In summary, we have observed magnon-pair excitations in two-dimensional K_2NiF_4 at a variety of temperatures and magnetic fields. The results imply very little energy renormalization at T_N and persistence well into the paramagnetic phase of zone-boundary magnons. The excellent quantitative success of Parkinson's calculation for the low-temperature magnon-pair line shape together with the qualitative association between the pair-mode spectrum and the wavelength dependence for magnon renormalization at finite temperatures give rise to the hope that a quantitative finite-temperature theory of magnon pairs may soon be developed. Such a theory is especially desirable because the magnon-pair Raman spectrum directly measures aspects of a four-spin correlation function,[4] from which several quantities of interest, such as the magnetic contributions to the specific heat[15] and to sound-wave attenuation,[16] may be extracted.

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EXTENSION OF THE VARIABLE MOMENT OF INERTIA MODEL TOWARD MAGIC NUCLEI

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Ground-state bands of the quadrupole type are found in even-even nuclei ranging from the most deformed to doubly magic. Their analysis indicates how an "internal stress" which tends to keep the nucleus spherical is overcome as the angular momentum increases.

An analysis[1] of ground-state bands in even-even nuclei led to a remarkable result: While the unified model contains the assumption that the moment of inertia of a deformed nucleus is, in first approximation, independent of the angular momentum, and that a nucleus which has a spherical ground state remains spherical[2] even if its surface undergoes 1-, 2-,⋯, n-phonon oscillations of the quadrupole type, the new analysis yielded the variable moment of inertia (VMI) "model." This model consists of two simple empirical laws:

(I) For rotational as well as "transition" and "vibrational" nuclei, the energy is given by

$$E(J) = \tfrac{1}{2}C(\mathcal{J}-\mathcal{J}_0)^2 + J(J+1)/2\mathcal{J} \quad (\hbar^2 = 1), \tag{1}$$

where $\mathcal{J}_J$ is chosen to minimize $E(J)$,

$$\partial E(J)/\partial\mathcal{J} = 0; \tag{2}$$

and

$$C > 0 \tag{3}$$

for each state.[3] The limit of validity for this law was taken to be $\mathcal{J}_0 > 0$, equivalent to the condition

$R_4 \equiv E_4/E_2 > 2.23$.

(II) The second empirical law relates the "transition moment of inertia," $\mathscr{I}_{02} = \frac{1}{2}(\mathscr{I}_0 + \mathscr{I}_2)$, to the intrinsic transition quadrupole moment, $Q_{02} = [(16/5)\pi B(E2)]^{1/2}$, for the $2^+ \to 0^+$ transition by

$$Q_{02} = k\sqrt{\mathscr{I}_{02}}, \tag{4}$$

where the coefficient k, determined by a least-squares fit, is found to be remarkably constant[4] for most nuclei[5]:

$$k = (39.4 \pm 2.6) \times 10^{-24}\ \mathrm{cm}^2\ \mathrm{keV}^{1/2}. \tag{5}$$

We present here an extension of the VMI model toward magic nuclei and speculate on the physical significance of the parameters in the model. Figure 1 shows the dependence of the moment of inertia on the angular momentum for a few representative cases. It is seen that for well-deformed, stable nuclei, e.g., for Hf^{180}, $\mathscr{I}$ is almost constant as J increases. For nuclei with the most deformed ground states, namely those occurring approximately in the center of their proton and neutron shells,[1] one finds $\mathscr{I}_0 \sim A^{5/3}$. This relation does not hold, however, for the transition nuclei, as is immediately evident from the coincidence of the curves for Xe^{120} ($Z = 54$, i.e., four protons beyond the closed shell) and Pt^{194} ($Z = 78$, four proton holes). For these nuclei the moments of inertia increase steeply from their value at $J = 0$. The most dramatic relative increase of $\mathscr{I}$ with J, however, is found in nuclei with very small values of $\mathscr{I}_0$, e.g., Cd^{110}. The realization that nuclei with a spherical ground state and $R_4 \sim 2.23$ are extremely "soft"[1] removes the difficulties encountered by the simplest form of vibrational model[2] (which assumes $\mathscr{I} = 0$ for all J), such as the appreciable static quadrupole moments of 2^+ states, and the smooth variation of Q_{02} and of ratios of $B(E2)$'s in the transition from spherical to rotational.[1] As was pointed out in Ref. 1, the parameter C, which determines the steepness of the "potential" $\frac{1}{2}C(\mathscr{I} - \mathscr{I}_0)^2$, is largest, in a given element, for the most stable nuclei.[6] Even for stable isotopes, C takes on smaller values for Xe and Pt which have relatively low-lying intrinsic odd-parity states, and for the heavy elements which have large symmetry energies. C becomes slightly negative for Fm^{254}, an isotope which has a small branching ratio for spontaneous fission.[7]

Excited states of many nuclei with $R_4 < 2.23$ have recently been populated, either by heavy-ion reactions or by inelastic scattering.[8-14] There appear to be regular "ground-state bands" in all even-even nuclei investigated. The good

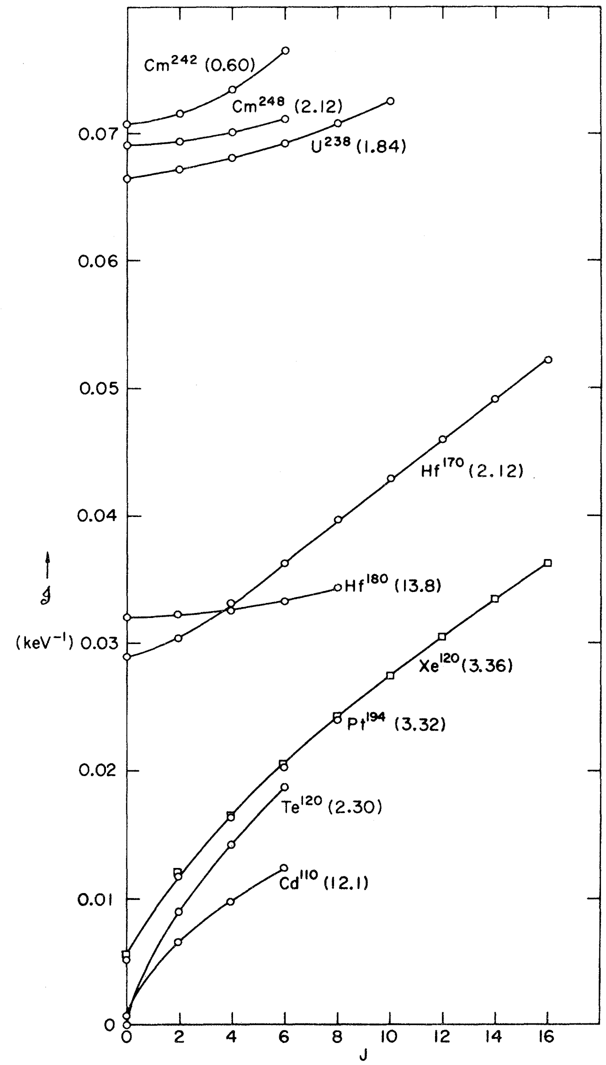

FIG. 1. Using Eqs. (1) and (2), moments of inertia $\mathscr{I}(J)$ have been computed by least-squares fitting of the energies of ground-state–band levels. Some representative examples for rotational (Hf^{180}, Hf^{170}, U^{238}, Cm^{242}, Cm^{248}), transition (Xe^{120}, Pt^{194}), and "vibrational" (Cd^{110}, Te^{120}) bands are shown. Values for the stiffness parameters C in units of 10^6 keV^3 are given in parentheses.

fits obtained for "vibrational" nuclei ($\mathscr{I}_0$ small, but positive) invite the extension of the VMI model to permit negative values of the parameter $\mathscr{I}_0$: One may then write Eq. (2) as

$$r^2(r-1) = -X, \tag{6}$$

with

$$r = \mathscr{I}/\mathscr{I}_0; \quad X = |J(J+1)/2C\mathscr{I}_0{}^3|.$$

For $r \neq 0$ one can write $r = 1 - X/r^2$ and find graphic solutions for r by plotting each side of the

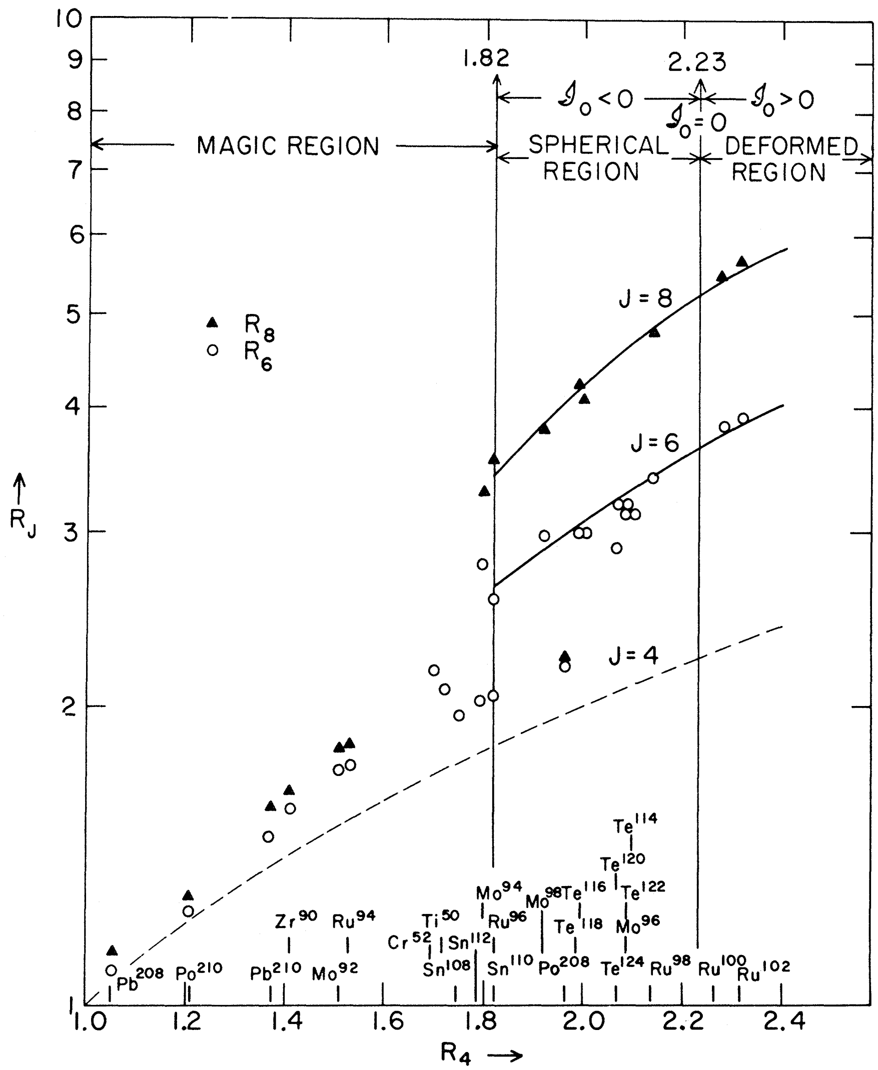

FIG. 2. This figure represents an extension of the VMI model to negative values of $\mathcal{J}_0$. The ratios R_6 and R_8 are plotted against R_4, where $R_J = E_J/E_2$. The solid curves are computed from Eqs. (1) and (2). The dashed line indicates R_4. Only the beginning of the deformed region is shown. It is apparent that the VMI branch is well described by the model.

equation separately. For all $X>0$ there is one real negative root which goes smoothly from $r \approx -\sqrt[3]{X}$ to $r\approx -\sqrt{X}$ as X goes from large to small. For this region one finds $\mathcal{J}_{J=0}=0$ and therefore, according to Eq. (1), $E_0=\frac{1}{2}C\mathcal{J}_0^2$. The limiting value for R_4 is again 2.23 as $\mathcal{J}_0 \to 0$. For large negative $\mathcal{J}_0$ one obtains $R_4 \to (20/6)^{1/2}=1.82$. We have included in Fig. 1 an example of a nucleus with $\mathcal{J}_0<0$, Te^{120}, which has $R_4=1.99$. Despite the negative $\mathcal{J}_0$, the $\mathcal{J}(J)$ curve resembles that of the isobar Xe^{120}.

From here on, the interval $2.23<R_4<3.33$ will be denoted as the deformed region, the interval $1.82<R_4<2.23$ as the spherical region, and the interval $1<R_4<1.82$, which contains only single and double magic nuclei (Ref. 1, Fig. 2, and Table I of this Letter), as the magic region.[15] In the deformed region the ground-state moment of

Table I. Energy values (in MeV) and R_4 values for ground-state bands in double magic nuclei.

Nucleus	E_2	E_4-E_2	E_6-E_4	E_8-E_6	R_4
O^{16} [a]	6.916	3.437			1.495
Ca^{40} [b]	3.903	1.372			1.351
Ni^{56} [c]	2.69	1.26			1.47
Pb^{208} [d]	4.070	0.235	0.100	0.195	1.058

[a] Ref. 16.
[b] Ref. 18.
[c] Ref. 19.
[d] Ref. 13.

inertia $\mathscr{I}_{J=0}=\mathscr{I}_0$, while in the spherical region $\mathscr{I}_{J=0}=0$. This means that as $\mathscr{I}_0$ changes sign, its relation to the character of the ground state changes in a manner reminiscent of a second-order phase transition. Negative values for $\mathscr{I}_0$ require the introduction of a new physical concept into the model. This is the notion of "internal stress" or "rigidity": The larger the negative value of $\mathscr{I}_0$, the more firmly the shell structure resists departure from spherical symmetry.

In Fig. 2 the established experimental values for R_6 and R_8 have been plotted against R_4 ($R_J = E_J/E_2$). We have included all cases with $R_4 < 2.23$ in which γ-ray cascades have been observed, including Ti^{50}, Cr^{52}, and Zr^{90}, whose levels are known from studies of radioactive decay.[16] The solid curves extend the predictions of the VMI model to the spherical region.[17] The experimental points lie along two branches: The first or VMI branch extends leftward from the deformed region. The points on this branch are well fitted by the extended VMI model and terminate precisely at $R=1.8$, the natural limit of the model. The nuclei on this branch in the spherical region are never more than four nucleons from single magic. The second or magic branch extends to the right from $R_4=1$ and consists entirely of double and single magic nuclei for $R_4 < 1.82$. One is tempted to include Po^{208} in this branch, even though it lies in the spherical region. The nuclei in this branch, especially the double magic nuclei[13,15,18,19] (see Table I) are characterized by considerably larger E_2's than their nonmagic neighbors, and by a drastic reduction in level spacing for the higher states in the band. This change of scale implies that a minimum of three parameters would be required in any scheme to fit these spectra. Such a scheme would have to take account of the qualitative inference that these nuclei undergo some kind of rearrangement or "melting" when they are excited, so that further rearrangement costs much less energy: The ground states of these nuclei are not only rigid, but also "brittle." The apparent phase change between the ground state and the higher states could be interpreted as a transition from "superconducting" to "normal" associated with the symmetry breaking effect of pair excitation.

To obtain a more detailed insight into the role of angular momentum in nuclear structure, knowledge concerning transition probabilities between higher members of ground-state bands is of great importance. At present such knowledge is quite scanty with the exception of recent results on deformed nuclei for $8^+\to 6^+$, $6^+\to 4^+$, and $4^+\to 2^+$ transitions.[20,21] The rotational model requires that $B(E2) \sim Q_{J\to J+2}{}^2 (J+1)(J+2)/(2J+3)(2J+5)$. The experimental results indicate that using this definition for Q, and the definition $\mathscr{I}_{J,J+2} = \frac{1}{2}(\mathscr{I}_J + \mathscr{I}_{J+2})$, Eq. (4), with the same coefficient k, holds within limits of error for all $4^+\to 2^+$ transitions measured. For the $6^+\to 4^+$ transitions the measured Q values are, on the average, a few percent lower, and for the $8^+\to 6^+$ transitions they are ~15% lower. A possible interpretation, both of Eq. (4) itself and of the results just quoted, is the following: As the angular momentum increases, the average radius for the neutrons extends more and more beyond that for the protons. This assumption, at the same time, permits an explanation of the puzzling results obtained from muonic x-ray and Mössbauer-effect experiments,[22] viz., that for many nuclei the charge radius between 0^+ and 2^+ states increases less than expected, and in a number of cases even appears to shrink.

The VMI model provides an economical description of ground-state bands for an extraordinary range of even-even nuclei. One may hope that the model will be an aid in both construction and criticism of microscopic models. To complete the experimental picture, we need a better knowledge of energies of ground-state-band members in lighter nuclei, and of transition probabilities for nuclei in the spherical region.

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[5]There is an indication that for the heaviest deformed nuclei, Q_{02} increases faster than Eq. (4) predicts (see Ref. 1, Fig. 9).

[6]Almost nothing is known about the behavior of C on the neutron-rich side of the valley of stability.

[7]It is of interest that for the unstable nucleus Be^8, whose 2^+ and 4^+ states are very wide, the average value of $R_4=3.94$, i.e., $C<0$.

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MEASUREMENT OF THE REACTION $\pi^+p\to K^+\Sigma^+$ AT LARGE MOMENTUM TRANSFERS*

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Cross section and polarization in the reaction $\pi^+p\to K^+\Sigma^+$ have been measured at 3, 4, and 5 GeV/c and for momentum transfers up to 3 (GeV/c)^2. Beyond the previously investigated forward peak the cross section shows a secondary maximum and a steep decrease towards large momentum transfers. The Σ^+ polarization is large and positive in this region.

The reaction $\pi^+p\to K^+\Sigma^+$ has been studied by our group in a previous experiment[1] which covered kaon laboratory angles between 3° and 17° and incident momenta from 3 to 7 GeV/c. The results from this experiment showed a diffractionlike forward peak and a dip or break at $-t\approx 0.5$ (GeV/c)^2 beyond which the cross section appeared to be rather flat. On the basis of these data it was not possible to decide whether the region of relatively slow variation in cross section indicated the presence of a secondary maximum followed by a steeper decrease or whether the cross section would continue its slow variation towards large momentum transfers. The present experiment was designed to answer this question and also to investigate whether there would be additional structure at larger momentum transfers which had been predicted by a Regge-pole fit[2] to previous data.

The present experiment covered kaon angles