meager data at present available makes even more clear the need for further experimental investigation of this area, the importance of which with regard to ideas on symmetry breaking is clear. We have two remarks in this respect. Firstly, the above results lead to very large effects in Reaction (1d) in the resonance region. For example, at 330 MeV the total cross section is predicted to be 276 μb for $x=0$, 179 μb for $x=-0.2$. Secondly, as stressed in Ref. (3) a study of the inverse reaction

$$\pi^- + p \rightarrow \gamma + n$$

completely avoids any of the uncertainties associated with the use of a deuterium target. Such an experiment has already been reported[15] at a somewhat higher energy $[E_\gamma(\text{lab}) = 520 \text{ MeV}]$ and agrees with the results from the Bonn experiment[13] quoted above at this energy. It is of great importance to carry out such measurements over the region of the first resonance.

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INERT HADRON REACTION FOR THE DETERMINATION OF THE POMERANCHUK TRAJECTORY SLOPE*

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A reaction with no expected secondary Regge poles ($\gamma p \rightarrow \varphi p$) is suggested as a means for direct experimental study of the Pomeranchuk exchange amplitude. From existing data on this reaction, we find a Pomeranchuk trajectory slope of $\alpha_P'(0) \simeq \frac{1}{2}$.

One of the most puzzling aspects of hadronic particle physics is the nature of the Pomeranchuk singularity.[1] Phenomenological studies of the Pomeranchuk exchange amplitude in common elastic scattering processes are unfortunately hampered by the occurrence of important secondary trajectory contributions. Consequently, the following three important questions concerning the Pomeranchukon are at least partially unanswered:

(1) Is the Pomeranchuk trajectory flat or sloping?

(2) What role do Regge cuts play in the Pomeranchuk amplitude?

(3) What is the dynamical origin of the Pomeranchuk contribution? (Is the Pomeranchuk built from background as semilocal duality models would suggest?)

Unambiguous answers to the preceding questions are unlikely to be forthcoming from studies of the common elastic scattering processes $\pi^\pm p$, $K^\pm p$, $\bar{p}p$, and pp. The purpose of this Letter is to point out the existence of an inert reaction in which all three channels are presumed to be decoupled from the secondary trajectories (P', ρ, ω, N_α, Δ_δ, etc.) and which can be used for direct experimental study of the characteristics of Pomeranchuk exchange. We will show that the limited existing data on this reaction already provide some suggestive answers to the questions posed above regarding the Pomeranchukon.

The elastic scattering process

$$\varphi + p \to \varphi + p \tag{1}$$

is possibly the only inert hadron channel that is accessible to experimental study. There is now considerable evidence that the φ meson is essentially decoupled from nonstrange hadrons. A compilation of the experimental evidence[2-6] regarding the φ decoupling from nonstrange particles is given in Table I. A mnemonic interpretation of this decoupling phenomenon is provided by a quarklike model with the φ built from $\lambda\bar{\lambda}$ strange quarks and with the N, ω, ρ, f^0, etc. particles built from nonstrange quarks. Such component structure automatically decouples all normal exchange trajectories from Reaction (1).

In the t channel ($\bar{N}N \to \bar{\varphi}\varphi$) of Reaction (1) only $C=+1$, $I=0$ states (the P and P') are allowed by charge conjugation. The P' trajectory, as identified with the f^0 meson,[7] has no $\lambda\bar{\lambda}$ component and thereby decouples in the t channel.[8] The approximately SU(3) unitary singlet nature[7] of the P (as inferred, for example, from the near equality of high-energy πp and Kp total cross sections) allows a P coupling to $\bar{\varphi}\varphi$ through its $\lambda\bar{\lambda}$ component. The s and u channels for Reaction (1) are identical. The φNN^* couplings for resonances or exchanges in these channels should be zero by the quark-component mnemonic or, equivalently, by the direct experimental evidence reviewed in Table I.

From these considerations we conclude that φp elastic scattering is exclusively due to the Pomeranchuk contribution. Furthermore, from the finite-energy sum rule (FESR) viewpoint, the Pomeranchuk exchange amplitude in Reaction (1) cannot be built from resonances because of the decoupling of direct-channel N^*'s. In the FESR for the φp elastic reaction there exists an absorptive part of the amplitude considerably below threshold (eg., $\varphi p \to K^+\Lambda$) which may contribute substantially to building the Pomeranchukon.

The φp elastic reaction also proves an exciting possibility for determining whether the Pomeranchuk amplitude is responsible for the fixed-t dip structure observed in many elastic hadronic processes. There are two alternate viewpoints concerning the origin of dip structure.[9] In the classical diffraction picture the Pomeranchuk contribution (i.e., diffraction scattering) provides dip structure. On the other hand, empirically the prominent dips occur only in elastic channels with direct-channel resonances (i.e., $\pi^\pm p$, K^-p, and $\bar{p}p$). In the conventional Regge model this dip structure is ascribed to secondary trajectory effects which are dual with the direct-channel resonances. Since the φp channel is decoupled both from direct-channel resonances and secondary trajectory exchanges, observation of any dip structure in Reaction (1) would favor the diffraction view point and vice versa.

The qualitative idea of vector-meson dominance for photon-induced processes provides an immediate handle for experimental study of Reaction (1). With the usual quark components, the amplitude for the reaction[8]

$$\gamma p \to \varphi p \tag{2}$$

is directly proportional to the amplitude for elastic scattering of transversely polarized φ mesons on protons:

$$\frac{d\sigma}{dt}(\gamma p \to \varphi p) = \frac{e^2}{4\gamma_\varphi^2}\frac{d\sigma}{dt}(\varphi_{\rm tr} p \to \varphi_{\rm tr} p). \tag{3}$$

Even if vector dominance turns out not to be

Table I. Compilation of experimental evidence on the decoupling of φ mesons from nonstrange hadrons.

Process	Kinematic region	Ratio of ω to φ cross section or ratio of coupling constants	Remarks	Reference
$\varphi \to \rho\pi$, $\omega \to \rho\pi$	Decay	~600	(Deduced from low rate of $\varphi \to \rho\pi$ decay)	2
$\pi^+p \to \Delta^{++}(\varphi,\omega)$	Small t	$\simeq$600 (3.9 GeV/c)	ρ exchange	3
$\pi^-p \to n(\varphi,\omega)$	Small t	30-60 (1.5-4 GeV/c)	ρ exchange or N^* direct channel	4
$K^-p \to \Lambda(\varphi,\omega)$	Small u	>20 (2-5 GeV/c)	N_α, N_γ baryon exchange	5
$\pi^+n \to p(\varphi,\omega)$	Small u	>40 (2.1 GeV/c)	N_α, N_γ baryon exchange or N^* direct channel	6

strictly true quantitatively, its approximate validity is probably sufficient for the qualitative study of Reaction (1) through Eq. (3). Consequently data on $\gamma p \rightarrow \varphi p$ bears directly on the nature of the Pomeranchuk amplitude.

A limited amount of data has been collected on Reaction (2) up to 18 GeV/c.[10,11] These data have been fitted by the form

$$d\sigma/dt = ae^{bt} \tag{4}$$

at each energy. For a Pomeranchuk Regge pole with a trajectory $\alpha_P(t) = 1 + \alpha_P'(0)t$ and an exponential residue e^{R^2t}, we expect

$$a = \text{const},$$

$$b = 2R^2 + 2\alpha_P'(0) \ln E_\gamma, \tag{5}$$

with E_γ in units of GeV.

The present experimental data[10,11] on Reaction (2) are consistent with a constant value for the parameter a but do not provide a definitive test. Assuming a = const, the values obtained for the parameter b, shown in Fig. 1, strongly suggest that $\alpha_P'(0)$ is nonzero. The preference shown by the data is for a slope $\alpha_P'(0) \sim \frac{1}{2}$. As stressed previously, data on Reaction (2) can perhaps provide the only unambiguous determination of the Pomeranchuk parameters. Other recent determinations based on Regge analysis of $\pi^\pm p$,[12] $K^\pm p$,[13] and pp[14] scattering with Pomeranchuk and secondary Regge poles and cuts also yield a Pomeranchuk slope in the range

$$\alpha_P'(0) \simeq 0.35 \text{ to } 0.5 \ (\text{GeV}/c)^{-2}. \tag{6}$$

Although the evidence on the Pomeranchuk slope from these analyses of common elastic channels is more indirect, the overall consistency of the various determinations gives strong support for a sloping Pomeranchuk trajectory.

Recent total cross-section data from Serpukhov[15] suggest that there are substantial cut contributions associated with iterations of a Pomeranchuk Regge pole.[16] In this circumstance the trajectory slope determined above represents an effective value from Pomeranchuk pole plus Pomeranchuk cuts. However, since the cuts are observed to interfere destructively with the pole, the lower limit on the slope of the primordial Pomeranchuk trajectory is greater than the effective slope in Eq. (6).[17] An upper limit of approximately $\alpha_P'(0) = 0.6$ on the primordial Pomeranchuk slope is obtained from the nonobservance of an SU(3)-singlet 2^+-meson state below 1.3 GeV. It is amusing to note that the primordial slope could be $\frac{1}{2}$, in which case both the slopes and intercepts of Regge trajectories could obey a "$\frac{1}{2}$-unit quantization" rule.[18]

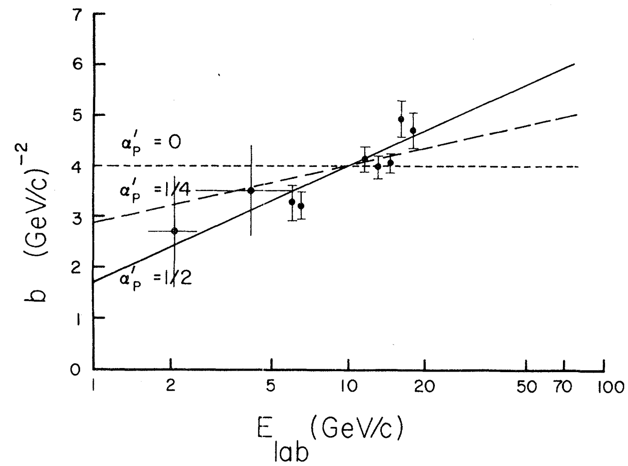

FIG. 1. Fitted slopes b as a function of $\ln(E_\gamma/1 \text{ GeV})$ for the reaction $\gamma p \rightarrow \varphi p$, using the form $d\sigma/dt$ = (const) $\times e^{bt}$. The maximum $|t|$ used in the fit was 0.7 (GeV/c)^2. [Data from Ref. 10 (solid circle) and Ref. 11 (solid triangle).] The derivative of b with respect to $\ln(E_\gamma)$ determines the slope of the Pomeranchuk trajectory [cf. Eq. (5)]. Curves for representative values of $\alpha_P'(0)$ are plotted.

A non-negligible slope for the Pomeranchukon has serious consequences for semilocal duality of direct-channel resonances and Regge-pole exchanges.[19] Semilocal duality implies that a Regge amplitude with a momentum-transfer–dependent phase will generate direct-channel resonance loops.[20,21] This is inconsistent with a sloping Pomeranchukon in Reaction (2) if, in fact, no direct-channel resonances are present. On the other hand, the existence of important direct-channel resonances in Reaction (2) would be contradictory to the coupling selection rules of the quark model. Experimental studies of the low-energy behavior of Reaction (2) would, therefore, be of great interest.

The qualitative behavior of the t dependence attributed to the Pomeranchuk exchange can also be studied using Reaction (2). Figure 2 contrasts the differential cross sections for the reaction $\gamma p \rightarrow \varphi p$ (P exchange) with $\gamma p \rightarrow \rho p$ (P, P' exchanges) and $\pi^- p \rightarrow \pi^- p$ (P, P', ρ exchanges) at low energy. The ρ-production data[10,11] indicate close similarity to the $\pi^- p$ elastic-scattering data.[22] In particular the dip-bump structure appears to be quite similar. If the dip structure is due to secondary trajectories (i.e., the P'), then the φ differential cross section should not exhibit

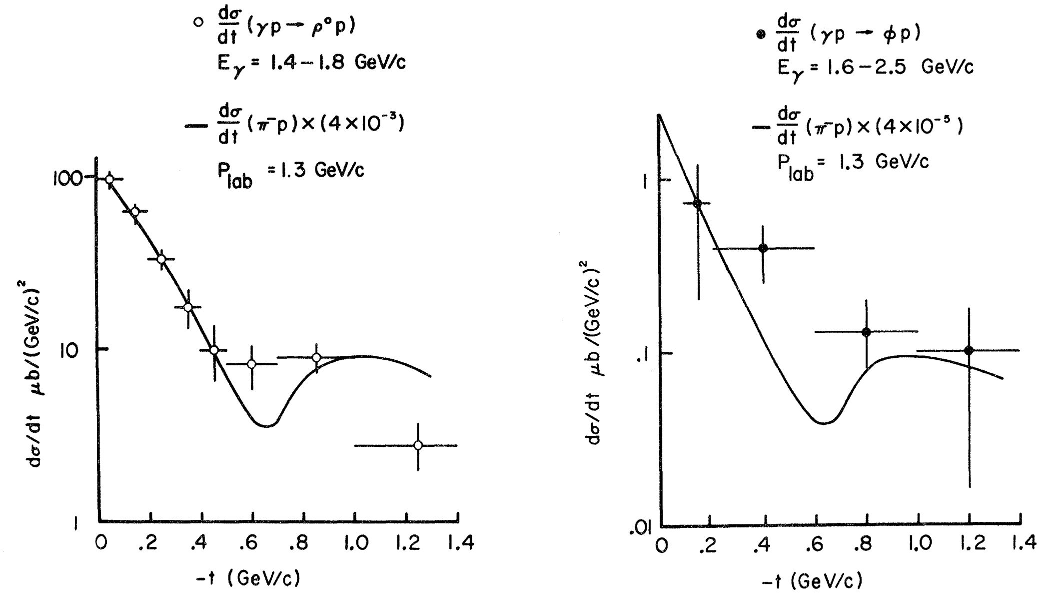

FIG. 2. Comparison of qualitative features of low-energy data on the reactions $\gamma p \to \rho^0 p$, $\gamma p \to \varphi p$, and $\pi^- p \to \pi^- p$.

structure. Although the φ data are statistically poor they nevertheless do suggest a qualitatively different behavior from $\gamma p \to \rho^0 p$.[10,11] High-energy data on $\gamma p \to \varphi p$ are consistent with a smooth expontial decrease in t, whereas the $\gamma p \to \rho p$ data show indications of a break structure similar to that observed in πp elastic scattering. A summary of the characteristics of these reactions is given below[10,11,22]:

	$\gamma p \to \varphi p$	$\gamma p \to \rho^0 p$	$\pi^\pm p \to \pi^\pm p$
t-channel exchange	P	P, P'	$P, P'\rho$
Direct channel	Decoupled	N, Δ	N, Δ
$d\sigma/dt$ at $t=0$ vs energy (expt)	Flat	Decreasing	Decreasing
Slope of $d\sigma/dt$ (expt)	Exponential	Break at $t \simeq -0.5$	Break at $t \simeq -0.5$
$db/d\ln E = \alpha'_{\text{eff}}(t=0)$ (expt)	$\sim\frac{1}{2}$	~ 0	~ 0

The qualitative features of the data outlined above are consistent with the duality of secondary trajectories (P', ρ) with direct-channel resonances. The fact that the effective trajectory slopes from $\gamma p \to \rho^0 p$ and $\pi^\pm p$ are quite different from the slope in Eq. (6) illustrates the complications introduced by secondary trajectories and reinforces the significance of a clean extraction of the Pomeranchuk parameters from $\gamma p \to \varphi p$.

In conclusion, we have found convincing evidence that the Pomeranchuk trajectory has a nonnegligible slope of the order of $\frac{1}{2}$. This rules out a broad class of theoretical models for elastic scattering which rely on a flat Pomeranchukon. As to the dynamical origin of the Pomeranchukon, a clear experimental resolution may be possible with precise data on $\gamma p \to \varphi p$. It well may be that the Pomeranchukon is dual to direct-channel N^* resonances on low-lying trajectories. If this is the case, resonance structure should appear in $\gamma p \to \varphi p$. On the other hand, if no resonances are observed, a connection between loops produced by the exchange of sloping trajectories[19] and loops of direct-channel resonances would be suspect in elastic channels.

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AMPLITUDES WITH MANDELSTAM ANALYTICITY AND DUAL STRUCTURE*

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A general representation for scattering amplitudes is proposed which is crossing symmetric, satisfies Regge behavior and duality, and possesses double spectral functions. As the width parameter goes to zero, the resonance poles on the second sheet move onto the real axis and the amplitude reduces to the Veneziano model. Partial waves generated from this amplitude exhibit correct threshold behavior in both the real and imaginary parts. A generalization to more-point functions is also proposed.

We would like to present here a representation for scattering amplitudes which have Mandelstam analyticity[1] (i.e., nonvanishing double spectral functions), and are Regge behaved and dual with complex resonance poles (on the second sheet) and no "ancestors". In the zero-width limit it reduces to the Veneziano formula.[2] Also, various recent generalizations[3,4] of the Veneziano formula are special cases of our representation. We also give some simple explicit examples of our spectral functions (satisfying the above constraints) which, moreover, have the correct threshold behaviors for all partial waves both for real and the imaginary parts, thus laying the ground work for unitarization. Extensions to more-point functions are possible. An explicit example for the five-point function and its Mandelstam analog is presented here.

Our starting point is the following basic form:

$$A(s,t)=\int_0^1 dx_1\int_0^1 dx_2\, x_1^{-\alpha_s}x_2^{-\alpha_t}\rho(x_1,x_2;s,t), \tag{1}$$

where $\alpha_s=a+bs$ is the linear part of the trajectory. If we choose for ρ in Eq. (1) the value

$$\rho_M(x_1,x_2;s,t)=\pi^{-2}\int dx\int dy\;\sigma(x,y)x_1^{\,x}x_2^{\,y}, \tag{2}$$