Mamnitskii, Zh. Eksperim. i Teor. Fiz. 55, 2017 (1968) [Soviet Phys. JETP 28, 1065 (1969)]; H. H. Schmidt, J. Opdycke, and C. F. Gay, Phys. Rev. Letters 19, 887 (1967).

[2]C. Edwards, J. A. Lipa, and M. J. Buckingham, Phys. Rev. Letters 20, 496 (1968), and private communication.

[3]A. T. Berestov, M. S. Giterman, and S. P. Malyshenko, Zh. Eksperim. i Teor. Fiz. 56, 642 (1969) [Soviet Phys. JETP 29, 351 (1969)].

[4]See R. B. Griffiths, Phys. Rev. 158, 176 (1967), for a discussion of scaling and for precise definitions of the exponents.

[5]P. Schofield, J. D. Litster, and J. T. Ho, Phys. Rev. Letters 23, 1098 (1969).

[6]M. R. Moldover, Phys. Rev. 182, 342 (1969).

[7]M. Barmatz, Phys. Rev. Letters 24, 651 (1970).

[8]More precisely, the quantity $a\mu_c$, rather than a, is determined from experiment. The same quantity also occurs in Eq. (5) below. Note also that the constant k in Eq. (1b) was called g in Ref. 5.

[9]M. Vicentini-Missoni, J. M. H. Levelt Sengers, and M. S. Green, J. Res. Natl. Bur. Std. (U.S.) 73A, 563 (1969). Unless noted otherwise, we shall use the values quoted in this work for all experimental constants.

[10]Using the values of the constants given in Ref. 9, we have estimated α from the jump in C_v for that model and find $\alpha \approx 0.035$ for Xe, He^4, and CO_2. This value is close to the value $\alpha \approx 0.04$-0.06 obtained in Ref. 9 by a least-squares analysis of PVT data, leaving α as a free parameter.

[11]G. T. Feke, K. Fritsch, and E. F. Carome, Phys. Rev. Letters 23, 1282 (1969).

[12]The general case, $\tilde{\rho} \neq \rho_c$, involves more complicated algebra and will be treated in a later publication.

[13]Throughout this work we neglect all nonlocal effects caused by the finite correlation length ξ [see Ref. 2]. For thermodynamic measurements these effects may be shown to be negligible in the experimental temperature range. For the sound velocity measurements, nonlocal effects are not entirely negligible very near T_c, even at the lowest frequencies used (see Ref. 7 and Fig. 3). A more thorough study of these effects will be carried out in a later publication.

[14]Professor Buckingham informs us that the data were obtained from heating curves, so that we expect nonequilibrium behavior to lead to experimental points which, very close to T_c, are too high. In addition, an increase in the value of T_c by 3×10^{-3} K ($\delta t \approx 10^{-5}$) would improve the agreement considerably for $T > T_c$. Finally, we note that the theoretical value of $\tilde{C}_v$ at T_c can be made to fit experiment exactly by choosing $\beta = 0.333$.

[15]E. Wasserstrom, to be published.

[16]For the lowest radial mode, the minimum of u is at $t_m = 4.2\times10^{-5} \approx 0.2t_0(h)$ (see Fig. 3). For the other modes, the value of t_m varies up to about $0.5t_0(h)$.

BALLOONING MODES IN AXISYMMETRIC TOROIDAL CONFIGURATIONS

G. Laval, E. K. Maschke, and R. Pellat

Association Euratom–Commissariat à l'Energie Atomique, Département de la Physique du Plasma et de la Fusion Contrôlée, Centre d'Etudes Nucléaires, 92 Fontenay-aux-Roses, France

(Received 18 November 1969; revised manuscript received 20 April 1970)

The stability of ballooning modes is investigated in the neighborhood of the magnetic axis of an axisymmetric toroidal configuration of the Tokamak type. It is shown that the stability criterion for these modes is the same as that for localized modes.

A purely poloidal configuration of the multipole type with closed magnetic field lines and an average magnetic well may become unstable when the ratio β of the kinetic pressure to the magnetic pressure exceeds a critical value.[1] The unstable perturbations have larger amplitude in the regions of unfavorable curvature of the field lines than in the regions of favorable curvature and are called ballooning modes. On the other hand, for toroidal configurations with rotational transform (Tokamak, Stellarator) the magnetohydrodynamic stability problem is usually studied with the so-called localized criterion.[2-4] The corresponding perturbations are much more localized near a magnetic surface than the ballooning modes, and in the case of an axisymmetric discharge having magnetic surfaces with circular cross sections it is found[5,6] that their stability is almost independent of β.

The purpose of the present work is to derive a stability criterion against ballooning modes for axisymmetric toroidal configurations with current parallel to the magnetic field lines to provide a rotational transform. Since all perturbations are easily stabilized by shear of the field lines, we restrict our calculations to the neighborhood of the magnetic axis where shear is negligible.

In cylindrical coordinates r, φ, z, the axisymmetric equilibrium magnetic field is

$$\vec{B} = \frac{T(r,z)}{r}\vec{e}_\varphi - \frac{1}{r}\vec{e}_\varphi \times \nabla\psi(r,z), \tag{1}$$

where $\vec{e}_\varphi$ is the unit vector in the φ direction. The surfaces $\psi(r,z)=\text{const}$ are the magnetic surfaces. It is convenient to introduce new coordinates ψ, φ, γ where γ is chosen such that the surfaces $\gamma=\text{const}$ are orthogonal to the surfaces of constant ψ and φ, and $\vec{e}_\gamma=\vec{e}_\psi\times\vec{e}_\varphi$. In this coordinate system all equilibrium quantities are periodic in γ with period 1; T and the plasma pressure p depend only on ψ, and the equilibrium equation takes the form

$$p'=-\frac{1}{J}\frac{\partial}{\partial\psi}(JB_\gamma^2)-\frac{TT'}{r^2}, \tag{2}$$

where

$$J=|\nabla\psi\times\nabla\varphi\cdot\nabla\gamma|^{-1}, \quad p'=dp/d\psi, \quad T'=dT/d\psi.$$

We use the energy principle of Bernstein et al.[1] The minimization of δW with respect to the component ξ_γ of the perturbation can be done exactly[2] and leads to the following expression:

$$\delta W=\frac{1}{\pi}\int d\psi\,d\gamma\,d\varphi\left\{\frac{1}{Jr^2B_\gamma^2}\left[\left(\frac{\partial}{\partial\gamma}+\frac{JT}{r^2}\frac{\partial}{\partial\varphi}\right)X\right]^2+\frac{r^2}{J}\left[\frac{\partial U}{\partial\gamma}-T\frac{\partial}{\partial\psi}\left(\frac{JX}{r^2}\right)\right]^2\right.$$
$$\left.+B_\gamma^2\left[\frac{\partial U}{\partial\varphi}+\frac{\partial X}{\partial\psi}-X\frac{\partial}{\partial\psi}\ln(JB_\gamma^2)\right]-KX^2\right\}, \tag{3}$$

where

$$X=rB_\gamma\xi_\psi, \quad U=\frac{1}{rB_\gamma}(\vec{B}\times\vec{e}_\psi)\cdot\vec{\xi}_\psi, \quad K=\frac{2p'}{B^2}\frac{\partial}{\partial\psi}\left(p+\frac{B^2}{2}\right)+\frac{j_\parallel}{B}\left[\frac{j_\parallel B}{r^2B_\gamma^2}-\frac{1}{J}\frac{\partial}{\partial\psi}\left(\frac{JT}{r^2}\right)\right],$$

and

$$B=|\vec{B}|, \quad j_\parallel=(\vec{j}\cdot\vec{B})/B.$$

We consider perturbations which vary rapidly in the dirctions perpendicular to the field lines, but along the field lines with the scale of the communication length between regions of favorable and unfavorable curvature. We write the perturbation in the form

$$X=\bar{X}(\psi,\gamma)\exp\{im(\varphi-\alpha)\} \tag{4}$$

and an analogous formula for U, where

$$\alpha(\psi,\gamma)=\int_{\gamma_0}^{\gamma}d\gamma\,JT/r^2-\mu\gamma, \quad \mu(\psi)=\oint d\gamma\,JT/r^2-2\pi k/m.$$

The factor $\bar{X}$ is assumed to be localized in the ψ direction with a localization length $\Delta\psi$ which is small compared to the scale of the equilibrium. k is an integer which will be chosen so that $|\mu|$ is as small as possible on a given magnetic surface around which $\bar{X}$ is localized.

In the derivation of the localized criterion the localization length $\Delta\psi$ in the ψ direction is supposed to be much smaller than the wavelength in the direction $\vec{e}_\psi\times\vec{B}$. In our case we assume that the latter wavelength is smaller than, or of the same order of magnitude as, $\Delta\psi$ (and both small compared to the scale of the equilibrium), as is usually done for ballooning modes. We can then minimize δW with respect to $\bar{U}$ by expanding the Euler equation for $\bar{U}$ in powers of $1/m$. Assuming μ to be of order $1/m$ and writing $\bar{U}=U_0+U_1+\cdots$, we obtain

$$U_0=\frac{i}{m}\frac{D}{D\psi}\bar{X}, \quad \frac{D}{D\psi}=\frac{\partial}{\partial\psi}-im\frac{\partial\alpha}{\partial\psi},$$

$$U_1=\frac{1}{m^2JB^2}\left\{T\frac{D}{D\psi}\frac{\partial\bar{X}}{\partial\gamma}-imJ\left(\frac{TT'}{r^2}+\frac{1}{J}\frac{\partial(JB_\gamma^2)}{\partial\psi}\right)\bar{X}+imT\left(\mu'\bar{X}+\mu\frac{D}{D\psi}\bar{X}\right)\right\}.$$

Using this result we find

$$\delta W=\int d\psi\,d\gamma\left\{\frac{1}{Jr^2B_\gamma^2}\left|\frac{\partial\bar{X}}{\partial\gamma}+im\mu\bar{X}\right|^2+\frac{r^2B_\gamma^2}{JB^2}\left|\frac{1}{m}\frac{D}{D\psi}\frac{\partial\bar{X}}{\partial\gamma}+i\mu\frac{D\bar{X}}{D\psi}\right|^2+\frac{i}{2m}\frac{\partial}{\partial\gamma}\left(\frac{j_\parallel}{B}-\frac{r^2B_\gamma^2}{JB^2}\mu'\right)\left(\bar{X}\frac{\partial\bar{X}^*}{\partial\psi}-\bar{X}^*\frac{\partial\bar{X}}{\partial\psi}\right)\right.$$
$$\left.-\left[\left(\frac{\partial}{\partial\gamma}\frac{j_\parallel}{B}\right)\frac{\partial\alpha}{\partial\psi}-\mu'\frac{\partial}{\partial\gamma}\left(\frac{r^2B_\gamma^2}{JB^2}\frac{\partial\alpha}{\partial\psi}\right)+\frac{2Jp'}{B^2}\frac{\partial}{\partial\psi}\left(p+\frac{B^2}{2}\right)\right]|\bar{X}|^2\right\}. \tag{5}$$

From this expression we can obtain the localized criterion if we pass to the limit $\Delta\psi = \epsilon \to 0$ for fixed m with $\bar{X} = X_0 + X_1$ where $\partial X_0/\partial\gamma = 0$ and $X_1 \sim \epsilon$, $\partial X_1/\partial\psi \sim 1$. This result can easily be understood if we remember the first derivation of the localized criterion.[2]

In the neighborhood of the magnetic axis we use a set of local coordinates ρ, θ, φ and we expand[6] in powers of ρ:

$$\psi = \psi_2\rho^2 + \cdots, \quad \psi_2 = a + b\cos 2\theta.$$

The perturbations are supposed to be localized around a surface $\psi = \psi_0$. The form of δW suggests the choice of the following ordering: $m\Delta\psi \sim \psi_0$, $\bar{X} = X_0 + X_1$ with $\partial X_0/\partial\gamma = 0$ and $X_1 \sim \psi_0^{1/2} \sim \rho$. We put $\partial X_1/\partial\gamma = Y + iZ$ where Y and Z are real. After an integration by parts with respect to γ we then obtain

$$\delta W = \int d\psi d\gamma \left\{ A(Y^2 + Z^2) + D\left[\left(\frac{1}{m}\frac{\partial Y}{\partial\psi}\right)^2 + \left(\frac{1}{m}\frac{\partial Z}{\partial\psi}\right)^2\right] + 2C\frac{1}{m}\frac{\partial Z}{\partial\psi}Y + 2E\frac{1}{m}\frac{\partial Z}{\partial\psi}X_0 + 2GYX_0 + FX_0^2 \right\}, \tag{6}$$

where

$$A = \frac{1}{J\gamma^2 B_\gamma^2} + \frac{\gamma^2 B_\gamma^2}{JB^2}\left(\frac{\partial}{\partial\psi}\int_{\gamma_0}^{\gamma}\frac{JT}{r^2}d\gamma\right)^2; \quad D = \frac{\gamma^2 B_\gamma^2}{JB^2}; \quad C = -\frac{2\gamma^2 B_\gamma^2}{JB^2}\frac{\partial}{\partial\psi}\int_{\gamma_0}^{\gamma}d\gamma\frac{JT}{r^2}; \quad E = -\left(\frac{j_\parallel}{B} - \oint d\gamma\frac{j_\parallel}{B}\right);$$

$$G = -\int_{\gamma_0}^{\gamma}d\gamma'\left[2p'J\frac{1}{r}\frac{\partial r}{\partial\psi} - \left(\frac{\partial}{\partial\gamma'}\frac{j_\parallel}{B}\right)\frac{\partial}{\partial\psi}\int_{\gamma_0}^{\gamma'}d\gamma''\frac{JT}{r^2}\right];$$

$$F = \int_{\gamma_0}^{\gamma}d\gamma\left[\left(\frac{j_\parallel}{B}\right)_1\left(\frac{\partial}{\partial\psi}\frac{JT}{r^2}\right)_{-1} + \left(\frac{j_\parallel}{B}\right)_2\left(\frac{\partial}{\partial\psi}\frac{JT}{r^2}\right)_{-2} - 2p'J\frac{\partial}{\partial\psi}\left(p + \frac{B^2}{2}\right)\right].$$

The order of these coefficients is as follows: $A \sim \rho^{-2}$, $C \sim \rho^2$, $D \sim \rho^2$, $E \sim \rho$, $G \sim \rho^{-1}$, $F \sim \rho^0$. We now make a Fourier transform with respect to $m\psi$:

$$X_0(\kappa) = \int X_0(\psi)\exp(im\kappa\psi)d\psi,$$

and similarly for all other quantities. Since m is large, the localization of the perturbation is insured if the Fourier transforms exist and are square integrable. We introduce the following variables:

$$P\cos\chi = \psi_0^{1/2}(\rho\cos\theta)_{\psi=\psi_0}, \quad P = (a+b)^{-1/2}; \quad Q\sin\chi = \psi_0^{1/2}(\rho\cos\theta)_{\psi=\psi_0}, \quad Q = (a-b)^{-1/2}; \tag{7}$$

and we put

$$\tan\eta = \frac{4\kappa\psi_0}{PQB_\varphi(\rho=0)} = 2\kappa\psi_0\frac{\iota}{2\pi}, \tag{8}$$

where ι is the rotational transform. The last expresssion is the ratio of the wavelengths of the perturbation in the directions $\vec{e}_\psi \times \vec{B}$ and $\vec{e}_\psi$. With the transformation

$$Y = (V_+ - iV_-)\psi_0^{-1/2}(\rho^2)_{\psi=\psi_0}\cos\eta, \quad iZ = (V_+ + iV_-)\psi_0^{-1/2}(\rho^2)_{\psi=\psi_0}\cos\eta, \tag{9}$$

we obtain δW in the form

$$\delta W = m\int d\kappa\int d\chi(\delta W_+ + \delta W_-) + m\int d\kappa d\gamma\, FX_0^2, \tag{10}$$

with

$$\delta W_\pm = (PQ/R)|V_\pm|^2[P^2\cos^2(\eta\pm\chi) + Q^2\sin^2(\eta\pm\chi)] + p'P^3Q^2X_0(V_\pm + V_\pm^*)\sin(\eta\pm\chi),$$

where R is the radius of curvature of the magnetic axis.

Obviously, after integration over χ the expression for δW does not depend on η. As we have noted before, the localized criterion is obtained for $\Delta\psi \to 0$ with m fixed, that is for $\kappa \to \infty$, $\eta \to \pi/2$. Therefore, the fact that δW does not depend on η means that in the neighborhood of the magnetic axis, the criterion for stability of ballooning modes ($\eta \neq \pi/2$) is identical to the criterion for localized modes ($\eta = \pi/2$). Thus, we have given a general proof of a stability condition for modes for which, up to now, no criterion had been established.

As we know, this result is not true for poloidal configurations (multipoles) in which ballooning modes may be unstable when localized modes are stabilized by an average magnetic well.[1] The present work

shows that in the neighborhood of the magnetic axis of a Tokamak-type configuration, ballooning modes are stabilized simultaneously with localized modes. This result is surprising since in the case of magnetic surfaces with circular cross section the localized criterion does not yield[5] a critical β. Indeed, the calculations of Adam and Mercier[7] have recently been completed[8] by taking into account second-order corrections in the curvature and it was shown that the stability limit is practically given by $\iota/2\pi < 1$. Hence, it seems that a critical β may perhaps be found only for perturbations with small values of m and a radial extension on the scale of the plasma radius.

It is a pleasure tc thank Dr. C. Mercier for profitable suggestions and discussions.

[1]I. B. Bernstein, E. A. Frieman, M. D. Kruskal, and R. M. Kulsrud, Proc. Roy. Soc. (London), Ser. A 244, 17 (1958); H. Furth, J. Killeen, M. N. Rosenbluth, and B. Coppi, in *Proceedings of a Conference on Plasma Physics and Controlled Nuclear Fusion Research, Culham, England, 1965* (International Atomic Energy Agency, Vienna, Austria, 1966), Vol. I, p. 103; R. Kulsrud, *ibid.*, p. 127.

[2]C. Mercier, Nucl. Fusion 1, 47 (1960), and Compt. Rend. 252, 1577 (1961).

[3]J. M. Greene and J. L. Johnson, Phys. Fluids 5, 510 (1962).

[4]C. Mercier, in *Proceedings of a Conference on Plasma Physics and Controlled Nuclear Fusion Research, Culham, England, 1965* (International Atomic Energy Agency, Vienna, Austria, 1966), Vol. I, p. 417.

[5]V. D. Shafranov and E. I. Yurchenko, Zh. Eksperim. i Teor. Fiz. 53, 1157 (1967) [Soviet Phys. JETP 26, 682 (1968)].

[6]C. Mercier, Nucl. Fusion 4, 213 (1964).

[7]J. C. Adam and C. Mercier, in *Proceedings of a Conference on Plasma Physics and Controlled Nuclear Fusion Research, Novosibirsk, U. S. S. R., 1968* (International Atomic Energy Agency, Vienna, Austria, 1969), Vol. I, p. 199.

[8]J. C. Adam, G. Laval, E. K. Maschke, and R. Pellat, to be published.

MODEL TO EXPLAIN LARGE CHANGES IN THE ELECTRONIC DENSITY OF STATES WITH ATOMIC ORDERING IN V_3Au

J. Labbé
University of Paris, Orsay, France

and

E. C. van Reuth
Naval Ship Research and Development Laboratory, Annapolis, Maryland 21402

(Received 12 January 1970)

The superconducting transition temperature of V_3Au has been experimentally shown to vary by a factor of 300, depending on the degree of atomic ordering. This has previously been attributed to a change in the electronic density of states $n(E)$ with ordering. A model is proposed here which provides a theoretical basis for this change in $n(E)$ with ordering.

Previous publications[1-3] have shown that the increase in T_c for the compound V_3Au by a factor of almost 300 is caused by atomic ordering. A study of the NMR properties[4,5] has shown that this change in T_c is accompanied by a change in the core-polarization term of the total Knight shift. This term, relatable to the d electrons, suggested an increase in the d-band density of states at the Fermi level, $n(E)$, with increasing superconducting transition temperature. We present here a model which theoretically predicts this increasing density of states with atomic ordering in V_3Au.

Figure 1 shows the observed temperature dependence of the Knight shift (above T_c) for a single V_3Au sample with varying T_c values caused by changing its degree of atomic ordering. With increasing T_c, two features are immediately obvious: First, the total Knight shift decreases at 4.2 K, and secondly, the total Knight shift shows an increasing temperature dependence. In Ref. 4 (hereafter R4), it was shown that both of these features are explicable when based on an increasing d-electron density of states at the Fermi level. Additionally, results shown in Figs. 3 and 5 of R4 were cited as further experimental proof[6] of the postulated increase in $n(E)$. Although it was clear that atomic ordering was causing an