

PHENOMENOLOGICAL MODEL FOR THE ELECTROMAGNETIC STRUCTURE OF THE PROTON*

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A phenomenological model of electron-proton scattering is developed which can account for the general qualitative and quantitative features of the data. Some interesting links between the elastic and inelastic form factors are discussed.

Electron scattering has proven to be a key tool in understanding the basic structure of matter. As evidenced from the history of atomic and nuclear physics, electron beams provide an ideal microscope for "seeing" the possible constituents of the target.¹ It is presumably with this in mind that several authors² have attempted to interpret the recent Stanford Linear Accelerator Center-Massachusetts Institute of Technology (SLAC-MIT) electron-proton scattering experiments³ in terms of basic proton constituents (called "partons"). In this paper we wish to describe a crude phenomenological model of the nucleon whose basic physical ideas have much in common with such "parton" models but whose calculational techniques are somewhat different. The model is able to account for the general quantitative features of both the new SLAC-MIT data as well as the older elastic data.⁴ Some interesting relations between these two types of experiments can also be deduced. It is hoped that in a later paper some correlations with purely hadronic processes can be discussed.

Before describing our model let us briefly review some of the salient features of the data. In the SLAC-MIT experiment high-energy electrons are scattered from a proton target and detected without regard to the final hadronic state. For a given momentum transfer squared (q^2) and energy loss ($E_1 - E_2$) the measured cross section in the laboratory system can be described in terms of two form factors, W_1 and W_2 :

$$\frac{d^2\sigma}{dq^2 d\nu} = \left(\frac{4\pi\alpha^2 E_2}{q^4 E_1} \right) [W_2(q^2, \nu) \cos^2 \frac{1}{2}\theta + 2W_1(q^2, \nu) \sin^2 \frac{1}{2}\theta], \quad (1)$$

where we have introduced the invariant $\nu \equiv p \cdot q = M(E_1 - E_2)$ and the laboratory scattering angle, θ , which is given by $q^2 = 4E_1 E_2 \sin^2 \frac{1}{2}\theta$. Elastic scattering corresponds to the special case where each W_i contains a factor $\delta(\nu + \frac{1}{2}q^2)$. The data show the following general features: (a) $\nu W_2(q^2, \nu)$ scales; i.e., for large values of $-q^2$ and ν , νW_2 becomes a function of the dimensionless

variable $x \equiv -q^2/2\nu$ only. (b) For a given value of $W^2 \equiv (p+q)^2$, $W_2(q^2, \nu)$ falls off slowly with q^2 ,⁵ in sharp contrast to the behavior of the elastic form factors. (c) $\nu W_2(x)$ is a smooth function of x which is large in the sense that the integral over ν at fixed q^2 is of the order of magnitude of the Mott cross section. (d) The equivalent virtual photoabsorption cross section is mostly transverse in character.

Now to our model. Point (c) strongly suggests that very massive virtual photons interact locally within the proton. This can be simply described by hypothesizing the existence of pointlike constituents. The nature of such objects ("partons") will not be specified much beyond this. They could be quarks or possibly the bare quanta of "old fashioned" field theory. It is not impossible

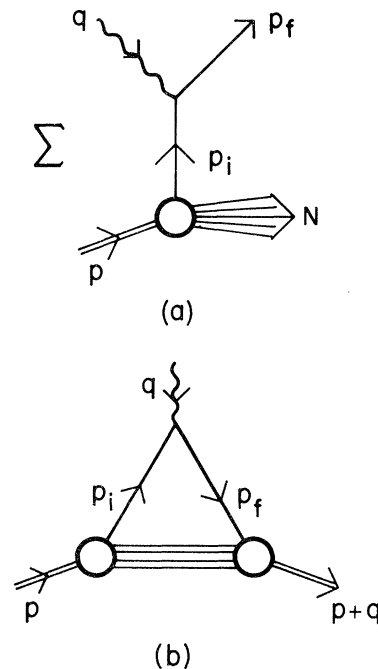


FIG. 1. (a) Model description of the inelastic scattering showing relevant kinematics; Σ implies an incoherent sum over possible partons. (b) Model description of the elastic scattering. The electron lines have been suppressed.

that they are actually real hadrons whose elastic form factors happen to approach constants asymptotically. This at least would eliminate the need for an explanation as to why they are not seen! If one is unwilling to accept such perversities in nature, then one must invoke the existence of some kind of final-state interaction which magically turns "partons" into real hadrons. Since the final state is presumably complete one might argue that this final-state interaction must turn one such complete set into another and should therefore be "of order unity," thereby not grossly affecting the general character of the results.

The scattering process is pictured symbolically in Fig. 1(a); this is not necessarily to be interpreted as a Feynman graph. The symbol \sum is meant to imply an incoherent sum over the possible different types of partons (e.g., over three different types of quarks or maybe over those hadrons whose elastic form factors do not vanish asymptotically).

It is well known^{2,3} that some of these partons must have nonzero spin in order that the transverse cross section (or, equivalently, W_1) be nonvanishing. However, for W_2 such contributions are very similar in nature to those from scalar partons (they are, in fact, identical for spin- $\frac{1}{2}$ particles). So, for simplicity, we shall concentrate on W_2 and assume that there is only one type of parton and that it has zero spin. The relevant cross section can be expressed in the form

$$\frac{d^2\sigma}{dq^2 d\nu} = \left(\frac{\pi\alpha^2}{q^4 E_1^2} \right) Q^2 \int \frac{d^3p_f}{(2\pi)^4 4p_f^0} [t_{\mu\nu} \Gamma^\mu \Gamma^\nu] f^2(p_f), \quad (2)$$

where Q is the effective parton charge, p_i (p_f) its initial (final) momentum, and $\Gamma^\mu = (2p_i + q)_\mu$ its electromagnetic vertex; $t_{\mu\nu}$ is a conserved tensor derived from the electron polarization sums:

$$t_{\mu\nu} = q^2 g_{\mu\nu} - q_\mu q_\nu + (k_1 + k_2)_\mu (k_1 + k_2)_\nu,$$

where k_1 (k_2) is the initial (final) electron momentum. The Lorentz scalar $f^2(p_f)$ is a function of the invariants $t \equiv p_i^2$, the square of the virtual parton mass, and $M_N^2 \equiv (p - p_i)^2$, the square of the residue mass. An approximate physical interpretation of $f(t, M_N^2)$ can be inferred from the observation that

$$f^2(t, M_N^2) \sim \Delta^2(t) \sigma(t, M_N^2), \quad (3)$$

where $\Delta(t)$ is the parton propagator and $\sigma(t, M_N^2)$ is a suitably normalized parton-proton total cross section: $f(t, M_N^2)$ is the amplitude for an unpolarized proton to break up into a virtual parton of mass \sqrt{t} leaving behind a residue of mass M_N . We evaluate the Lorentz scalar $t^{\mu\nu} \Gamma_\mu \Gamma_\nu = 4t^{\mu\nu} p_{i\mu} p_{i\nu}$ in the proton rest frame and assume that for this part of the scattering process, the parton is near its mass shell. Such a condition is, in fact, required in the numerator of the integral in Eq. (2) if we enforce gauge invariance (i.e., in some average sense, $p_i^2 = \mu^2$, where μ is the rest mass of the parton). It is now straightforward to isolate W_2 :

$$W_2(q^2, \nu) \cong \frac{\mu^2}{4(2\pi)^3} \frac{Q}{W|\vec{q}|_{\text{c.m.}}} \int_{M^2}^{(M_N^2)_{\text{max}}} dM_N^2 \int_{t_{\text{min}}}^{t_{\text{max}}} dt f^2(t, M_N^2). \quad (4)$$

When $-q^2$ and ν become asymptotic, one can show that the following limits are valid provided $2\nu(1-x) \gg M^2$: $W|\vec{q}|_{\text{c.m.}} \rightarrow \nu$, $t_{\text{max}} \rightarrow -2\nu \rightarrow -\infty$, $(M_N^2)_{\text{max}} \rightarrow W^2 \rightarrow \infty$, and, for a given M_N^2 ,

$$t_{\text{min}} \rightarrow \left(\frac{-x}{1-x} \right) [M_N^2 - (1-x)M^2]. \quad (5)$$

Hence, if $f(t, M_N^2)$ is sufficiently convergent, and the condition $2\nu(1-x) \gg M^2$ is satisfied, then the integral in (4) can only depend upon x when $-q^2$ and ν become large and we are naturally led to the scaling law for νW_2 . By studying Eq. (4) as a function of t_{min} one can already see that it has many of the properties one would like. For instance, it is nonvanishing at $x=0$, vanishes at $x=1$ (since, in that case, $t_{\text{min}} \rightarrow -\infty$), and is maximized when $x = 1 - \langle M_N \rangle / M$, where $\langle M_N \rangle$ is some suitably defined average value of M_N . As a specific example, suppose that the major t variation of the integrand is dominated by the propagator function $\Delta(t)$ which, for simplicity, we take to be $(t - \mu^2)^{-1}$; then, from Eq. (4), we have

$$\nu W_2(x) \cong \int_{M^2} dM_N^2 \sigma(M_N^2) \frac{(1-x)}{xM_N^2 + (\mu^2 - xM^2)(1-x)}. \quad (6)$$

This expression is not expected to be valid near $x=1$ since, as indicated above, that region is sensi-

tive to the asymptotic t behavior of $f(t, M_N^2)$ and there is no reason to expect the domination of $\Delta(t)$ to continue out to such large values of t . This observation will be important when we discuss the elastic form factors. A value for μ can be extracted from the data by observing that Eq. (6) leads to the condition that the combination $x(1-x)^{-1}F(x)$ is maximized when $x = \mu/M$.⁶ A graphical plot of this function yields the value $\mu \sim 400$ MeV; this can be interpreted as some average value of the parton masses. By choosing a specific form for σ it is not difficult to obtain a good fit to the data. We thus see how the dominance of the parton propagator leads to a natural understanding of the general shape of νW_2 .

In order to understand the size of νW_2 we turn to the problem of sum rules. These are derived by demanding that the elastic form factors, which we crudely represent by diagrams of the type in Fig. 1(b), normalize correctly to the proton charge at $q^2=0$. Using similar approximations to those used in the "calculation" of W_2 , we write (again assuming, for simplicity, only one type of parton)

$$(2p+q)_\mu F_1(q^2) \cong Q \int \frac{d^4 p_i}{(2\pi)^4} (2p_i+q)_\mu f(p_i) f(p_i+q). \quad (7)$$

At $q=0$ this leads to the normalization condition

$$Q \int p_i^\mu f^2(p_i) \frac{d^4 p_i}{(2\pi)^4} = p^\mu. \quad (8)$$

W_2 can be expressed in a similar form:

$$W_2(q^2, \nu) = 2\mu^2 Q^2 \int \frac{d^4 p_i}{(2\pi)^4} f^2(p_i) \delta[(p_i+q)^2 - \mu^2]. \quad (9)$$

Now in the asymptotic region the argument of the δ function becomes $p_i^2 - \mu^2 + 2q_0(p_i^0 - |\vec{p}_i \cdot \vec{q}| / |\vec{p}_i| |\vec{q}|) + q^2$ and the δ function can be unambiguously removed by an integration over q^2 at fixed ν :

$$\int_0^{-2\nu} dq^2 W_2(q^2, \nu) = 2\mu^2 Q^2 \int \frac{d^4 p_i}{(2\pi)^4} f^2(p_i). \quad (10)$$

The normalization condition (8) together with the requirement that (7) be gauge invariant leads, after some manipulation, to the sum rule

$$\int_0^1 \nu W_2(x) dx = Q(\mu/M)^2. \quad (11)$$

If we use the value of μ determined from above (~ 400 MeV) and set $Q=1$, we obtain ~ 0.16 for this integral, which is in remarkable agreement with experiment.^{3,7} The sum rule (11) differs considerably from the analogous nonrelativistic (NR) one.¹ To see how this comes about we note that in the NR region we can safely set $p_i \sim \mu$ and $\vec{p}_i \sim 0$ almost everywhere. However, $|\vec{p}_i|$ is no longer necessarily small compared with $|\vec{q}|$ so an integration over q^2 at fixed ν will now introduce the ambiguous factor $|\vec{p}_i/\vec{q}|$ and no interesting sum rule can be derived. On the other hand, since $p_i^0 \sim \mu \gg \vec{q}$, the δ function can instead be removed by an integration over q_0 at fixed q^2 ; only the harmless factor $(2p_i^0)^{-1} \sim (2\mu)^{-1}$ will be introduced. A comparison with the normalization condition (8) then reproduces the "classical" sum rule:

$$\int_0^\infty W_2(q^2, \nu) d\nu \cong Q.$$

Finally, we shall show that, within this model, the asymptotic behavior of $F_1(q^2)$ is correlated with the threshold ($x \rightarrow 1$) behavior of $\nu W_2(x)$.⁸ To see this, we note that Eq. (7) tells us that the asymptotic behavior of $F_1(q^2)$ is governed by the large- t behavior of $f(t, M_N^2)$. However, as remarked above, this also determines the behavior of νW_2 near $x=1$.⁹ To be more specific, suppose that

$$f(t, M_N^2) \xrightarrow{t \rightarrow -\infty} t^{-n} g(M_N^2);$$

then from (7) we see that

$$F_1(q^2) \xrightarrow{q^2 \rightarrow -\infty} (-1/q^2)^n,$$

while from (4),

$$\nu W_2(x) \xrightarrow{x \rightarrow 1} (1-x)^{2n-1}.$$

If the elastic form factors were to fall exponentially, e.g., $F_1(q^2) \rightarrow e^{-a(-q^2)}$, the relation is not quite as clean; we would then expect $\nu W_2 \rightarrow (1-x)^{1/2} e^{-a'/(1-x)^{1/2}}$ with a' different from a .¹⁰

We have, of course, throughout this paper tacitly assumed that contributions from processes in which the physical proton directly mutates into a single parton are unimportant. These correspond to renormalization graphs in perturbation theory and unless canceled would lead to a nonvanishing $F_1(q^2 \rightarrow -\infty)$. We must therefore demand that the direct proton-parton transition amplitude be zero. If we are willing to postulate nonvanishing asymptotic elastic form factors for

mesons (thus identifying partons with mesons) the question of a parton-proton transition amplitude becomes irrelevant. Such a conjecture is clearly more natural for mesons than for baryons since we know experimentally that $F_i(q^2 \rightarrow -\infty) = 0$ for the nucleons. An immediate experimental consequence of this hypothesis is that in the inelastic experiments the fast particles in the final state should be mostly mesons. It should be noted that the absence of such a direct coupling implies the absence of "s-channel" type contributions to the W_i . In the small- q^2 region, however, such contributions need no longer be small since they will not now be damped by the physical baryon elastic form factors. For this reason one should not expect the real photoabsorption limit to correspond necessarily to the $x=0$ point in the scaling limit.

We conclude with a few brief remarks about gauge invariance. Although we have a very crude model we have consistently required the resulting hadronic current tensor to be conserved. It is definitely a nontrivial task within this model to derive an expression which is manifestly gauge invariant and which does not require current conservation to be forced upon it in an "unnatural" way. There were several important problems such as this that were not fully discussed in this paper; in particular, we did not attempt a calculation of the parton spin contributions which determine W_1 . In a forthcoming paper we intend to discuss such problems in detail and to elaborate and extend some of the ideas presented here. Meanwhile I would like to thank several of my colleagues for valuable discussions, in particular, Dr. F. A. Berends, Professor J. I. Friedman, Dr. R. E. Peierls, and Professor V. Weisskopf.

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¹See, e.g., N. F. Mott and H. S. W. Massey, *Theory of Atomic Collisions* (Clarendon Press, Oxford, England, 1949), *passim*; R. Hofstadter, *Nuclear and Nucleon Structure* (Benjamin, New York, 1963), *passim*.

²For a review of such models see F. Gilman, in *International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, 1969*, edited by D. W. Brabben (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970), pp. 177-194.

³R. E. Taylor, *ibid.*, pp. 251-260.

⁴J. G. Rutherglen, *ibid.*, pp. 163-176.

⁵As pointed out in Ref. 2 this follows from (a).

⁶It is interesting to note that in the nuclear, or non-relativistic, limit where $\sigma(M_N^2)$ peaks at some particular value of $M_N (\sim M - \mu)$, our results agree with the standard one, namely that νW_2 is maximized at $x \sim \mu/M$. However, our model suggests that in the extreme relativistic region it is $x(1-x)^{-1}\nu W_2(x)$, rather than $\nu W_2(x)$, which peaks near $x \sim \mu/M$.

⁷It should be pointed out that the precise factor on the right-hand side of Eq. (11) is not meant to be taken too seriously. However, the calculation leading to Eq. (11) does point to the possibility that there are important corrections to the NR result. The latter would suggest that the constituents have fractional charges (see Ref. 2).

⁸While we were writing this paper, a preprint by S. D. Drell and T.-M. Yan [Stanford Linear Accelerator Center Report No. SLAC-PUB-699 (unpublished)] reached us. They prove a similar result using their field-theory model (see Ref. 2). The result can also be conjectured from a sidewise dispersion relation for the F 's.

⁹We must, of course, ensure that $|t_{\min}| \ll |t_{\max}|$, i.e., $2\nu(1-x) \gg M^2$, which is a condition for scaling.

¹⁰Note that our expression for νW_2 , Eq. (6), is not inconsistent with the results of this paragraph, since when $x \rightarrow 1$ we expect the behavior of $f(t, M_N^2)$ to be modified by the asymptotic t dependence of $\sigma(t, M_N^2)$. For nonasymptotic values of t (which effectively means x not near 1) $\sigma(t, M_N^2)$ is assumed to behave smoothly with t so that $f(t, M_N^2)$ is dominated by $\Delta(t)$.