MAGNETIC MOMENT OF THE POSITIVE MUON*

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The magnetic moment of the positive muon has been measured relative to that of the proton by determining its precession frequency in a magnetic field measured by proton NMR. With both protons and muons bound in a spherical sample of water, the result is $\omega_{\mu'}/\omega_{p'} = 3.183\,362 \pm 0.000\,030$ (9.4 ppm). Including uncertainties due to the chemical environment of the muon, the result for free muons and protons is $\omega_\mu/\omega_p = 3.183\,330 \pm 0.000\,044$ (14 ppm) which is consistent with recent measurements of the hyperfine splitting of muonium.

We report a new measurement of the magnetic moment of the positive muon and a comparison with a previous measurement.[1] We also discuss a possible re-evaluation of the results of both experiments to include an uncertainty[2] in the muon's chemical interactions in the stopping material.

The magnetic moment is determined by measuring the precession frequency of muons at rest in a known magnetic field. Longitudinally polarized muons are obtained from forward (in the c.m.) decays in flight[3] along a 158-MeV/c pion beam. As shown in Fig. 1, the beam enters the precession magnet, passes through a $1\frac{1}{8}$-in. copper absorber which filters out pions and muons from backward c.m. decays, and stops in a water target. The vertical magnetic field causes the muon spin to precess in a horizontal plane with angular velocity $\omega_\mu = 2\mu_\mu H/\hbar$. The muon magnetic moment is given by $\mu_\mu = g(e_\mu/2m_\mu c)\hbar/2$,

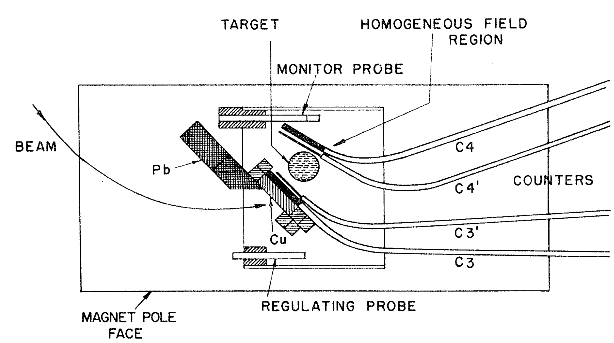

FIG. 1. Apparatus for stopping muons and detecting their decays inside the 18- by 40-in. magnet gap. The magnetic field of 12.0 kG was normal to the plane of the figure, and was regulated by a feedback circuit which sensed the output of the NMR regulating probe.

where g represents the muon g factor. The distribution of time intervals between the stopping of a muon and the detection of its decay positron at a fixed angle in the precession plane is

$$dN = e^{-t/\tau}[1 + a\cos(\omega_\mu t + \alpha)](dt/\tau) \times d\Omega_{\rm det}/4\pi, \qquad (1)$$

where τ is the muon lifetime (2.2 μsec) and the cosine term arises from the $\vec{S}_\mu \cdot \vec{p}_e$ correlation in muon decay; $d\Omega_{\rm det}$ is the detector solid angle. The coefficient a is reduced from its maximum value of 0.3 to about 0.04 in this experiment because of incomplete polarization of the muon beam, further depolarization upon stopping in the target, and effects of time resolution and solid angle of the $C_{4'}$-C_4 telescope. The angle α is a constant which depends on the relative azimuth of the muon polarization and the counter telescope and also on delays in the electronic logic. The muon stopping signal is given by the coincidence $33'\bar{4}'\bar{4}$, and the positron timing signal by a $4'4\bar{3}'\bar{3}$ coincidence occurring within 4.0 μsec of the muon stop.

The method of determining the precession frequency was developed at the University of Chicago[4] and Columbia University,[1] and a detailed description can be found elsewhere.[5,6] The time interval from muon stopping to decay is measured modulo the period of a reference oscillator whose frequency, $\omega_{\rm ref}/2\pi$, is very close to the muon precession frequency. The phase of this oscillator, $\theta_{\rm ref}$, corresponds to $\omega_\mu t$ of (1), so a distribution of decay times versus $\theta_{\rm ref}$ has the cosine form of (1) and includes the angle α. If the 4.0-μsec gate in which decay positrons are accepted is divided into two intervals, 0.05-2.0 μsec (early positrons) and 2.0-4.0 μsec (late

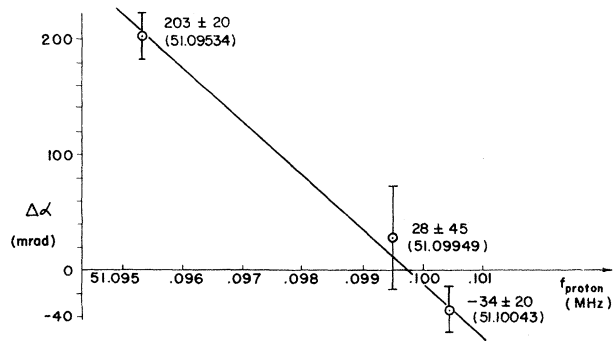

FIG. 2. The difference $\Delta\alpha$ between the initial phases of the "early" and "late" data distributions as a function of the magnetic field, measured by the proton NMR frequency of the monitor probe. The least-squares straight line intersects the f_p axis at 51.099 74 ± 0.000 36 MHz.

positrons), and the distributions recorded separately for each group, then the initial phase α will be the same for each group only if $\omega_\mu = \omega_{\rm ref}$. Otherwise there is a phase shift $\Delta\alpha$ between the two,

$$\Delta\alpha = \alpha_{\rm late} - \alpha_{\rm early} = (\omega_{\rm ref} - \omega_\mu)T_E, \quad (2)$$

where T_E is the length of the early gate, 1.95 μsec. To remove the dependence on gate length T_E, the magnetic field, as measured by the NMR frequency f_p of protons in a mineral-oil probe, is varied in the region of $\omega_\mu = \omega_{\rm ref}$, and $\Delta\alpha$ is found for each field setting by Fourier analysis of the distributions. A plot of $\Delta\alpha$ vs f_p, Fig. 2, intersects $\Delta\alpha = 0$ where ω_μ is equal to the known frequency $\omega_{\rm ref}$. The value found for f_p at $\Delta\alpha = 0$ is 51.099 74 ± 0.000 36 MHz, and $\omega_{\rm ref} = (162.666\,21 \pm 0.000\,06\ \text{MHz}) \times 2\pi$, giving a value of $\omega_\mu/\omega_p = 3.183\,308 \pm 0.000\,022$ (statistical error).

This initial result is subject to several corrections. An experimental technique devised by the Columbia group was used to measure possible systematic errors in the phase measurements. At a frequency of 162 MHz, a phase error of 20 mrad will produce a 10-ppm change in the measured frequency. To calibrate the clock, dummy data are generated with a distribution similar to (1) but with a known "precession" frequency. The muon stopping time is simulated by a pulse generator and the decay time by a second, random-time, random-voltage, pulse generator. Each pulse is linearly mixed with a common rf voltage at the calibration "precession" frequency and then triggers a discriminator. These two pulses are now time correlated at a known frequency as the rf modulates the probability of a discriminator trigger. The probability of time interval t between the two pulses is

$$P(t)dt \sim \exp\left(\frac{t}{T_{p2}}\right) \times \left\{1 + \sum_{j=1}^{\infty} a_j \cos(j\omega_{\rm rf} t + \theta_j)\right\} dt. \quad (3)$$

T_{p2} is the average period of the random pulser and is set equal to 2.2 μsec. The summation is a Fourier-series representation of a correlation of arbitrary shape, but as only the first term is nonzero in the Fourier analysis of the data, (3) reduces to

$$P(t) \sim \exp[-t/(2.2\ \mu\text{sec})] \times \{1 + a_1 \cos(\omega_{\rm rf} t + \theta_1)\}. \quad (4)$$

This is of the same form as Eq. (1), except that ω_μ is replaced by the known $\omega_{\rm rf}$. Calibration of the clock at intervals during data taking indicated a constant systematic error in $\Delta\alpha$ of -6 ± 4 ppm. Therefore the initial ratio ω_μ/ω_p must be decreased by 6 ± 4 ppm.

The average magnetic field in the stopping target was lower than the field at the position of the monitor NMR probe. The precision field mapping in the target region was performed by means of a movable NMR probe containing a 0.093-molar solution of $NiSO_4$ in water. The average proton frequency over the target volume was lower than that of the stationary monitor probe by 23.5 ± 1.0 ppm. The difference in bulk magnetic susceptibility corrections for the movable probe and the stopping target, including effects of shape and of paramagnetic ions in the probe,[7] was 0.5 ± 0.2 ppm. Combining these two corrections, the initially calculated ratio ω_μ/ω_p must be increased by 23.0 ± 1.0 ppm to account for the difference in magnetic fields.

Errors have been calculated for a number of small effects: (a) time variations in the magnetic field (±2 ppm); (b) error in reading the magnetic field due to the asymmetry in the 10-ppm-wide oscilloscope display of the NMR signal (±2 ppm); (c) muon stoppings outside the target volume, mostly in counters 3′ and 4′ (±2 ppm); (d) nonuniformity in the stopping distribution of muons in the target (±3 ppm); and (e) muon stoppings in the target container wall (±0.3 ppm).

The net effect of these corrections and errors is $+17 \pm 6.2$ ppm. All the errors quoted are one standard deviation. To find $\omega_{\mu'}/\omega_{p'}$, where the prime indicates that the precession frequency is that of a particle in a spherical sample of water, this net correction is made to the initial ω_μ/ω_p.

This results in $\omega_{\mu'}/\omega_{p'} = 3.183\,362 \pm 0.000\,030$ (9.4 ppm), including both the systematic and statistical errors.

To find the value ω_μ/ω_p for the free muon and proton, the uncertainty in the muon environment must be taken into account. The magnetic field seen by the muon includes the effects of diamagnetic shielding by the electrons of the molecule in which the muon comes to rest (chemical shift). If the muon replaces a proton in a water molecule the effect of this shift is small, as it is essentially the same for both ω_μ and ω_p and cancels when the ratio is taken. However, Ruderman[2] has suggested that the muon may form $(H_2O\text{-}\mu\text{-}H_2O)^+$ and that the muon shielding in such a molecule could be reduced by as much as 20 ppm. Since these two models of muon chemistry yield chemical shifts differing by 20 ppm, and since there is at present no a priori way to choose between them, a correction of -10 ± 10 ppm is made to the bound value, yielding $\omega_\mu/\omega_p = 3.183\,330 \pm 0.000\,044$ (14 ppm).

The Columbia group assumed that the bound and free ratios were equal. Their result was $\omega_{\mu'}/\omega_{p'} = 3.183\,38 + 0.000\,04$ (13 ppm), in agreement with this experiment. Combining the results of the two experiments yields

$$\omega_{\mu'}/\omega_{p'} = 3.183\,369 \pm 0.000\,024 \text{ (7.5 ppm)}$$

and, after correction for the uncertainty in the chemical shift,

$$\omega_\mu/\omega_p = 3.183\,337 + 0.000\,040 \text{ (12.5 ppm)}.$$

The muon-moment measurement can be checked for consistency with measurements of the hyperfine splitting of muonium. The theoretical expression for f_{hfs}, which has been thoroughly discussed by Hughes,[8] can be simplified to

$$f_{\text{hfs}} = 2.632\,9441 \times 10^7 (\omega_\mu/\omega_p)\alpha^2 \text{ MHz},$$

using more recent values for the constants,[9] and m_μ/m_e as calculated[10] from our combined value of ω_μ/ω_p. Using the value of Taylor, Parker, and Langenberg[9] for α^{-1} (137.036 08 ± 1.9 ppm) and the combined ω_μ/ω_p above, one finds

$$f_{\text{hfs}}(\text{calculated}) = 4463.284 \pm 0.056 \text{ (13 ppm)},$$

where the error is determined almost completely by the error in the muon moment.

Recent measurements have been made of f_{hfs} by groups at Yale[11] and Chicago.[12] Their results are

$$f_{\text{hfs}}(\text{Yale}) = 4463.248 \pm 0.031 \text{ MHz (7 ppm)},$$

$$f_{\text{hfs}}(\text{Chicago}) = 4463.317 \pm 0.021 \text{ MHz (5 ppm)},$$

in good agreement with the foregoing calculation.

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