those involving all pions.[7]

We wish to thank our scanners and measurers for their diligent efforts. We are grateful to Dr. James Sanford, Dr. Hugh Brown, and Dr. Lou Voyvodic and the staffs of Brookhaven National Laboratory and Argonne National Laboratory for their cooperation in obtaining this film. One of us (G.A.S.) is grateful to Professor L. W. Alvarez, in whose group he was a guest during the early part of this work.

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[4]The Monte Carlo model was based on phase space plus $K^*(890)$ production. We have found this model of phase space plus resonance to be an excellent approximation to the several all-pionic annihilation processes we have studied in this experiment.

[5]The reactions included in Fig. 1, other than (1)-(6), are $K_1^0K^\pm\pi^\mp$, $K_1^0K_1^0\pi^0$, $K_1^0K_1^0\pi^-$, $K_1^0(K^0)\pi^-$, $K_1^0K^-\pi^0$, $K^+K^-\pi^+\pi^-$, $K_1^0K_1^0\pi^+\pi^-$, $K_1^0K^\pm\pi^\mp\pi^0$, $K_1^0(K^0)\pi^+\pi^-$, $K_1^0K_1^0\pi^-\pi^0$, $K_1^0K^\pm\pi^\mp\pi^-$, $K_1^0K^\pm\pi^\mp\pi^-\pi^+\pi^0$, and $K_1^0(K^0)\pi^+\pi^-\pi^+\pi^-$.

[6]A. G. Frodesen *et al.*, Nucl. Phys. B10, 307 (1969); C. D'Andlau *et al.*, Nucl. Phys. B5, 693 (1968); J. Barlow *et al.*, Nuovo Cimento 50, 701 (1967).

[7]At the 1969 Boulder Conference, the combined data of the Michigan and Michigan State University bubble chamber groups were presented for Reaction (1). A suggestive enhancement in the cross section for $K_1^0K_1^0\omega^0$ was pointed out. [H. Ring *et al.*, cited by M. Derrick, in *Proceedings of the Boulder Conference on High Energy Physics, 1969*, edited by K. T. Mahanthappa, W. D. Walker, and W. E. Brittin (Colorado Associated Univ., Boulder, Colorado, 1970), p. 291; see also H. Ring *et al.*, *ibid.*, p. 495.] With the addition of Reaction (3), which has a substantial ω^0 signal, our group no longer observes any significant enhancement for $K\bar{K}\omega$. Removal of the ω^0 events in Reactions (1), (3), and (5) has little, if any, bearing on the $K^*K\pi\pi$ effects presented in this paper.

OBSERVATION OF $N^*(1720)$ DECAY TO $\Delta(1236)\pi$ FROM π^+p INTERACTIONS AT 13.1 GeV*

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Evidence is presented for the production of $N^*(1720)$ ($M=1719\pm6$ MeV, $\Gamma=63\pm12$ MeV) and subsequent decay to $\Delta(1236)\pi$ in π^+p interactions at 13.1 GeV. Analysis of the sequential decay $N^*(1720)\rightarrow\Delta^{++}\pi^-\rightarrow p\pi^+\pi^-$ limits the spin to values greater than $\frac{3}{2}$.

Enhancements in the $p\pi^+\pi^-$ mass spectrum near 1700 MeV have been observed in production experiments from πp, Kp, and pp interactions,[1,2] but there has been disagreement on the mass, width, and decay modes. The experiment with the best statistical significance has been recently published by Barnes et al.[2] From K^-p interactions at 4-7 GeV they reported a mass of 1712 MeV, width of 70 MeV, and a decay to $p\pi\pi$ with no $\Delta(1236)\pi$ intermediate state.

In this Letter, we submit data of comparable statistical accuracy from the reaction

$$\pi^+p\rightarrow\pi^+p\pi^+\pi^- \quad (1)$$

at 13.1 GeV. The mass and width of the $N^*(1720)$ are shown to be 1719 and 63 MeV, respectively, in excellent agreement with the values stated above. In contrast, evidence is presented for a dominant $\Delta\pi$ decay mode of the enhancement reported here. Also, we give an analysis of the sequential decay $N^*(1720)\rightarrow\Delta^{++}\pi^-\rightarrow p\pi^+\pi^-$ showing that J is greater than $\frac{3}{2}$.

The events used in this analysis were obtained from a 230 000-picture exposure of the Stanford Linear Accelerator Center 82-in. hydrogen bubble chamber to a 13.1-GeV π^+ beam. Of the 60 000 four-prong events measured, approximately 10 200 fit Reaction (1). The $p\pi_b{}^+\pi^-$ mass spectrum below 3.2 GeV is shown in Fig. 1(a), where $\pi_b{}^+$ was chosen as the positive pion with the larger four-momentum transfer from the incident pion. The $N^*(1720)$ appears as an 8-standard-deviation peak over a smooth background. A fit using an s-wave Breit-Wigner shape plus a smooth hand-drawn background yields

$$M=1719\pm6 \text{ MeV}, \quad \Gamma=63\pm12 \text{ MeV}.$$

The shaded portion of Fig. 1(a) contains only

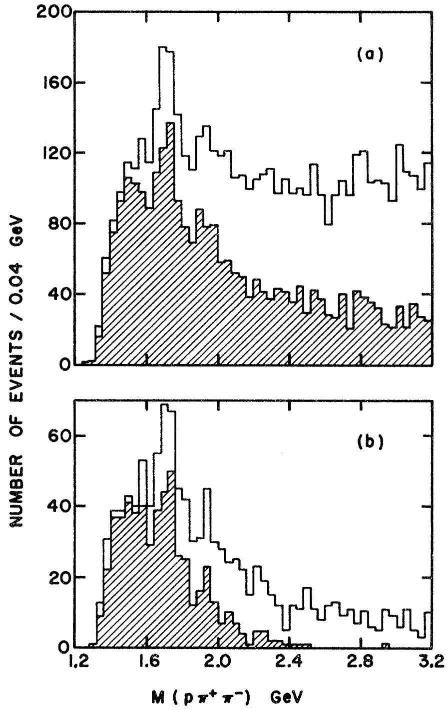

FIG. 1. (a) The $p\pi^+\pi^-$ mass spectrum below 3.2 GeV. The shaded histogram contains events with $1136 < M(p\pi^+) < 1336$ MeV. (b) Same as (a) for the "clean" sample of events.

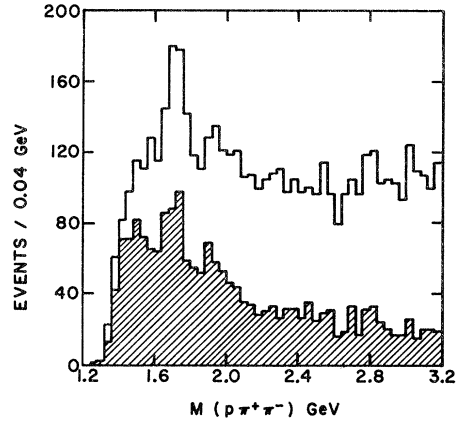

FIG. 2. The $p\pi^+\pi^-$ mass spectrum below 3.2 GeV. The shaded histogram contains events in a narrow Δ^{++} mass band, $1160 < M(p\pi^+) < 1280$ MeV.

those events for which $M(p\pi_b{}^+)$ lies in the Δ^{++} band.[3] A histogram of the events not in the Δ^{++} band (not shown) shows no enhancement at 1720.

A large fraction of the events used in Fig. 1(a) is the result of the intermediate states $\Delta^{++}\rho^0$, $\Delta^{++}f^0$, $p\pi^+\rho^0$, and $p\pi^+f^0$, and is background to the reactions of interest here. These background channels do not contribute to that subsample of events (hereafter referred to as "clean") for which the angle between the π^- and $\pi_a{}^+$ in the $p\pi_b{}^+\pi^-$ rest system is greater than 90°. The $p\pi_b{}^+\pi^-$ system is peripherally produced (98% of the events have a c.m. production angle θ_c such that $\cos\theta_c > 0.9$); consequently, the clean sample does not contain Δ^{++}'s produced at small four-momentum transfer nor events with $M(\pi_a{}^+\pi^-)$ less than about 1500 MeV in the region of the $p\pi_b{}^+\pi^-$ enhancement. This "clean" subsample was used to produce Fig. 1(b); again, events in the Δ^{++} band are shown shaded. Mass and width values obtained from a fit to Fig. 1(b) are consistent with those obtained from Fig. 1(a).

To determine the fraction of $N^*(1720)$'s which decay to $\Delta^{++}\pi^-$, two methods were used. In the first, the fraction of events remaining after a narrow cut on the $p\pi^+$ mass $[1160 < M(p\pi_b{}^+) < 1280$ MeV$]$ was calculated assuming (a) a phase-space distribution:

$$f_{\mathrm{ps}} = \int_\Delta d\varphi / \int d\varphi = 0.268; \quad (2)$$

and (b) a $\Delta^{++}\pi^-$ intermediate state:

$$f_\Delta = \frac{\int_\Delta |f_{\mathrm{BW}}(m,\Gamma)|^2 d\varphi}{\int |f_{\mathrm{BW}}(m,\Gamma)|^2 d\varphi} = 0.615, \quad (3)$$

where $\int_\Delta d\varphi$ represents integration over the narrow $p\pi^+$ mass band of phase space; $\int d\varphi$, integration over the entire phase space; and $|f_{\mathrm{BW}}(m, \Gamma)|^2$, the Breit-Wigner function for the Δ^{++} resonance.[4] The branching ratio for $[N^*(1720) \to \Delta^{++}\pi^-]/(\Delta^{++}\pi^- + p\pi^+\pi^-)$, a, was determined from the relation

$$N_\Delta/N = (1-a)f_{\mathrm{ps}} + af_\Delta, \quad (4)$$

where N_Δ is the experimentally observed number of events above background after the narrow cut, and N is the observed number of events above background with no cut. The $p\pi_b{}^+\pi^-$ mass spectrum is shown again in Fig. 2; the shaded area contains the events remaining after the narrow $p\pi_b{}^+$ mass cut. Using smooth hand-drawn background estimates on Fig. 2, the value of 0.70 ± 0.15 was obtained for a.

For the second method, an approach similar to that used in Ref. 2 was adopted. The maximum-likelihood method was used to obtain the fractions of Δ^{++} events, ϵ, and a parameter associated with the Δ^{++} decay, b, for a series of 40-MeV bins in the region of the $N^*(1720)$.[5] The total number of events in each bin was multiplied by the value of ϵ for that bin to produce the mass spectrum for $\Delta^{++}\pi^-$, which is shown in Fig. 3(a). By dividing the number of $\Delta^{++}\pi^-$ events above

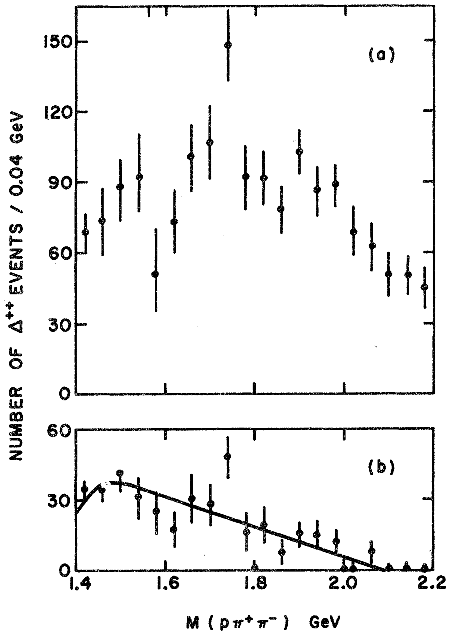

FIG. 3. (a) Number of Δ^{++} events vs $M(p\pi^+\pi^-)$. (b) Same as (a) for the "clean" sample of events. The solid curve is the result of a double-Regge-pole model calculation.

background in the $N^*(1720)$ enhancement by the number of total events above background, the value of 0.75 ± 0.20 was obtained for the branching ratio.

This second method was used to produce the $\Delta^{++}\pi^-$ mass spectrum for the "clean" sample shown in Fig. 3(b). An estimate of the $\Delta^{++}\pi^-$ background was calculated from a double-Regge-pole model.[6] The solid curve represents this background. Although the curve was normalized to the number of events in the plot, it is apparent that a significant enhancement above background exists at 1720 MeV.

Table I. Parameters obtained from the production and decay angular distributions for the sequential decay $N^*(1720) \to \Delta^{++}\pi^- \to p\pi^+\pi^-$.

L	Coefficient of Y_{L0}
0	0.282
2	0.115 ± 0.023
4	0.039 ± 0.024
6	0.002 ± 0.023
8	0.007 ± 0.024
$\lvert A_{3/2}\rvert^2 - \lvert A_{1/2}\rvert^2$	-0.162 ± 0.085

The observed ratio of Δ^{++}/Δ^0 events on the Dalitz plot of the $N^*(1720)$ is consistent with the ratio 9:1 expected for an isospin-$\frac{1}{2}$ object. Such an isospin assignment would allow the decay $N^*(1720) \to \Lambda K$. However, the ΛK^+ mass spectrum from the final state $\pi^+\Lambda K^+$ shows no enhancement near 1720 MeV; using all events in this mass region, the upper limit on the branching ratio $[N^*(1720) \to (\Lambda K^+)_{\rm tot}]/[N^*(1720) \to p\pi^+\pi^-]$ is 0.025.[7]

Assuming that the $N^*(1720)$ enhancement is the result of a single resonant state, a spin-parity analysis of the sequential decay $N^*(1720) \to \Delta^{++}\pi^- \to p\pi^+\pi^-$ was carried out. The angular distribution of the Δ^{++} for a particle of spin J decaying to $\Delta^{++}\pi^-$ may be written[8]

$$I(\theta) = \Big\{ \sum_{L_e}^{2J-1} (2L+1)^{1/2} \big[|A_{3/2}|^2 C(JLJ; \tfrac{3}{2}0\tfrac{3}{2}) + |A_{1/2}|^2 C(JLJ; \tfrac{1}{2}0\tfrac{1}{2}) \big] t_{L0} Y_{L0}(\theta) \Big\} / \sqrt{\pi}, \quad (5)$$

where θ is the angle between the target proton and the Δ^{++} in the $p\pi^+\pi^-$ rest frame, A_λ is a helicity amplitude, C is a vector addition coefficient, t_{L0} is a statistical tensor, Y_{L0} is a spherical harmonic, and L_e indicates that the sum is taken over even values of L. The decay distribution of the Δ^{++} may be written

$$I(\psi) = \tfrac{1}{2} - \tfrac{1}{2}(3\cos^2\psi - 1)(|A_{3/2}|^2 - |A_{1/2}|^2), \quad (6)$$

where ψ is the angle of the decay proton with respect to the line of flight of the Δ^{++} in the Δ^{++} rest system. The helicity amplitudes are constrained by the relation

$$|A_{1/2}|^2 + |A_{3/2}|^2 = \tfrac{1}{2}. \quad (7)$$

Projection operators were used on the "clean" sample of events to obtain the coefficients of Y_{L0} and $|A_{3/2}|^2 - |A_{1/2}|^2$.[9] Results are shown in Table I. Coefficients for $L \geq 6$ are consistent with zero, indicating that $J > \frac{5}{2}$ is excluded. There is, however, one special case for which the coefficient of Y_{60} may vanish, i.e., if $J^P = \frac{7}{2}^-$ and only the lower-l wave contributes to the decay. In this case the term in square brackets in Eq. (5) equals 0 for $L = 6$ and $|A_{3/2}|^2 - |A_{1/2}|^2 = 1/7$ (because of the small Q value of the decay, the contribution from the $l = 2$ wave may well dominate that for $l = 4$ for $J = \frac{7}{2}$).

In order to determine the helicity amplitudes and the density matrix elements of the $N^*(1720)$, a maximum-likelihood fit to the sequential decay was made assuming $J=\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$. The correlated distribution may be written[8]

$$I(\theta,\psi)=(2J+1)\{\sum_{\lambda>0}^{3/2}|A_\lambda|^2\sum_{m>0}^{J}\rho_{mm}Z_{mm}{}^{J\lambda+}(\theta)-[\sum_{\lambda>0}^{3/2}C(\tfrac{3}{2}2\tfrac{3}{2};\lambda 0\lambda)|A_\lambda|^2 \times\sum_{m>0}^{J}\rho_{mm}Z_{mm}{}^{J\lambda+}(\theta)]5^{1/2}(3\cos^2\psi-1)/2\},\qquad(8)$$

with $Z_{mm}{}^{J\lambda+}(\theta)=[d_{m\lambda}{}^J(\theta)]^2+[d_{m-\lambda}{}^J(\theta)]^2$ where the $d_{m\lambda}{}^J(\theta)$ are rotation matrices,[10] and ρ_{mm} are the density-matrix elements of the $N^*(1720)$ in the Jackson frame. The density-matrix elements are constrained so that

$$\sum_{m>0}^{J}\rho_{mm}=\tfrac{1}{2}.\qquad(9)$$

Results for $J=\frac{5}{2}$ and $J=\frac{7}{2}$ are presented in the first two columns of Table II. Goodness of fit for $J=\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$ was tested by the χ^2 method. The confidence levels were 0.0001, 0.40, and 0.63 for $J=\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$, respectively. The production and decay angular distributions are shown in Fig. 4, with results of the correlated fit represented by solid lines for $J=\frac{7}{2}$ and by dashed lines for $J=\frac{5}{2}$.

The fitting procedure was repeated assuming that only the lower-l wave contributes to the decay; thus, the values of $|A_{1/2}|^2$ and $|A_{3/2}|^2$ were fixed. The confidence levels were 0.20 and 0.63 for $J=\frac{5}{2}$ and $J=\frac{7}{2}$, respectively. These results are presented in the last two columns of Table II.

No attempt has been made to correct for background in the spin and parity analysis. With the limited statistics available from 50-MeV bands on each side of the $N^*(1720)$ the production and decay distributions are consistent with isotropy.

In conclusion, an enhancement in the $p\pi^+\pi^-$ mass spectrum at 1719 MeV with dominant $\Delta\pi$ decay mode was observed. In contrast, Barnes et al. recently reported an enhancement with mass and width values in excellent agreement with those found in this Letter for the $N^*(1720)$, but they observed no $\Delta\pi$ decay. The spin of the $N^*(1720)$ is greater than $\frac{3}{2}$, but both $\frac{5}{2}$ and $\frac{7}{2}$ give acceptable fits to the angular distributions.

We would like to express our appreciation to the many people at Stanford Linear Accelerator Center, particularly Dr. Joseph J. Murray, for their assistance in obtaining these excellent

Table II. Parameters obtained from the maximum-likelihood fit to the correlated decay $N^*(1720)\to\Delta^{++}\pi^-\to p\pi^+\pi^-$.

	$J=\frac{5}{2}$	$J=\frac{7}{2}$	$J^P=\frac{5}{2}^+$	$J^P=\frac{7}{2}^-$
$A_{1/2}{}^2$	0.37 ± 0.04	0.34 ± 0.04	0.30	0.3214
$A_{3/2}{}^2$	0.13 ± 0.04	0.16 ± 0.04	0.20	0.1786
$\rho_{1/2\,1/2}$	0.39 ± 0.06	$0.38^{+0.09}_{-0.11}$	0.32 ± 0.09	0.38 ± 0.09
$\rho_{3/2\,3/2}$	0.11 ± 0.06	$0.08^{+0.14}_{-0.08}$	0.18 ± 0.09	$0.08^{+0.12}_{-0.08}$
$\rho_{5/2\,5/2}$	$0.00^{+0.03}_{-0.00}$	$0.04^{+0.07}_{-0.04}$	$0.00^{+0.03}_{-0.00}$	$0.04^{+0.07}_{-0.04}$
$\rho_{7/2\,7/2}$		$0.00^{+0.03}_{-0.00}$		$0.00^{+0.03}_{-0.00}$

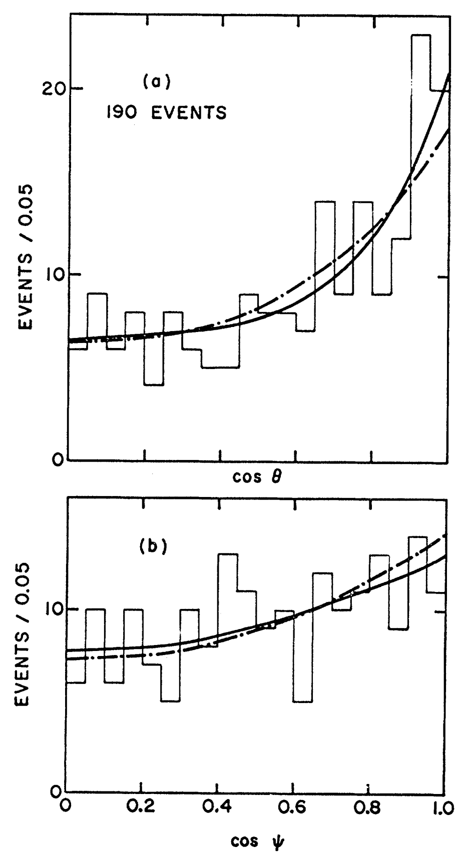

FIG. 4. (a) The folded angular distribution of the Δ^{++} relative to the target proton in the $N^*(1720)$ rest system. (b) The folded angular distribution of the decay proton relative to the Δ^{++} line of flight in the Δ^{++} rest system. The solid curves represent the most likely fit to the correlated decay for $J=\frac{5}{2}$.

photographs. Also, we would like to thank our diligent scanners and measurers for their careful work.

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[1]A summary of the $N\pi\pi$ mass enhancements observed in production experiments is given by J. G. Rushbrooke in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September 1968,* edited by J. Prentki and J. Steinberger (CERN Scientific Information Service, Geneva, Switzerland, 1968), p. 158. See also V. Allis-Borelli *et al.*, Nuovo Cimento 47A, 232 (1967); R. Ehrlich *et al.*, Phys. Rev. Letters 21, 1839 (1968); K. F. Galloway *et al.*, Phys. Letters 27B, 250 (1968); J. I. Rhode *et al.*, Phys. Rev. 187, 1844 (1969).

[2]V. E. Barnes *et al.*, Phys. Rev. Letters 23, 1516 (1969).

[3]The Δ^{++} band is defined here as $1136 < M(p\pi_b^+) < 1336$ MeV.

[4]The form of the Breit-Wigner function used was

$$|f_{BW}(M,\Gamma)|^2 = \frac{M}{q}\,\frac{\Gamma(q/q_0)^3}{(M^2-M_0)^2+(M_0\Gamma)^2(q/q_0)^6}$$

where M_0 and Γ are the mass and width of the Δ^{++} and q (q_0) is the breakup momentum of the $p\pi$ system with effective mass M (M_0).

[5]The likelihood function used was $\prod_k\{\epsilon|M_\Delta|^2/\int|m_\Delta|^2 \times d\varphi + (1-\epsilon)/\int d\varphi\}_k$ with $|m_\Delta|^2 = |f_{BW}(M,\Gamma)|^2(1+b\cos^2\psi)/2(1+b/3)$ where ψ is the angle between the decay proton and the Δ^{++} line of flight in the Δ^{++} rest frame. $\int d\varphi$ represents integration over the $p\pi^+\pi^-$ Dalitz plot.

[6]Results of the double-Regge-pole model calculations are being prepared for publication.

[7]D. H. Miller (Purdue University), private communication.

[8]J. D. Jackson, *High Energy Physics* (Gordon and Breach, New York, 1965), p. 327; J. Button-Shafer, Phys. Rev. 139, B607 (1965); S. M. Berman and M. Jacob, Phys. Rev. 139, B1023 (1965).

[9]The $N^*(1720)$ is defined here as $1620 < M(p\pi^+\pi^-) < 1820$ MeV.

[10]M. E. Rose, *Elementary Theory of Angular Momentum* (Wiley, New York, 1957).