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ROTATIONAL DESCRIPTION OF STATES IN CLOSED- AND NEAR-CLOSED-SHELL NUCLEI*

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All ground-state bands observed in even-even nuclei, including closed-shell nuclei, are accurately predicted by a two-parameter description based on the cranking model. A hitherto unobserved discontinuity in a plot of $E(I)/E(2)$ vs $E(4)/E(2)$ is also predicted by the model.

Experimental data recently reported[1-4] appear to confirm that the gradual change of properties of ground-state bands which takes place with decreasing deformation continues well beyond the "vibrational" region and includes indeed all nuclei.[5,6] The purpose of this note is to propose a unified rotational description for all these bands based on an extension of the cranking model[7] due to Harris.[8]

In 1959 Mallmann[5] pointed out that ground-state bands in even-even nuclei (i.e., lowest excited states with spins $I=2,4,6,\cdots$ and even parity) exhibited a regular behavior when the energy ratios $E(I)/E(2)$ were plotted against $E(4)/E(2)$. The data presented by Mallmann spanned the interval

$$1 \lesssim E(4)/E(2) < 3.33. \tag{1}$$

A successful phenomenological description of these regularities in the more limited interval

$$2.23 \leq E(4)/E(2) \leq 3.33 \tag{2}$$

has been provided by the variable moment-of-inertia (VMI) model.[9] It has been shown[9] that the equations of this model are mathematically equivalent to the equations derived by Harris[8] from an extension of the cranking model[7] to the next higher order in ω^2 (ω is the angular velocity). In both approaches two parameters are involved, the moment of inertia in first order $\mathscr{I}_0$ and the coefficient C of the next-higher-order term. In (2) the upper limit corresponds to $C=0$ (the case of hard, well-deformed nuclei) while the lower limit corresponds to $\mathscr{I}_0=0$ (the case of "soft" nuclei, spherical in the ground state). In the following we discuss the solutions which are obtained when $\mathscr{I}_0$ and C are allowed to be negative.[10] We choose to follow Harris's approach because, although the solutions obtained with both models are mathematically equivalent, they are very difficult to interpret in terms of the VMI model.[11] Furthermore, since Harris's equations are derived from the cranking model,[7] one may hope to obtain $\mathscr{I}_0$ and C, which are here adjusted to fit the data, from a microscopic calculation. Such a calculation would be a valuable complement to the following analysis.

Harris's model[8] is based on the two equations

$$E_R = \tfrac{1}{2}\omega^2(\mathscr{I}_0 + 3C\omega^2) \tag{3}$$

and

$$[I(I+1)]^{1/2} = \omega(\mathscr{I}_0 + 2C\omega^2). \tag{4}$$

From Eqs. (3) and (4) one sees that, as Harris has pointed out, a different "effective" moment of inertia enters into the calculations of energy and angular momentum. It is convenient (as discussed below) to introduce a phenomenological definition of the moment of inertia $\mathfrak{F}(I)$ in terms of energy and spin as

$$E_R(I) = I(I+1)/2\mathfrak{F}(I). \tag{5}$$

Using Eqs. (3), (4), and (5), the angular velocity ω is eliminated and the following equations are obtained which determine the moment of inertia

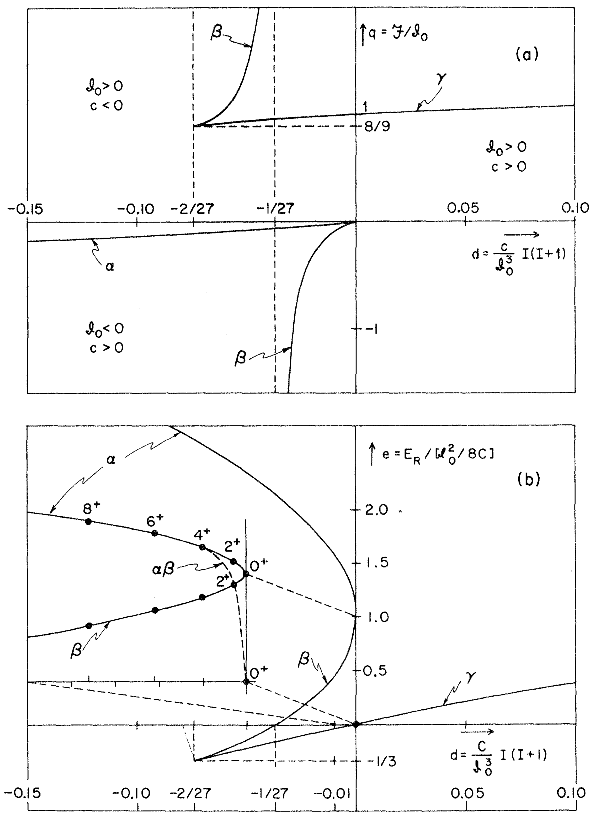

FIG. 1. The three real roots α, β, and γ of the cubic equations (6a) and (6b) for the moment of inertia and the energy are plotted in (a) and (b), respectively. The insert in (b) shows an expanded plot (for $-10^{-2} < d < 0$) of solutions α and β when $\mathcal{J}_0 < 0$ and $C > 0$. The dots indicate the physical solutions obtained for $C/\mathcal{J}_0^{\,3} = -10^{-4}$ and $I = 0$, 2, 4, 6, and 8. The $\alpha\beta$ "trajectory" indicated in dashed line corresponds approximately to the ground-state band of Mo^{92}, while the sequence of solutions lying on the α "trajectory" corresponds approximately to the ground-state bands of Mo^{94} and Ru^{96}.

and the energy[12]:

$$(1+27d)q^3-(1+36)q^2+8dq-16d^2=0, \qquad (6a)$$

$$e^3-2e^2+(1+36d)e-4(1+27d)d=0, \qquad (6b)$$

where $q=\mathfrak{F}/\mathcal{J}_0$, $e=E_R/(\mathcal{J}_0^{\,2}/8C)$, and $d=(C/\mathcal{J}_0^{\,3})I(I+1)$.

The real roots of Eqs. (6a) and (6b), which we denote as α, β, and γ, are shown in Fig. 1. Three real roots coexist only in the interval $-2/27 < d < 0$. For $d > 0$ one obtains the energy levels described in Ref. 9. For $I=0$, the γ root leads to $\mathfrak{F}(0)=\mathcal{J}_0$ and $E_R(0)=0$, while the α and β roots lead to $\mathfrak{F}(0)=0$ and $E_R(0)=\mathcal{J}_0^{\,2}/8C$. Thus the energy of the ground state in cases α and β is different from zero, and the level energy is $E(I)=E_R(I)-E_R(0)$.

Ground-state bands are obtained from the model as a sequence of solutions $E_R(I)$ with $I=0,2,4,\cdots$ and constant $\mathcal{J}_0$ and C. Once the pair ($\mathcal{J}_0$, C) is fixed, only those points on the α, β, and γ curves which correspond to integer values of I will represent possible solutions. This situation is similar to that found in high-energy physics where physical bound states and resonances are described by a Regge-pole trajectory passing through integer values of the spin. We can thus think of the α, β, and γ curves as analogous to Regge trajectories. Formally, however, other "trajectories" connecting solutions which lie on different curves are also possible as long as $\mathcal{J}_0$ and C are the same. In the insert of Fig. 1(b) one such possible "trajectory", labeled $\alpha\beta$, is indicated. As shown below it describes remarkably well the ground-state bands in closed-shell nuclei. The α trajectory, also indicated in Fig. 1(b), gives a very good agreement with the energy spectrum of "vibrational" nuclei. In this note we only report the results obtained with these two solutions together with those corresponding to the γ root reported earlier.[9]

The ratios $E(I)/E(2)$ are plotted versus $E(4)/E(2)$ for $I=6$ and 8 in Fig. 2, together with the experimental data.[13] Solution α spans the interval $1.825 \le E(4)/E(2) \le 2.231$ and connects smoothly with the solution γ whose interval of validity is (2). The solution $\alpha\beta$, on the other hand, extends from $E(4)/E(2)=1$ to ∞. A remarkable agreement for the doubly and singly closed-shell nuclei in the interval $1 \le E(4)/E(2) \le 1.825$ is obtained with this solution[14] and the existence of a sharp discontinuity is apparent at $E(4)/E(2) = 1.825$. This value corresponds to the lower limit of the α-solution interval. The experimental data shown in Fig. 2 strongly support this prediction of the model and reveal a singular phenomenon, hitherto unobserved, which deserves further study. In the interval $1.825 \le E(4)/E(2) \le 2.231$ the data are seen to scatter between the α and $\alpha\beta$ curves. This spread may suggest that states of nuclei in this interval share, to some degree, the two solutions. Beyond $E(4)/E(2) = 2.231$ the experimental data are, again, accurately described by the γ solution with the exception of Ba^{136} and Ti^{48}. The 6^+ isomeric state in Ba^{136} has been interpreted in terms of shell-model configurations.[15] On the other hand, it is well predicted by the $\alpha\beta$ solution (Fig. 2). The significance of this agreement, however, cannot be assessed until more data become available.

Finally we briefly mention that the definition

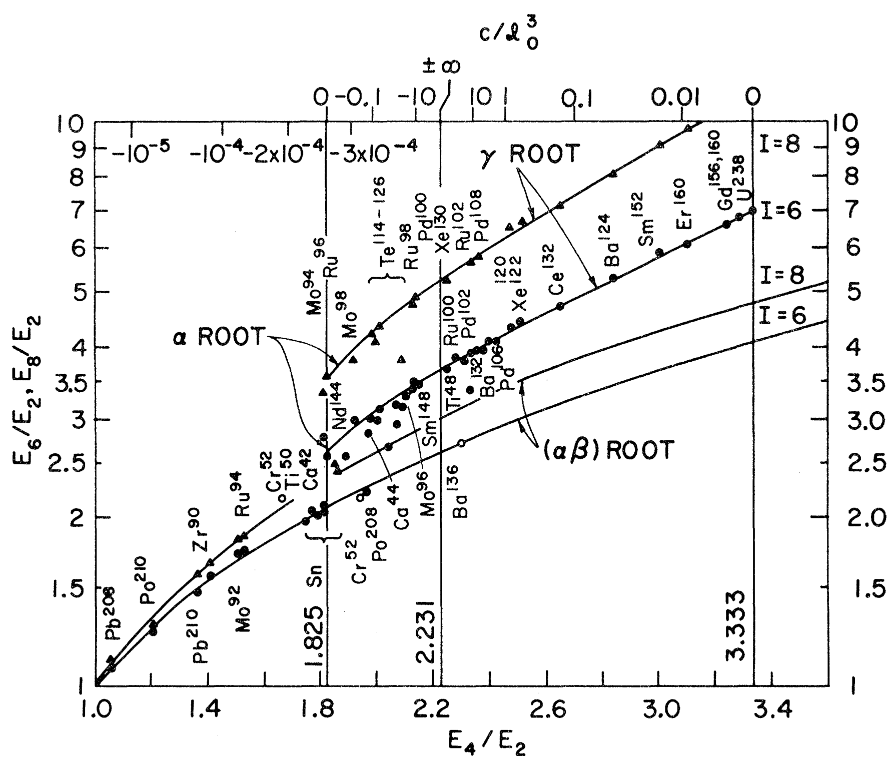

FIG. 2. Energy ratios $E(I)/E(2)$ as a function of $E(4)/E(2)$ for $I=6$ and 8. Solid circles and triangles indicate experimental values of this ratio for $I=6$ and 8, respectively. Open circles are discussed in the text and in Ref. 14. The energy ratios are a function of only one parameter, $C/\mathscr{J}_0^3$. Values of this parameter are indicated at the top of the figure. The lower scale corresponds to the $\alpha\beta$ solution, while the upper scale corresponds to the α and γ solutions. For nuclei with $E(4)/E(2) > 2.23$ the parameter $C/\mathscr{J}_0^3$ represents the "softness" of the nucleus in the ground state, i.e. $[\mathfrak{F}^{-1}(d\mathfrak{F}/dI)]_{I=0}$.

(5) of the moment of inertia allows us to extend the validity of the observed correlation[9] between intrinsic quadrupole moments and moments of inertia $\{Q_{02}^2 \sim B(E2, 2^+ \to 0^+) \sim [\mathfrak{F}(0)+\mathfrak{F}(2)]/2\}$ to closed- and near-closed-shell nuclei and, at the same time, provides a justification for the experimental fact pointed out by Grodzins[6] that $B(E2, 2^+ \to 0^+)$ values are proportional to $1/E(2)$ [i.e., to $\mathfrak{F}(2)$] throughout the nuclear chart. Since $\mathfrak{F}(0)=0$ and $\mathfrak{F}(0)=\mathscr{J}_0 \simeq \mathfrak{F}(2)$ for nuclei with $E(4)/E(2)<2.23$ and ≈ 3.33, respectively, the model predicts a product $B(E2)\times E(2)$ twice as large in the latter case. Although the experimental values of this product scatter considerably, they do seem to form two groups of values consistent with the prediction.

To summarize, ground-state bands in all even-even nuclei are accurately fitted by a rotational description. Several intriguing points emerge from this phenomenological analysis such as the prediction and experimental confirmation of a sharp break in Mallmann's curves at $E(4)/E(2) = 1.825$, the nonzero ground-state energy in the case of the α solution, and the striking validity of the "abnormal trajectory" $\alpha\beta$. The understanding of these puzzling facts, as well as the meaning of $\mathscr{J}_0<0$, remains a challenging problem. Furthermore, it is also of interest to find the extent to which this model applies to the description of other nuclear excited states which do not belong to the ground-state band.

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[13]Figure 2 includes all data available to us for $E(4)/E(2) < 2.23$. Data corresponding to larger values of this ratio which have already been discussed elsewhere (see Ref. 9 and references therein) are not included except for some representative points. The data have been taken from Refs. 1-4; C. M. Lederer *et al.*, *Table of Isotopes* (Wiley, New York, 1967); A. Luukko *et al.*, Nucl. Phys. A135, 49 (1969); I. Bergstrom *et al.*, Nucl. Phys. A123, 99 (1969); J. H. Bjerregaard *et al.*, Nucl. Phys. A113, 484 (1968).

[14]The only deviation from the $\alpha\beta$ solution in the interval $1 \leq E(4)/E(2) \leq 1.825$ corresponds to Cr^{52}. In this nucleus two 4^+ states are observed lying close together. If the correct member of the ground-state band is the second 4^+ excited state $[E(4)/E(2) = 1.94]$ then a good agreement is obtained (see Fig. 2).

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SIMPLE MODEL FOR TACHYONS

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The Frenkel-Kontorova model of a dislocation might be useful for the study of tachyons. The model has tachyonlike solutions which correspond to unstable states. This may explain why tachyons are unlikely to be observed.

Some time ago, Bilaniuk, Deshpande, and Sudarshan[1] re-examined the question of whether or not the special theory of relativity forbids a particle to travel faster than light. They concluded that the existence of faster-than-light particles, or tachyons, is in no way precluded by the theory. Recently, Bilaniuk and Sudarshan[2] remarked that we should either find the tachyons or explain why they can never be observed. The question raised considerable interest and heated discussions.[3] To my mind, the arguments of Bilaniuk and Sudarshan are sound and have, so far, stood up to criticism. Yet the question of why nobody has ever observed a tachyon still remains open.

For the sake of argument, let us assume that tachyons can, in fact, exist. It is not clear whether tachyons can directly interact with tardons (slower-than-light particles). However, tardons most certainly do interact with luxons (particles traveling at the speed of light). The symmetry in the relationships between tardons and luxons on one hand and tachyons and luxons on the other hand implies that tachyons must interact with luxons and that such interactions should be observable.

A transcendent tachyon, i.e., one traveling at an infinite speed, would be perceived as a rigid wall.[2] Along this line of thought, it appears that a nearly transcendent tachyon should produce effects similar to a slow, massive object. A careful analysis of the tachyon effective mass would seem to be a worthwhile enterprise.

Some insight into the nature of tachyons may be obtained from the one-dimensional model of a dislocation developed by Frenkel and Kontorova.[4] In this model, a chain of atoms moves under the combined influence of harmonic forces between nearest neighbors and of a sinusoidal interaction with a rigid substrate. The solutions describe particles and antiparticles in one dimension. The outstanding feature of the Frenkel-Kontorova model is that it exhibits a striking analogy to the theory of relativity, with the propagation speed of interaction in the chain, or the speed of sound, playing the role of c. This result is independent of the particular law of interaction with the substrate.

Frank and van der Merwe[5] showed that the model also yields supersonic solutions which they termed "positive and negative antidislocations," though they might be better called "tachydislocations and antitachydislocations." These behave