TOROIDAL FUSION PLASMA WITH POWERFUL NEGATIVE BIAS*

Thomas H. Stix†
Weizmann Institute of Science, Rehovot, Israel
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Imposition of a powerful electrostatic bias allows a dense uniform-pressure plasma to be held in a static equilibrium in a toroidal closed-magnetic-line system. A small wall current of 3.5-MeV ***d-t*** alpha particles easily maintains the 6-MV/cm electric field necessary for controlled thermonuclear reaction conditions. As ions can be confined in an enormously deep energy well, the system lends itself also to production of highly stripped heavy nuclei.

Since inception around 1952 the main thrust of controlled-nuclear-fusion research has been toward the confinement of neutral plasmas by strong static magnetic fields. In the present Letter it is pointed out that the imposition of a large electrostatic bias introduces a new degree of freedom into the design of plasma equilibria and allows several interesting objectives to be immediately attained. In particular, a dense, static, uniform-pressure plasma can be confined in a pure-toroidal closed-line magnetic field with toroidal electrostatic potential surfaces. In the proposed controlled thermonuclear reaction (CTR) device the ions are held in an energy well of enormous depth and the necessary bias is spontaneously maintained by a small wall current of 3.5-MeV d-t alpha particles. The stability of the electron-plasma heavy-ion plasma accelerator (HIPAC) system, analyzed and documented by Daugherty et al.,[1] lends considerable encouragement that the proposed configuration may also be well behaved.

We start the analysis of the toroidal equilibrium state by examining the equations for single-particle motion. In cylindrical coordinates (r, θ, z), symmetry in the azimuthal (θ) direction provides that the angular momentum p_θ is a constant of the motion. With static fields the particle energy is also a constant and we make use of the approximate invariance of the magnetic moment μ. The particle energy H can then be written

$$H(\mu, J, r) = q\varphi(r, z) + \mu B(r) + p_\theta^2/2mr^2 + mv_D^2/2, \quad (1)$$

where q is the particle charge. The motion of the particle guiding center is restricted to the surface H=const; the guiding-center drift velocity along this surface is given to lowest order in Larmor radius by

$$\vec{v}_D = -(c/qB)\nabla H \times \hat{\theta} - (mc/qB)(\hat{\theta}\cdot\nabla\times\vec{v}_D)\vec{v}_D. \quad (2)$$

We have chosen $\vec{B} = \hat{\theta}B(r)$. The second term introduces[2] the effect of centrifugal force due to curved drift orbits.

We now consider a region of space filled with density $n(r)$ of particles of the same μ and J. The conservation law $\partial n/\partial t + \nabla\cdot(n\vec{v}_D) = 0$ has an exact time-independent solution[3] [we make use of $\nabla\times(\hat{\theta}/r) = 0$],

$$n(\mu, J, r) = (B/r)[1 + \Omega^{-1}(\hat{\theta}\cdot\nabla\times\vec{v}_D)]f(H), \quad (3)$$

with $\Omega \equiv qB/mc$. Inasmuch as the guiding center current differs from the perpendicular particle current only by the divergence-free magnetization current density $\nabla\times\vec{M}$, the guiding-center density in (3) is also the particle density to this order of approximation.

Self-consistency of the low-β static equilibrium comes from the solution of the Poisson equation

$$\nabla^2\varphi = -4\pi e \sum_{q,\mu,J} Zn_i(\mu, J, r) - n_e(\mu, J, r), \quad (4)$$

in which the density terms are to be evaluated using the previous three equations.

To avoid the undesirable buildup of electrostatic potentials large compared with kT/e in the interior of the proposed CTR plasma requires charge neutrality on the right-hand side of Eq. (4) to one part in 10^6 or 10^7. Attempts to fill the plasma according to the density prescription for ions and electrons in (3) can easily lead to deviations from charge neutrality many times larger than this. Furthermore, to obtain the necessary accuracy of computation, ion Larmor-radius effects must be included to at least one higher order. A simple solution appears, however, for the case in which H is independent of z. Then the density functions $\sum n(\mu, J, r)$ can be arbitrary functions of r even when finite Larmor-radius corrections are added, and ion- and electron-charge densities can be equated. Physically the situation corresponds to the equilibrium of a straight cylinder or cylindrical shell of plasma which extends to $z = \pm\infty$.

We terminate this solution at a closed toroidal equipotential surface S. Inside S we assume that exact charge neutrality exists, hence the potential φ is constant within S. We now locate the electron $H=$ const surfaces which are tangent to S at the outside of the torus, and turn out elsewhere to lie slightly outside of S, Fig. 1. These surfaces are then loaded with electrons with the purpose of depressing $\varphi(S)$ significantly below the wall potential.

We can now trace out the full constant-H surfaces for the plasma particles. Figure 1 shows the intersection of representative surfaces with a $\theta=$ const plane. Inside S the projection of H $=$ const is a vertical line. For ions the line turns outward immediately above its intersection with S, and hugs the outside of S until it reaches the bottom intersection. The constant-H contours for electrons are similar but circumnavigate the inner side of S. The H contours are the guiding-center orbits. Since these contours are closed one knows immediately that static flow is characterized by $\nabla\cdot\vec{j}=0$.

For the CTR plasma under consideration the electric field just outside S is so strong that the thermal ions barely penetrate it. It is actually a better picture of their motion then to consider that S is a perfectly reflecting wall for thermal ions and that there is no net ion diamagnetic flow.

The electron Larmor radius, on the other hand, is sufficiently small that we may use drift theory to calculate the electron surface current. The flux of electron guiding centers in the body of the plasma corresponds to a flow $-ne\vec{v}_D/c=\hat{z}(2p_e/Br)$. When the drifting electrons reach the surface S their charge adds to the surface current $j_s{}^*$ in accordance with the equation $d(rj_s{}^*)/dr=-2p_e/B$. For $p_e=$ const and $B\sim r^{-1}$, integration yields

$$\vec{j}_s{}^*=\vec{j}_p{}^*+\vec{j}_c{}^*$$
$$=\hat{\theta}\times\hat{n}\left[-\frac{p_e}{B(r)}+\frac{p_e r_{\max}}{B(r_{\max})r}+\frac{\text{const}}{r}\right], \qquad (5)$$

where $\hat{n}$ is the unit normal vector directed into S. There are two components to $\vec{j}_s{}^*$. The current $\vec{j}_p{}^*$, due to "plasma" electrons alone, goes to zero at the outer boundary of S, $r=r_{\max}$, and is represented by the first two terms on the right. In addition $\vec{j}_s{}^*$ may include an arbitrary amount of divergence-free sheath current, $\vec{j}_c{}^*$, represented by the last term in (5).

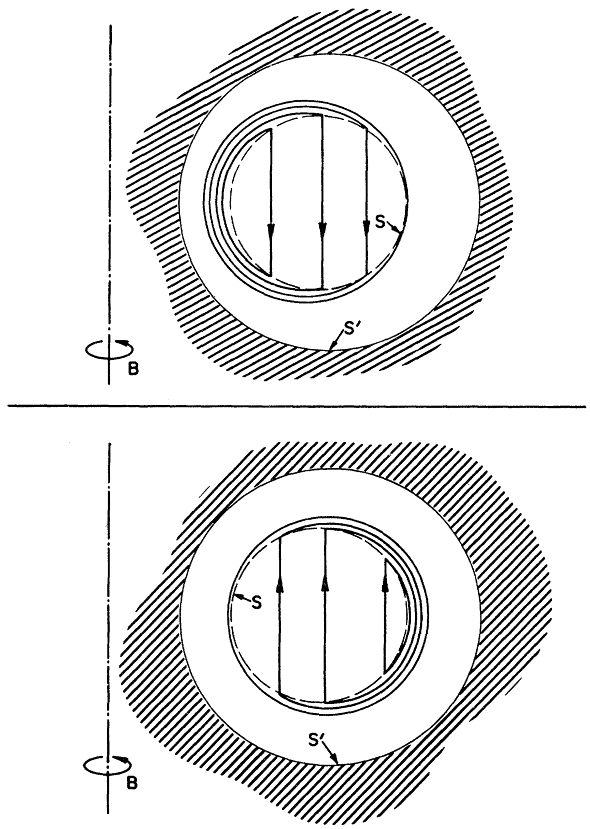

FIG. 1. Projections of representative constant-energy $H(\mu,J,r,z)$ surfaces in a $\theta=$ const plane. Equipotential surface S is negative with respect to conducting wall S'. Upper figure is for electrons, lower figure for ions.

The structure of $\vec{j}_s{}^*$ can be examined on a finer scale. We may write

$$\vec{j}_s{}^*=-(eB/c)\int n_e\vec{v}_D ds, \qquad (6)$$

where s measures distance in the current sheath parallel to $\hat{n}$. Now the ions penetrate deeply into the sheath and may even cross the sheath. We evaluate the electrostatic ion pressure balance

$$\hat{n}p_i=q\int n_i\vec{E}ds\approx(qB/c)\int n_i(\hat{\theta}\times\vec{v}_D)ds \qquad (7)$$

to find that the ion pressure can be expressed algebraically in terms of an equivalent sheath current.

Continuing, we note that the magnetization current $(\nabla\times\vec{M})$ associated with the magnetically confined electrons gives rise to a diamagnetic surface current $\vec{j}_D{}^*B=\hat{\theta}\times\hat{n}p_e$. We combine this current with $\vec{j}_s{}^*$ in (6) to obtain the total surface current $\vec{j}^*$,

$$\vec{j}^*B=(\vec{j}_D^*+\vec{j}_S^*)B=\hat{\theta}\times\hat{n}(p_e+p_i)+(B/c)\int\sigma\vec{v}_D ds=\hat{\theta}\times\hat{n}[8\pi p-(\vec{E}\cdot\hat{n})_{\rm plasma}{}^2+(\vec{E}\cdot\hat{n})_{\rm vac}{}^2]/8\pi, \quad (8)$$

where we have used $4\pi\sigma=\nabla\cdot\vec{E}$ and $\vec{v}_D\approx\vec{E}\times\vec{B}c/B^2$. The equation for $\vec{j}^*$, derived mainly from drift theory, is identical to the conventional fluid-force equation, integrated across the sheath, and reveals the necessary inclusion here of electrostatic forces to describe the complete pressure balance. The negative sign on the right occurs because the field lines are parallel to the pressure gradient and the electric stress is tensile in character.

To complete the description of the equilibrium we must satisfy the boundary conditions for the Poisson equation (4). The additional electrons loaded into the H contours just outside S make the task easy. We can find an equipotential surface outside S by proceeding from all points of S a short distance inversely proportional to $-E_n=4\pi\sigma^*$. On this new surface S' we place our conducting boundary. To compute the separation between S and S' we combine (5) and (8) with $\vec{j}_D^*B=\hat{\theta}\times\hat{n}p_e$ and use $B\sim r^{-1}$ to find

$$(\vec{E}\cdot\hat{n})^2=8\pi[-p+(\text{const}/r^2)]. \quad (9)$$

We recall from the discussion of (5) that the constant depends on the amount of electron injection into the H contours just outside S. It is clear from (9) that the electric field will be stronger at $S(r_{\rm min})$ than at $S(r_{\rm max})$. If we choose the constant in (9) such that $E_{\rm max}=E(r_{\rm min})=NE(r_{\rm max})=NE_{\rm min}$ for a toroidal surface S of major and minor radii R and a, respectively, we find to lowest order in a/R

$$E_{\rm max}=3.8\times10^{-3}\left[\frac{N^2}{N^2-1}\,\frac{a}{R}\,nkT\right]^{1/2}, \quad (10)$$

where $E_{\rm max}$ is in V/cm, $n=n_e+n_i$ is in $\rm cm^{-3}$, and kT is in eV.

For controlled fusion a typical set of conditions could read $n_e=n_i=10^{14}\ \rm cm^{-3}$, $kT=10^5$ eV, $a=50$ cm, $R=500$ cm, and $N=2$. $E_{\rm max}$ is then 6.2×10^6 V/cm and if the plasma is biased to negative 1.5 MV, the maximum and minimum distances between S and S' (between the plasma and the conducting wall) are 0.5 and 0.25 cm, respectively. In a magnetic field of 150 kG, the rms Larmor radius for thermal tritons would be 0.5 cm, but this figure is not particularly significant since these ions are able to penetrate a distance of only 100 kV into the 1.5-MV energy barrier at the surface S prior to their reflection. The rms electron Larmor radius is 0.7×10^{-2} cm while the Debye shielding length is 2.4×10^{-2} cm. The surface charge on S necessary to produce $E_{\rm max}$ corresponds to 3.4×10^{12} electrons/cm^2.

We turn now to the problem of charge injection and the maintenance of the strong electrostatic bias on the plasma. Three methods come to mind:

(A) Alpha-particle emission. The Larmor radius of the 3.52-MeV alpha-particle fusion product from the d-t reaction is 1.8 cm. Not only will alphas emitted from reactions taking place near the surface S hit the wall, but alpha particles from the volume reactions will be drifted into the wall. The resulting loss of positive charge will increase the negative charge of the plasma to the point where such wall absorption becomes energetically impossible, namely, when the negative bias of the plasma reaches almost 1.76 MV. This process will serve to maintain the necessary plasma bias.[4]

(B) Magnetic injection. Injection of electrons at S' during a period of rising magnetic field causes these electrons to be carried on the magnetic lines of force into the toroid volume. The method was proposed by Daugherty et al.[1] and has been used successfully by the HIPAC research team.

(C) Rf magnetic injection. A variant of the HIPAC scheme could be used to inject electrons into a steady-state toroidal magnetic volume. Consider the injection of cold electrons from a short annular region of S'. External $m=1$ radio-frequency coils cause the local B_θ to vary at the frequency of the E/B drift in the region between S and S' (about 10 MHz). Injected electrons with proper phase will then always see a rising magnetic field as they pass through the rf B_θ. A positive trapping bias on the wall S' at the injection annulus can be used to maintain their parallel bunching. Physically, the cold injected electrons are diffused in configuration space by the rf field. Those electrons which reach the vicinity of S are then diffused in velocity space by scattering against the hot sheath electrons, and the configuration-space density of trapped injected electrons at S is correspondingly reduced.

The very weak diffusion of the untrapped electrons caused by the rf can be sharply reduced by placing two synchronous injectors side by side but 180° out of phase. Orbit deviations then tend to cancel for the untrapped electrons which transit both injection fields by moving parallel to $\vec{B}$.

One method for plasma heating is important here for nonfusing plasmas: For a sufficiently large negative bias, ions injected from S' will fall inside the volume of S before they can execute a Larmor circle; they arrive inside S with kinetic energy corresponding to the plasma bias and undergo a cycloidal motion which carries them back out to S' on each cycle. Scattering, however, will reduce their perpendicular energy and cause them to be trapped inside S' and eventually, for many of them, inside S.

The rate of classical diffusion of the plasma across the magnetic field may be obtained from a simple model of the sheath. As usual, the $\vec{v}\times\vec{B}$ Lorentz force is equated, for both ions and electrons, to the ion-electron collisional drag force. We assume further that $\vec{v}_i \ll \vec{v}_e \approx c\vec{E}\times\vec{B}/B^2$ and that the uniform electron density in the plasma, $n_e = Zn_0$, extends out into the sheath, but that the ion density falls off as $n_0 \exp(-Ze\varphi/kT_i)$. The equation for diffusion flux, $n_i v_{di} = n_e Z^{-1} v_{de}$, and Poisson's equation are then simply

$$n_i v_{di} = -F_0 e^{-\psi}\psi'; \quad \psi'' = 1 - e^{-\psi}, \tag{11}$$

where $F_0 = \eta_\perp n_0^2 kT_i c^2/(B^2\lambda_D)$, $\lambda_D^2 = kT_i/4\pi n_0 Z^2 e^2$, $\psi \equiv Ze\varphi/kT_i$, and the prime denotes $\lambda_D d/ds$. Integrating Poisson's equation once we are able to find $(n_i v_{di})_{\max} = 0.3185F_0$ and $(dn_i/ds)_{\max} = 0.3185 \times n_0\lambda_D^{-1} = 0.7634 n_i \lambda_D^{-1}$ at the same point. The model CTR plasma diffuses across $\vec{B}$ at $(n_i \times v_{di})_{\max}/n_0 = 0.07$ cm/sec.

The stability of the system has not been analyzed. In the model plasma almost a megajoule of electrostatic energy is perilously available to drive an instability. On the other hand we may note that the ions sit comfortably in a deep energy well, that the uniform density and temperature within S will inhibit volume instabilities, and that the high-velocity drift motion in the sheath tends to shear and smooth out flutes and ripples. The small distance between the plasma boundary and the wall and the related strong image forces should also aid stability. Perhaps the strongest encouragement comes from the well-documented stable character of experimental HIPAC plasma. In this regard we remark that the diocotron instability[5] figure of merit, $q = \omega_{pe}^2/\omega_{ce}^2 = 22^{-1}$ in the volume plasma, is satisfactorily quite small. A thickness of 0.03 cm would lead to the same q value in the sheath at $r_{\min}$. Finally we may observe that the constancy of energy and angular momentum in a system of toroidal symmetry indicates that uniform vertical or radial fields of a few gauss can be tolerated without destroying single-particle confinement.

With regard to the application of this system to the production of highly stripped ions to be used, for instance, in heavy-nucleus accelerators, the calculation in Ref. 1 indicates that an exposure time corresponding to $n_e\tau \sim 10^{10}$-10^{11} sec cm^{-3} is required to attain 50% removal of electrons for elements in the range $Z = 20$-92. Here n_e designates the density of 13.6-keV electrons and τ is the ion exposure or confinement time. The excellent ion confinement that can be obtained with a deep electrostatic well[6] facilitates achievement of the necessary $n_e\tau$. Electron heating to $kT_e = 10$ keV typically, perhaps by electron-cyclotron resonance techniques, would have to be provided.

In summary, an analysis has shown that a dense, uniform-pressure plasma can be confined in equilibrium by a purely toroidal magnetic field together with toroidal electrostatic potential surfaces. For typical CTR conditions a 1.5-MV negative plasma bias across a minimum scrape-off distance of 0.25 cm introduces a complete pressure balance and confines the plasma ions in an energy well of enormous depth. A small wall current of 3.5-MeV alpha particles[4] provides spontaneous maintenance of the strong bias. The system is also suitable for production of highly stripped heavy ions.

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†John Simon Guggenheim Memorial Foundation Fellow, on leave from the Plasma Physics Laboratory, Princeton University, Princeton, N. J.

[1]See, for instance, J. D. Daugherty, L. Grodzins, G. S. Janes, and R. H. Levy, Phys. Rev. Letters 20, 369 (1968).

[2]J. D. Daugherty and R. H. Levy, Phys. Fluids 10, 155 (1967).

[3]Ref. 2 gives $n \sim B^2(\cdots)$ rather than $(B/r)(\cdots)$. The small error is due to the $\nabla\times\vec{M}$ contribution to $\vec{j}$ which introduces a possible contribution to $\nabla\times\vec{B}$ parallel to ΔH.

[4]The occurrence of a significant number of d-d and d-He^3 reactions will increase the plasma bias due to the wall current of 3.02- and 14.7-MeV protons, respectively. The scrape-off distance and N will then be changed also, in accordance with Eq. (10).

[5]O. Buneman, R. H. Levy, and L. M. Linson, J. Appl. Phys. 37, 3203 (1966).

[6]T. H. Stix, Phys. Rev. Letters 23, 1093 (1969).