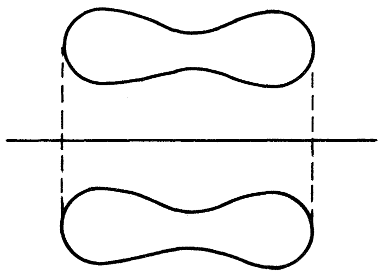

FIG. 3. Doublet.

namic stability for either of these complex shapes; in particular, the relevance of shear or a well is unknown.

To conclude, we repeat that present Tokomak experiments appear to be in a transient state with regard to diffusion. No scaling or extrapolation to new conditions should disregard this fact. The experimentally achieved value of β is a transient (much larger than $\hat{\beta}_l$); any proposed increase in β moves still farther from equilibrium unless the geometry is altered to increase $\hat{\beta}_l$. The limiting profile is, at present, beyond experimental reach, but it could become relevant in an elongated plasma with shorter relaxation time. If so, a significant part of Tokomak scaling would become accessible and relatively insensitive to many parameters.

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[1]H. Grad and J. Hogan, Bull. Am. Phys. Soc. 14, 1017 (1969).
[2]A. A. Galeev and R. Z. Sagdeev, Zh. Eksperim. i Teor. Fiz. 53, 348 (1967) [Soviet Phys. JETP 26, 233 (1968)].
[3]H. Grad and J. Hogan, to be published.
[4]H. Grad, Phys. Fluids 10, 137 (1967).
[5]M. D. Kruskal and R. M. Kulsrud, Phys. Fluids 1, 265 (1958).
[6]E. Maschke, Association Euratom–Commissariat à l'Energie Atomique Report No. EUR-CEA-FC-429, 1967 (unpublished).
[7]D. Pfirsch and A. Schluter, Max Planck Institute, Munich, Report No. MPI/PF/7/62, 1962 (unpublished).
[8]H. Grad and H. Rubin, in *Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958* (United Nations, Geneva, Switzerland, 1958), pp. 190-197.
[9]B. B. Kadomtsev and V. D. Shafranov, Dokl. Akad. Nauk, SSSR 167, 65 (1966).
[10]L. A. Artsimovich, *et al.*, in *Proceedings of a Conference on Plasma Physics and Controlled Nuclear Fusion Research, Novosibirsk, U. S. S. R., 1968* (International Atomic Energy Agency, Vienna, Austria, 1969), Vol. I, p. 157.
[11]R. Lust *et al.*, Phys. Fluids 4, 891 (1961).
[12]T. H. Jensen and T. Ohkawa, Bull. Am. Phys. Soc. 14, 1015 (1969).

PLASMA STABILIZATION BY FEEDBACK

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A simple model is discussed which illustrates the general features of plasma stabilization by an external feedback system. This indicates that different phase relations in the feedback loop are needed to stabilize differing classes of electrostatic instability.

Several experiments have been performed[1-6] in which an otherwise unstable plasma is stabilized by some form of feedback system—that is, by some external electrical system which "senses" the onset of instability and injects an appropriate "suppression" signal.

The nature of the plasma and the way in which the feedback is applied differ considerably in the various experiments and the quantitative realization of feedback stabilization depends on the detailed arrangements. However, a simple model can account for most of the qualitative features of these experiments and also illustrates some significant differences in the application of feedback stabilization to different types of electrostatic instability.

In this model the feedback mechanism is one which senses the potential at r' and in response to this signal charges up a suppressor element at r. The charge on the suppressor is related to the potential of the sensor by a response function

$$g(\omega)G(r, r'),$$

where G is real and $g(\omega)$ is a complex function of the frequency ω which is defined by the amplification $[|g(\omega)|]$ and phase difference $[\arg g(\omega)]$

introduced by the feedback circuit. Since the feedback must be real and causal (i.e., no output signal can precede its input), $g(\omega)$ satisfies the Kramers-Kronig dispersion relations[7] and is such that $g(\omega)=g^*(-\omega)$ with $g(\omega)\to$ real constant as $\omega\to\infty$.

In the absence of feedback the response of the plasma to an electric potential $\varphi(r)e^{-i\omega t}$ can be represented by a generalized conductivity tensor $\kappa_\omega(r,r')$ such that

$$J_\omega(r)=\int\kappa_\omega(r,r')E_\omega(r')dr'. \tag{1}$$

This leads to the dispersion equation

$$\nabla\cdot\int dr'\epsilon_\omega(r,r')\nabla\varphi(r')=0, \tag{2}$$

where

$$\epsilon_\omega(r,r')=I\delta(r-r')-\frac{4\pi}{i\omega}\kappa_\omega(r,r'). \tag{3}$$

Equation (2) determines the oscillation frequencies and the stability of the system through the eigenvalue ω.

When the suppressor is in operation it introduces "external" charge given by

$$\rho_{\text{ext}}(r)=g(\omega)\int dr'G(r,r')\varphi(r'), \tag{4}$$

so that with the application of feedback the oscillation frequencies are given by

$$\nabla\cdot\int dr'\epsilon_\omega(r,r')\nabla\varphi(r')+g(\omega)\int dr'G(r,r')\varphi(r')=0. \tag{5}$$

Before the effect of the feedback can be discussed it is important to distinguish two different types of instability which can arise from Eq. (2). The first type involves only a single mode of oscillation, which may have positive or negative energy. Growth of this instability is accompanied by an exchange of energy between the oscillation and the plasma medium, e.g., by dissipation. The second type of instability involves two modes of oscillation, one of positive and the other of negative energy. These oscillations become degenerate at the threshold of instability, which can be regarded as the exchange of energy between the two oscillations without any net transfer to the plasma medium. Following Hasegawa[8] we refer to the first instability as dissipative and the second as reactive and we consider the two cases separately.

(i) Dissipative Instabilities. – To discuss general dissipative instabilities we first observe that the power absorbed by the plasma from an oscillating electric field is

$$\text{Re}\int\vec{E}^*\cdot\vec{J}d\tau=\text{Re}\int drE^*(r)\times\kappa_\omega(r,r')E(r')dr', \tag{6}$$

and if the plasma were neutrally stable this would vanish. The necessary and sufficient condition for this[9] is that $\kappa(r,r')$ be anti-Hermitian or equivalently that $\epsilon_{ij}(r,r')$ should be Hermitian. Thus, if the plasma is only weakly unstable we can write $\epsilon=\epsilon_h+\epsilon_a$ where ϵ_h is Hermitian and ϵ_a will be small compared with ϵ_h. Then standard perturbation theory can be applied, treating both ϵ_a and the suppressor term $g(\omega)$ as small perturbations to ϵ_h, to determine the effect of feedback. If, in the absence of feedback, there is a mode of oscillation with frequency ω_0 and growth rate γ_0 then in the presence of the feedback these are modified to

$$\omega=\omega_0+\kappa g\cos(\theta+\Psi), \tag{7}$$

$$\gamma=\gamma_0+\kappa g\sin(\theta+\Psi), \tag{8}$$

where

$$g\equiv|g(\omega_0)|,\quad\theta\equiv\arg g(\omega_0),$$

and

$$\kappa e^{i\Psi}=\frac{\iint\varphi^*(r)G(r,r')\varphi(r')dr\,dr'}{\iint\nabla\varphi^*(r)(\partial/\partial\omega)\epsilon_h(r,r')|_{\omega=\omega_0}\nabla\varphi(r')dr\,dr'}.$$

To stabilize the plasma, therefore, one needs

$$-\kappa g\sin(\theta+\Psi)>\gamma_0.$$

There is thus an optimum phase delay θ in the feedback loop at which a minimum amplification is needed to provide stability. However with larger values of amplification stabilization can be achieved with a phase delay lying within up to $\pm90^\circ$ of the optimum, i.e., over a total range of 180°. For phase delays outside this range the effect of feedback will be to enhance the instability. As a function of the phase delay, feedback will have zero effect on the frequency of the instability whenever its effect on the growth rate (suppression or enhancement) is a maximum. These general features are well exhibited in the experiments of Simonen, Chu, and Hendel[5] and of Keen,[6] where the fluctuation level serves as a measure of γ_0, to which it is related by nonlinear limiting processes.

One may also observe that the optimum phase delay changes by 180° according as

$$W=\iint\nabla\varphi^*\frac{\partial}{\partial\omega}\epsilon_h(r,r')\nabla\varphi(r')dr\,dr' \tag{9}$$

(which is real) is greater or less than zero. This is a consequence of the negative energy character of oscillations with $W<0$.

(ii) Reactive Instabilities. – These can also be treated by perturbation theory but there are several important differences from the dissipative case. A reactive instability does not depend on dissipation and we may neglect ϵ_a in discussing them. Then in the absence of feedback the eigenvalues are either real or occur in complex-conjugate pairs. The threshold for onset of reactive instability occurs when

$$W=\iint \nabla\varphi^*(r)\frac{\partial}{\partial\omega}\epsilon_h(r,r')\nabla\varphi(r')dr\,dr'=0, \quad (10)$$

which represents the fact that the threshold is reached when a positive- and a negative-energy mode become degenerate. To apply perturbation theory to this case we treat both the deviation from threshold and the suppressor term as small quantities, i.e., we expand about the marginally stable state. Then, making use of the above condition one obtains for the perturbed eigenvalue

$$(\omega-\omega_0)^2=-\gamma_0^2+g(\omega)\hat{\kappa}, \quad (11)$$

where now

$$\kappa e^{i\Psi}=\hat{\kappa}=\frac{2\iint\varphi^*(r)G(r,r')\varphi(r')dr\,dr'}{\iint\nabla\varphi^*(r)(\partial^2/\partial\omega^2)\epsilon_h(r,r')|_{\omega=\omega_0}\nabla\varphi(r')dr\,dr'}. \quad (12)$$

ω_0 is the oscillation frequency at threshold and γ_0 the growth rate without feedback.

Conditions under which feedback may produce stability can be determined from (11). Thus if $\omega_0\gg\gamma_0$ stabilization requires that

$$[\kappa g\cos(\theta+\Psi)]_{\omega=\omega_0}>\gamma_0^2,$$
$$[\kappa g\sin(\theta+\Psi)]_{\omega=\omega_0}=0. \quad (13)$$

We see, therefore, that while stabilization can be achieved if the amplification in the feedback system exceeds some threshold value, the phase delay θ in the feedback loop must be much more precisely specified for this (reactive) case than it was for the dissipative case. This very stringent requirement may account for the great difficulty experienced in experiments aimed at stabilizing reactive instabilities by feedback.[10]

The conditions for stabilization when $\omega_0\lesssim\gamma_0$, that is when the instability is an almost purely growing mode, are slightly less stringent. In this case they are

$$[\kappa g\cos(\theta+\Psi)]_{\omega=0}>\gamma_0+\omega_0^2,$$

and

$$[\kappa g\sin(\theta+\Psi)]_{\omega=0}>0. \quad (14)$$

There is again a minimum amplification which will produce stability at the optimum phase delay, but with larger values of amplification stabilization can be achieved with phase delays lying in a $90°$ range on one side of the optimum.

In conclusion, in stabilizing plasmas by feedback, the phase relationship between sensor and suppressor needed to stabilize reactive instabilities differs from that needed for dissipative instabilities. In particular, the phase relationship needed to stabilize a high-frequency ($\omega_0\gg\gamma_0$) reactive instability is especially stringent.

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[1]V. V. Arsenin, V. A. Zhiltsov, and V. A. Chuyanov, in *Proceedings of a Conference on Plasma Physics and Controlled Nuclear Fusion Research, Novosibirsk, U. S. S. R., 1968* (International Atomic Energy Agency, Vienna, Austria, 1969).

[2]V. V. Arsenin, V. A. Zhiltsov, V. K. Likhtenshtein, and V. A. Chuyanov, Zh. Eksperim. i Teor. Fiz. Letters 8, 69 (1968) [Soviet Phys. JETP Letters 8, 41 (1968)].

[3]R. R. Parker and K. I. Thomassen, Phys. Rev. Letters 22, 1171 (1969).

[4]B. E. Keen and R. V. Aldridge, Phys. Rev. Letters 22, 1358 (1969).

[5]T. C. Simonen, T. K. Chu, and H. W. Hendel, Phys. Rev. Letters 23, 568 (1969).

[6]B. E. Keen, Phys. Rev. Letters 24, 259 (1970).

[7]See for example, L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon, London, 1960), p. 256.

[8]A. Hasegawa, Phys. Rev. 169, 204 (1968).

[9]J. E. Drummond, R. A. Gerwin, and B. G. Springer, J. Nucl. Energy: Pt. C 2, 98 (1961).

[10]E. G. Murphy, private communication.