huggins[4] has suggested that the temperature T varies instead in order to balance u_s by means of the thermomechanical effect, but this has also been found to be incorrect.[5] In the usual formulation of film hydrodynamics μ depends only on z and T, and so Eq. (1) appears to be violated.

We shall argue below that the difficulty arises from the common assumption that the film behaves just like the bulk liquid except for the van der Waals field, i.e.,

$$\mu = \mu_L - \alpha/z^3, \tag{2}$$

where μ_L is the chemical potential of the liquid and α is constant. This assumption becomes untenable when the surface tension γ, usually neglected in the liquid, becomes a more important source of forces than the pressure, that is, for thickness less than $z_0 \approx \gamma/P \approx 10^{-4}$ cm. The films of interest are in the range $z \lesssim 5\times10^{-6}$ cm,[6] and $P \approx 3\times10^3$ dyn/cm^2 is the vapor pressure of helium at $\approx 2^\circ$K.