RADIO INTERFEROMETRIC TEST OF THE GENERAL RELATIVISTIC LIGHT BENDING NEAR THE SUN*

D. O. Muhleman, R. D. Ekers, and E. B. Fomalont
Owens Valley Radio Observatory, California Institute of Technology, Pasadena, California 91109
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We have used radio interferometry to measure the deflection of a radio source as it passed the sun. This deflection is a separable combination of refraction in the coronal electron plasma and the general relativistic effect. The observed deflection was $1.04^{+0.15}_{-0.10}$ times that predicted by Einstein's theory of general relativity. The observation also yielded a reasonable value for the coronal electron density.

Any beam of radiation is deflected during its passage near the sun as a result of the general relativistic (GR) effect and of refraction in the coronal electron plasma. We used radio interferometry to measure the change in relative position of two discrete radio sources, 3C273 and 3C279 (separation 9.5°), as the sun passed near them. During our observations (2-10 October 1969) the angular distance between the sun and 3C279 varied between 19.9 and 4.5 solar radii ($R_\odot$). The coronal refraction in our experiment (at 2388 MHz) was everywhere a relatively small fraction of the deflection predicted by Einstein's theory, viz. 1.75″ at the solar limb. The fringe spacing of our interferometer was 1.2″ which made it ideal for this experiment since the measured shift never exceeded one fringe and was unambiguous. Fortunately, activity on the sun's surface was relatively low during our observational period.

The expected angular deflection can be accurately computed using geometrical-optics techniques in a spherically symmetric refracting medium of index (in isotropic coordinates)

$$n(r)=1+\frac{2GM_\odot}{rc^2}-\frac{2\pi e^2}{m_e\omega^2}N_e(r), \quad (1)$$

where $N_e(r)$ is the electron-density profile in the corona and interplanetary medium[1] (G = gravitational constant, $M_\odot$ = solar mass, c = light speed, e = electronic charge, m_e = electronic mass, and ω = angular frequency). The electron density can be well represented in the form

$$N_e(r)=A/r^6+B/r^{2.33}, \quad r>3R_\odot, \quad (2)$$

which was obtained from van de Hulst,[2] Blackwell and Petford,[3] and Blackwell, Dewhirst, and Ingham,[4] who have analyzed the coronal light intensities measured during many solar eclipses. The coefficients A and B depend on the solar cycle and vary by as much as a factor of 5 during an 11-yr cycle. The exponent in the B term (which is the more important term for our observations) is uncertain to about ±0.2 and is consistent with space-probe in situ measurements, radio astronomical scintillation observations, and solar wind theory (see Newkirk[5]).

Since the maximum possible values of the refractivities in Eq. (1) are 10^{-6} and 5×10^{-7} (at $4R_\odot$) for the GR and plasma terms, respectively, the calculations can be carried to first order in these terms with high accuracy. Consequently, the deflections from the individual terms in Eqs. (1) and (2) can be combined linearly. Putting in the numerical factors, the total deflection in seconds of arc is

$$\delta(p)=\frac{1.75}{p}\gamma\left\{\tfrac{1}{2}+\tfrac{1}{2}\left(1-\frac{p^2}{r_\oplus^2}\right)^{1/2}\right\}$$
$$-0.86\times10^{-5}Ap^{-6}-4.99\times10^{-6}Bp^{-2.33}, \quad (3)$$

where p is the ray impact parameter relative to the sun in units of $R_\odot$, A and B are the electron densities in cm^{-3} at unit distance, and $r_\oplus$ is the earth's distance $\simeq 215R_\odot$. The value of γ (unity for Einstein's theory) was estimated with A and B from the observations.

The interferometer consisted of two steerable parabolic reflectors, one 64-m and one 26-m, at the Goldstone Tracking Station. The antennas used have a separation of 21.566 km in azimuth 155°. The interferometer components are shown schematically in Fig. 1(a). The overall bandwidth was 396 kHz and all image signals were suppressed. The local-oscillator signals were coherently synthesized from independent rubidium frequency standards at each site.

The 0.455-MHz intermediate-frequency signal was transmitted via a microwave link to the 26-m antenna site where it was correlated with the signal from the 26-m antenna and recorded on magnetic tape. Thus although the interferometer used independent local oscillators it differed from the usual very long baseline interferometry

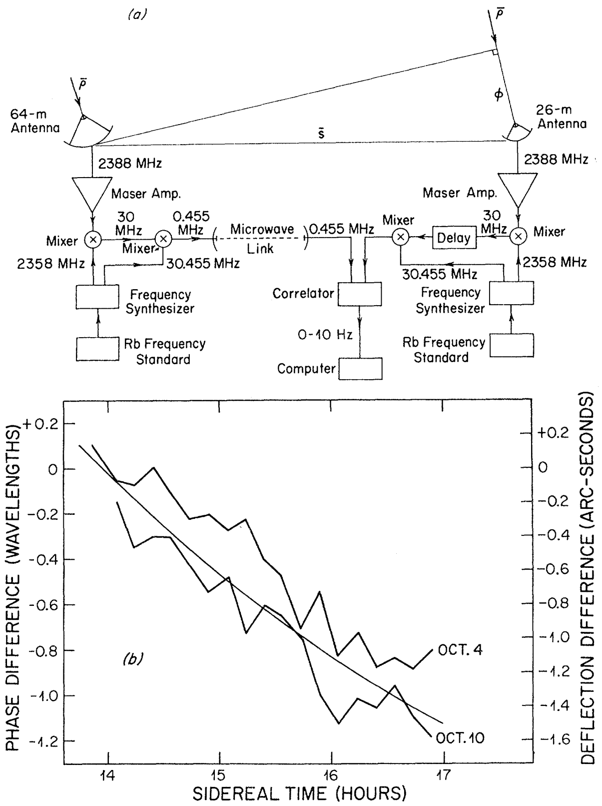

FIG. 1. (a) Schematic block diagram of the interferometer system. (b) Observations of the relative phase difference between the two radio sources on 4 and 10 October 1969. The smooth curve shows the baseline from the solution for a, b, and c obtained from all of the data.

configuration[6] in which the intermediate-frequency signals are recorded on magnetic tape at each site and later brought together and correlated. The real-time correlation technique allows increased bandwidth and vastly simplifies data reduction.

A variable delay, consisting of a binary set of sonic delay elements which could be varied from 0 to 126 μsec in steps of 2 μsec, was inserted in the 30.0-MHz intermediate frequency from the 26-m antenna. This was necessary to equalize the total path lengths from each antenna before correlating to stay near the central "white light" fringe. It was possible to use a single delay setting for each observation.

Measurements were taken alternately of 3C273 and 3C279. To minimize the effects of phase variations between the independent local oscillator systems it was necessary to make the switches as fast as possible. Sources were changed every 5 min and on the average this gave $3\frac{1}{2}$ min on the source and $1\frac{1}{2}$ min between sources. This scheme maximized the number of differences which could be measured while providing sufficient time on each source to overcome the noise error and to establish a mean oscillator drift rate. Observations were made between 13.0 and 17.0 h (local sidereal time) on 2, 4, 6, and 10 October. During this time the baseline was optimally aligned to detect the radial deflection of the radio signal, and the altitude of the sources was greater than 20°. A total of 88 pairs of observations were obtained.

For a two-element radio interferometer with a vector baseline separation $\vec{S}$ (in wavelengths), the path-length difference $\varphi(t)$ of radiation from a point source at position $\vec{p}$ (a unit vector) is [see Fig. 1(a)]

$$\varphi(t) = \vec{p}\cdot\vec{S} + \Psi(t). \tag{4}$$

Other path length differences (atmospheric effects, oscillator drifts, cable length changes) can be lumped into the instrumental phase term $\Psi(t)$, which is a slowly varying function of time.

If two radio sources are observed at times t_1 and t_2, then their phase difference $\Delta\theta(t)$ is

$$\Delta\theta(t) = a\cos t + b\sin t + c + \vec{S}\cdot[\vec{\delta}_1(t) - \vec{\delta}_2(t)] + \Psi(t_1) - \Psi(t_2), \tag{5}$$

where $t = \frac{1}{2}(t_1 + t_2)$ and $\vec{\delta}_1(t)$ and $\vec{\delta}_2(t)$ are the deflections for sources 1 and 2. This assumes that t_1 and t_2 are sufficiently close (<5 min) that the only significant nonlinear variation over the time interval is in the instrumental phase term. All source position errors and baseline errors are now included in the constants a, b, and c. The drifts in the instrumental phase terms $\Psi(t_1)$-$\Psi(t_2)$ were removed by using sets of three observations, 3C273-3C279-3C273, to determine the phase differences between 3C279 and 3C273.

Substituting the deflection of each source due to the sun from Eq. (3) into Eq. (5), we have

$$\Delta\theta(t) = a\cos t + b\sin t + c + 1.75\gamma\vec{S}\cdot(\vec{e}_1 p_1^{-1} - \vec{e}_2 p_2^{-1}) - 0.86\times10^{-5}A\vec{S}\cdot(\vec{e}_1 p_1^{-6} - \vec{e}_2 p_2^{-6}) - 4.99\times10^{-6}B\vec{S}\cdot(\vec{e}_1 p_1^{-2.33} - \vec{e}_2 p_2^{-2.33}), \tag{6}$$

where $\vec{e}$ is the unit vector from the sun through the source, and the term $p^2/r_\oplus^2$ is neglected. Thus the phase difference is expressed as a linear function of six unknowns—a, b, c, γ, A, and B. A least-squares procedure was used to determine their values. The average rms error per point determined from the final residuals was 0.087 wavelength, almost exactly that expected from a laboratory measurement of the stability of the independent oscillators.

The measurements on two days are shown in Fig. 1(b), the difference in the values at corresponding local sidereal times being due to the difference in GR and coronal bending for days when the source was on opposite sides of the sun. The slope on the observations is caused by the errors in baseline and source position, which were the same for all days. Least-mean-squares fits of Eq. (6) showed that the instrumental parameters a, b, and c were well separated from the remaining parameters in all cases (very low correlations). However, γ was moderately correlated with A and B which, in turn, were almost completely correlated with each other, i.e., only one coronal parameter can be estimated. As a measure of the sensitivity of our experiment to coronal effects we determined a solution for γ using the data from the three days for which the impact parameter was greater than 9 solar radii and assumed no coronal effects. The result was $\gamma = 0.95 \pm 0.10$. Inclusion of the slight coronal bending operative on those days could only increase this value. In order to take advantage of the considerable a priori knowledge of A and B from eclipse measurements, we limited our solutions to γ, B, a, b, and c while we adjusted fixed values of A over a wide range of reasonable values. The least-squares solutions for γ and B are shown in Table I. Although each solution is well determined, the sums of squared residuals do not differ significantly for the various solutions. The correlation between the γ, B solutions and A is clearly evident and it is not possible to determine which of the values of γ is the most likely from the measurements alone. However, each pair of A and B yields significantly different electron density profiles and the ambiguity can be resolved with the aid of the solar eclipse data. The profiles for solutions 1, 2, 3, and 4 are shown in Fig. 2 along with the density profiles from Refs. 2 and 3, both of which had to be adjusted downward by a factor of 2 to agree with the mean density of any of our profiles. The

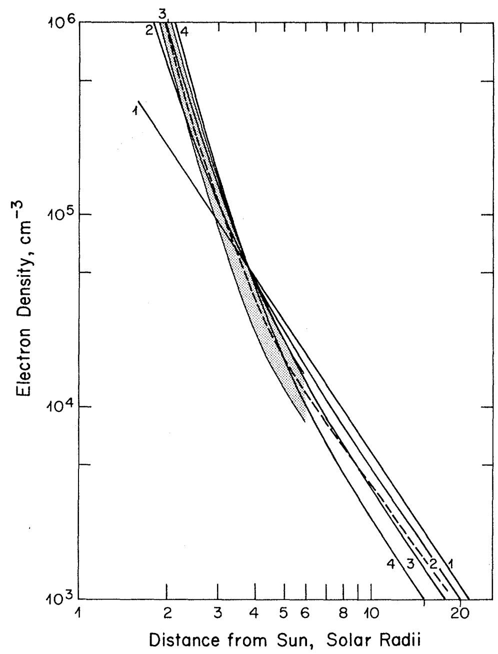

FIG. 2. Electron density profiles for solutions 1, 2, 3, and 4 of Table I. The electron density profile from eclipse measurements of Blackwell and Petford (Ref. 3) is indicated by the dashed line. Van de Hulst (Ref. 2) profiles for minimum and maximum solar activity are indicated by the shading. The eclipse profiles are scaled by a factor of $\frac{1}{2}$.

Table I. Least-squares solutions for the relativity and coronal parameters.

Solution	Fixed parameter A (cm^{-3})	Solutions B (cm^{-3})	γ	N_e (Earth's orbit) (cm^{-3})
1	0	1.2×10^6	1.09	4.4
2	2.8×10^7	9.8×10^5	1.07	3.6
3	5.85×10^7	7.6×10^5	1.04	2.7
4	8.75×10^7	5.2×10^5	1.02	1.9
5	1.18×10^8	3.2×10^5	1.00	1.2
6	1.46×10^8	1.0×10^5	0.97	0.4

form of our solution 3 is in best agreement with the eclipse measurements and, consequently, we adopt it as our best result: $\gamma = 1.04$, which yields 1.82 sec at the solar limb. The error arising just from this fitting procedure is about ±0.05 in γ or ±0.1″.

The total uncertainty of our result was estimated in several ways, all of which yielded essentially the same results. The most reliable method was a Monte Carlo simulation of equivalent data samples generated by adding Gaussian noise (of variance equal to that of the measurements) to observables computed from solution 3. A set of 200 such numerical experiments was treated with the same procedures as that for the actual measurements. This gave an error in γ from noise alone of ±0.08, and an additional error resulting from the correlation with the coronal parameters of +0.13, −0.06. The skewness arises from applying the obvious constraint that $B > 0$. Our final result is $\gamma = 1.04^{+0.15}_{-0.10}$ or $\Psi = 1.82^{+0.24}_{-0.17}$ sec. All quoted errors are standard deviations. These figures are in good agreement with that predicted by Einstein's theory of gravitation and compare quite favorably to the optical determinations of light bending (see Trumpler[7] and Mikhailov[8]). The value of Ψ predicted by Dicke,[9] 1.63″ ($\gamma = 0.93$), to be consistent with his determination of the quadripole moment of the sun, lies one standard deviation from our most probable value. Thus, while our result is in better agreement with Einstein's value, the disagreement with Dicke's value cannot be regarded as very significant. Our result is in good agreement with an independent radio-interferometric experiment performed by Seielstad, Sramek, and Weiler.[10]

It should be emphasized that our solution is only slightly dependent on our treatment of the coronal plasma bending. Furthermore, although we have utilized the form of the electron density profile from eclipse measurements in selecting our final model, we believe that our model is a significant representation of the corona. The disagreement with classical methods by a scale factor of 2 is not serious considering the well-known difficulties in reducing eclipse light intensities to electron densities. The extrapolation of our model to the orbit of the earth is shown in Column 5 of Table I and the results are in excellent agreement with the space-probe in situ determinations of 4-6 electron/cm^3 at 1 A.U.

These results were obtained with 16 h of observing time. The accuracy could be improved by about a factor of 3 with a modest increase in observational time distributed more optimally over the solar occultation period of 3C279.

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