MEASUREMENT OF THE SOUND-VELOCITY ANISOTROPY IN ORIENTED SINGLE CRYSTALS OF bcc HELIUM-3†

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The longitudinal and transverse sound velocities have been measured in x-ray–oriented single crystals of bcc He^3 at a molar volume of 22.6 cm^3. General agreement with the predicted anisotropy is found; however, the transverse velocities fall significantly below the expected values. A θ_D of 23.7°K calculated using the elastic moduli is in excellent agreement with the value extrapolated from specific-heat measurements.

Sound-velocity anisotropies of approximately 100 to 200 m/sec have been predicted for the longitudinal and transverse modes in the bcc and hcp phases of He^3 and He^4.[1-4] Anisotropies of this order have been measured in oriented crystals of He^4[5-7] but have not been found in unoriented samples of bcc He^3 or He^4.[8,9] In this Letter we report the first measurements of the longitudinal and transverse velocities in oriented single crystals of bcc He^3.

A single-ended, pulse-echo method at a frequency of 10 MHz was used to measure the sound velocities. The single-ended system was chosen because it lent itself to the design and construction of a freezing cell that was physically very simple (see Fig. 1). It was hoped that this geometry would enhance the chances of growing high-quality crystals. The two major reasons for selecting Lucite for the cell body material were its low x-ray absorption coefficient and its low thermal conductivity. The latter property was important in that it forced the heat of fusion to be carried away through the growing crystal, helping insure single-crystal growth. Other qualities contributing to the choice of this material were its hardness, which permitted fine polishing; its strength, which allowed keeping the wall thickness to 0.040 in.; and its transparency, which permitted visual inspection of crystal growth.

The pressure necessary to produce the solid was generated in a "gasifier"[10] system located in a second Dewar. The "gasifier" bulb had a volume of 6 cm^3 and was joined to the cell with stainless steel capillary tubing. The system was nearly at its capacity in producing the working pressure of 64.5 atm.

The x-ray beam entering through the top of the cryostat was collimated to $\frac{1}{16}$ in. a few cm above the cell which was mounted near the bottom of the evacuated center bore of the Mössbauer-type Dewar.[11] The radiation scattered by the crystal was detected with a Polaroid x-ray camera mounted under the Dewar. Unlike the case of hcp He^4[7] when three to six Laue reflections were usually visible, most often only two or three dots were detected which in all cases could be classified as either 200 or 101 reflections. The large difference in the intensities of these two reflections due to the temperature factor made identification of the dots an easy matter; however, in some cases, there was ambiguity in the orientation. For example, if there were only two reflections visible and both were of the 101 type and separated by 60° or 120°, there were two orientations of the crystal possible, related by a rotation of 60° about the [111] axis.

Most crystals were grown at nearly constant pressure in approximately one hour by carefully lowering the temperature of the cold finger. The crystals tended to grow with the [101] axis generally parallel to the direction of crystal growth. In the hope that changing the growing rate might help stimulate other orientations, different growing rates were tried but led to no success. In fact, it was found that very slow growing rates produced crystals which showed a great amount

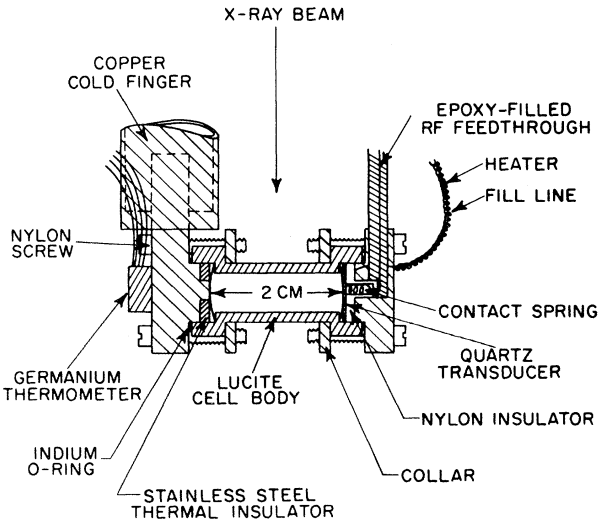

FIG. 1. Details of the freezing cell.

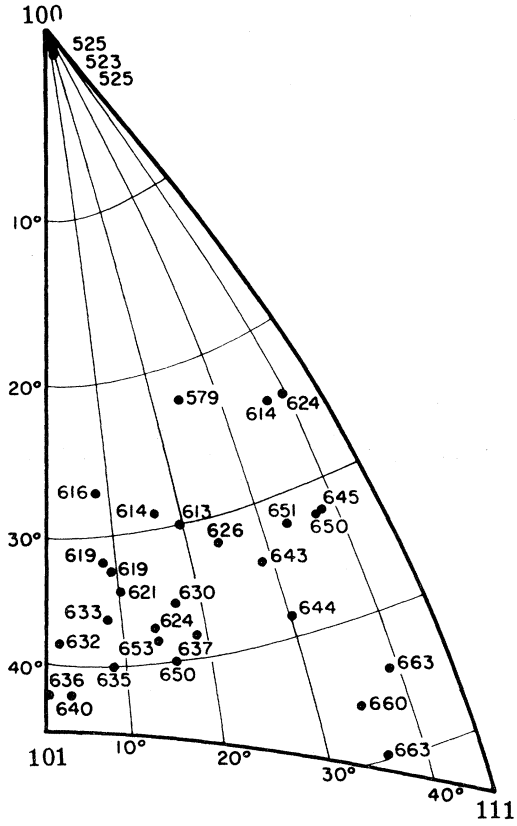

FIG. 2. Longitudinal sound velocities in m/sec plotted on a stereographic projection of 1/48 of the reference sphere. Each dot corresponds to a different crystal.

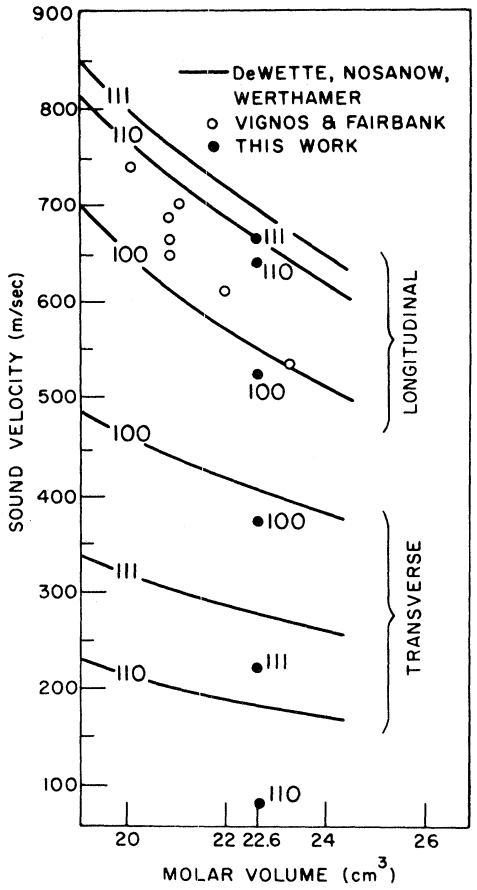

FIG. 3. Comparison of sound velocities derived from the measured elastic moduli with the theoretical calculations of Ref. 1. Longitudinal measurements of Ref. 9 on unoriented crystals are also indicated.

of polygonization. Samples grown at rapid rates were more often polycrystalline and/or led to a weak electronic signal. In all cases, the crystals appeared to be optically clear.

The longitudinal sound-velocity data are plotted on a stereographic projection of 1/48 of the reference sphere in Fig. 2. Each dot corresponds to a different crystal and represents the direction of sound propagation. The number associated with each of the dots gives the corresponding measured velocity in meters per second. In principle, it is possible to determine all of the elastic constants knowing only the longitudinal velocities, but it should be noted that this mode is very insensitive to changes in C_{44}. The longitudinal data accurately determine only C_{11} and $(C_{12}+2C_{44})$. The measured transverse sound velocities corresponding to the higher-frequency phonon branch in ten crystals show that shear waves propagate in the [100] direction at 375 m/sec. Using this information, we calculate that

$$C_{11}/\rho = 2.76\times10^{9}\ \mathrm{cm^2/sec^2},$$

$$C_{12}/\rho = 2.62\times10^{9}\ \mathrm{cm^2/sec^2},$$

$$C_{44}/\rho = 1.41\times10^{9}\ \mathrm{cm^2/sec^2}.$$

From the scatter in the data, the accuracy of each of the values is estimated to be better than ±2%.

A comparison with the theory of DeWette, Nosanow, and Werthamer[1] is shown in Fig. 3. Although in qualitative agreement with the predicted anisotropy, the measured velocities fall below the expected values with considerable discrepancy in the transverse modes. The Debye temperature can be calculated using the expression

$$\theta_D = (h/k_B)(18\pi^2 N)^{1/3} I^{-1/3},$$

where

$$I = \frac{1}{4\pi}\int\left(\frac{1}{v_0^{\,3}}+\frac{1}{v_1^{\,3}}+\frac{1}{v_2^{\,3}}\right)d\Omega,$$

and v_0, v_1, and v_2 are the longitudinal and transverse (2) velocities. Fedorov[12] has derived an expression for I in terms of the elastic moduli alone. Using this approximation, we find that our moduli lead to a θ_D of 23.7°K in excellent

agreement with the value extrapolated from heat-capacity measurements by Sample and Swenson.[13] This compares with 29°K as calculated by DeWette, Nosanow, and Werthamer.

We find no indication of any anomaly in the phonon dispersion curves which might explain the low-temperature specific-heat anomaly.[13-16] And indeed, this anomaly has recently been attributed by Varma[17] as arising from a phonon-mediated long-range spin interaction, although some criticism has been raised by Guyer.[18]

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FLUIDS WITH SEVERAL PHASE TRANSITIONS

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For a fluid in which the interaction potential has a hard core plus a negative part, softening of the hard core can produce a second transition if a first already exists. We give a general argument for the occurrence of the second transition in the lattice gas, plus explicit results for one-dimensional fluid models with two first-order transitions. One such model also provides an example of the breakdown of the law of rectilinear diameters.

For a model of a fluid in which the pair potential $\varphi(r)$ has a hard core plus a negative part, the occurrence of a first-order transition depends upon dimensionality and upon the range of the attraction.[1] In this note we further conclude that in such models already exhibiting a phase transition, one can induce a second transition by simply softening the hard core judiciously.

We begin with a general argument relevant to a lattice gas of any dimensionality and then give explicit exact results for a one-dimensional fluid.

I. The ν-dimensional lattice gas.—The lattice gas is a cell model of a fluid in which the interparticle potential depends only upon the relative location of the cells containing the particles. The hard core is provided for by exclusion of multiple occupancy of a cell; suppose in addition that the positive potential V_0 between pairs of particles in adjacent cells is also infinite. Let particles in non-neighboring cells interact via a negative potential capable of producing a first-order transition[1] with a critical point at (ρ_1, T_1). Using the cell volume as the volume unit, the maximum density of the system is clearly $\leq \frac{1}{2}$.

Now consider V_0 finite but arbitrarily large.