DYON THREE-TRIPLET MODEL OF HADRONS*

M. Y. Han and L. C. Biedenharn
Department of Physics, Duke University, Durham, North Carolina
(Received 19 November 1969)

Schwinger's composite model of hadrons, based on dyons, is formulated as a dyon three-triplet model with the postulated symmetry group SU(3)[electric] ⊗ SU(3)[magnetic]. The hadron spectrum is dominated by the superstrong magnetic-charge exchange forces; we find the first "exotic" states to be in the meson channel of the $(2D2\bar{D})$ system. This model possesses the good features of the quark model and explains its paradoxes.

Recently Schwinger presented a speculative —but highly ingenious—theory of matter based upon the postulated existence of elementary, dually charged (possessing both fractional electric and magnetic charge) constituents—called dyons.[1] We wish to point out that Schwinger's construction can be given in two distinct versions, one of which is isomorphic (in the structural sense) to the three-triplet model of hadrons proposed earlier.[2] In the context of Schwinger's work, the three-triplet model assumes now a much more explicit form (the superstrong force assumed in the original model is to be identified with Schwinger's enormously strong "magnetic force").

In particular, the superstrong magnetic-charge exchange forces should dominate the gross structure of the hadron spectrum. (Such exchange forces are required[1] in order to suppress the large CP-nonconserving mechanism inherent in dually charged particles.) On the basis of two semiquantitative models this exchange interaction appears to be compatible with the observed spectrum, in terms of one scale parameter—that of the superstrong exchange interactions.

The three-triplet model with double SU(3) symmetry was originally proposed[2] to modify the Gell-Mann–Zweig quark model on three critical counts: the avoidance of observable fractional charges; the reconciliation of Fermi statistics with the totally symmetric SU(6) baryon wave function; and the automatic realization of only triality-zero SU(3) states.[3] There are nine fundamental fermions of spin $\frac{1}{2}$ out of which all hadrons are to be made. This nonet is governed by a double SU(3) symmetry in the sense that the particles, $T=t_{A''}{}^{A'}$ [A' and A'' are SU(3)′ and SU(3)″ indices, respectively], are "quarks" with respect to the first SU(3)—called SU(3)′—and "antiquarks" with respect to the second SU(3) —called SU(3)″; they transform as the representation $(\underline{3}', \underline{3}^{*}{}'')$ of the group $G \equiv \mathrm{SU}(3)' \otimes \mathrm{SU}(3)''$. The hypercharge and charge quantum numbers, which have fractional values with respect to the individual SU(3) groups, add up to give integral values with respect to the total group G.

Mesons are triplet-antitriplet states $(T\bar{T})$, and baryons are three-triplet states (TTT). The hierarchy of interactions involved two postulates: (a) SU(3)″ interactions are "superstrong," much stronger than those of SU(3)′; (b) SU(3)″ singlets are the lowest-lying levels. Accordingly we have the low-lying mesons and baryons uniquely selected as the states $(\underline{1}', \underline{1}'')$, $(\underline{8}', \underline{1}'')$, $(\underline{1}', \underline{1}'')$, $(\underline{8}', \underline{1}'')$, and $(\underline{10}', \underline{1}'')$, respectively.

An essential feature of Schwinger's discussion was to show that dually charged objects allowed charge quantization in units of $e/3$ for electric charge, and $g/3$ for magnetic charge with the dimensionless interaction strength $g^2/\hbar c \sim 36 \times 137 \cong 5000$. Magnetic charges interact via long-range, superstrong, forces. It is very natural simply to identify SU(3)′ with the symmetry group of the fractional electric charges, and SU(3)″ as the symmetry group of the fractional magnetic charges of the dyons. We can now spell out the structure of the dyon-three-triplet model.

There are nine fundamental dyons (and nine antidyons) denoted D and $\bar{D}$; under the symmetry group $G \equiv \mathrm{SU}(3)^{(e)} \otimes \mathrm{SU}(3)^{(m)}$, they transform as the representation $(\underline{3}, \underline{3}^{*})$, that is to say that dyons are quarks in the electric-charge space and antiquarks in the magnetic-charge space. The three-triplet model and this dyon model are isomorphic:

$$T \leftrightarrow D,\quad \mathrm{SU}(3)' \leftrightarrow \mathrm{SU}(3)^{(e)},\quad \mathrm{SU}(3)'' \leftrightarrow \mathrm{SU}(3)^{(m)}.$$

The values of the electric and the magnetic charges of dyons are both fractional and are identical to the values of Q' and Q'', respectively, of the three-triplet model. [This identification reverses Schwinger's magnetic-charge assignment.] The hadrons are made up of dyons in the same way as before; namely, mesons $(D\bar{D})$ and baryons (DDD). The restriction that physically observable particles be magnetically neutral would automatically select magnetic SU(3) singlets, as required – but we prefer not to impose this restriction but to deduce it (see below).

The novel – and crucial – aspect of the dyon concept is that an elementary dyon violates CP invariance.[1] One must therefore consider the possibility that the nucleons possess an electric dipole moment. We wish to explore this feature in order to show how it implies qualitative results in hadron spectroscopy.

In the dyon three-triplet model the nucleons are assigned the $SU(6)^{(e)} \otimes SU(3)^{(m)}$ labels $(\underline{56}^{(e)}, \underline{1}^{(m)})$. Using the fact that the dyons are dually charged, one may calculate the electric dipole moment of the nucleons in precisely the same way as in the conventional magnetic-moment calculation for the quark model.[4] This leads to the estimate $(E1)_{\rm nucleon} \sim g\hbar/Mc \approx 10^{-12}q$ cm, which is grossly at variance with the experimental result[5]: $(E1)_{\rm neutron} < 10^{-22}q$ cm. To rescue the situation, Schwinger suggested the possibility of magnetic-exchange currents. We will estimate this possibilty in two ways.

(a) Let us assume that conventional methods for phenomenological treatment of exchange currents apply. Then the electric dipole moment may be written $\vec{d} = \frac{1}{2}\int dv\, \vec{r} \times \vec{J}^{(m)}$, where $\vec{J}^{(m)} = \vec{J}^{\rm conv} + \vec{J}^{\rm pol} + \vec{J}^{\rm mx}$ (convection, polarization, and magnetic-exchange currents, respectively). For order-of-magnitude estimates we use the conservation law for magnetic charge $(\rho^{(m)})$: $\vec{\nabla}\cdot\vec{J}^{\rm mx} = (i\hbar c)^{-1}[H_{\rm exch}, \rho^{(m)}]$. This equation, strictly speaking, is no restriction on the static $E1$ moment, but for purposes of estimate, we argue that this is not crucial, and find $J^{\rm mx} \approx (g/\hbar c) \times \langle H_{\rm exch}\rangle$. Taking the polarization moment $(\approx g\hbar/M_Dc^2)$ to cancel the exchange moment, we arrive at the estimate $\langle H_{\rm exch}\rangle \approx M_Dc^2 \approx 6$ GeV (Schwinger's "guesstimate").

(b) On the other hand, we may proceed differently and argue this way: The polarization moment arises through the Zitterbewegung of the dyon, and this in turn involves frequencies $\nu \approx \hbar/2M_Dc^2$. To "turn off" the effects of this Zitterbewegung, the magnetic-exchange interaction must be so strong that in a time short compared to ν^{-1} the magnetic charge on a given dyon is exchanged so many times that it appears to be the average (baryon) magnetic charge (zero). We arrive then at the alternative estimate $\langle H_{\rm exch}\rangle >> M_Dc^2$.

Admittedly, these are very tentative estimates, but we feel it premature to try to improve the situation by adducing detailed phenomenological forms for the exchange current. Based on these estimates, however, we shall give semiquantitative features of the hadron spectrum.

The SU(3) group admits of two types of exchange operators: the familiar two-body exchange operators P_{ij}, and the three-body exchange operators P_{ijk}. For a first orientation, let us take $H_{\rm exch}$ to be

$$H_{\rm exch} = \sum_{i<j} f(r_{ij})P_{ij} + \sum_{i<j<k} g(r_{ij})P_{ijk} \qquad (1)$$

$$\cong A\sum_{i<j} P_{ij} + B\sum_{i<j<k} P_{ijk}. \qquad (2)$$

The contribution of $H_{\rm exch}$ to the hadron spectrum is $M \approx \langle H_{\rm exch}\rangle \cong A\langle P_{ij}\rangle + B\langle P_{ijk}\rangle$, where, for general U(3) labels $[pqr]$,

$$\langle[pqr]|\sum_{i<j} P_{ij}|[pqr]\rangle = \tfrac{1}{2}[p(p-1)+q(q-3)+r(r-5)], \qquad (3)$$

$$\langle[pqr]|\sum_{i<j<k} P_{ijk}|[pqr]\rangle = \tfrac{1}{6}[p(p-1)(p-2)+q(q-2)(q-4)+r(r-4)(r-5)-3(pq+pr+qr)]. \qquad (4)$$

For the hadron spectrum in case (a) ("weak" magnetic-exchange force), we take only a two-body interaction, the magnitude of which is comparable to that of the free-dyon mass. Let us write $M \approx \langle H_{\rm dyon}{}^0\rangle + \langle H_{\rm exch}\rangle$. For an aggregate of dyons alone, or antidyons alone, the quantity $\langle H_{\rm exch}\rangle$ is given by $A\langle P_{ij}\rangle$, with U(3) conjugate labels for the antidyons. To extend these considerations to dyon-antidyon composite systems, the first point to note is that the exchange magnetic current, which for baryons led to the dyon exchange operators, no longer has a particle exchange interpretation in the $(mDn\bar{D})$ system. Nevertheless the exchange magnetic current still operates between dyon and antidyon: We will therefore identify the structural form of $H_{\rm exch}$ (DD) to be preserved. For a general $(mDn\bar{D})$ system we find

$$\langle H_{\rm exch}\rangle = A\,\{\langle P_{ij}\rangle_{mD} + \langle P_{ij}\rangle_{n\bar{D}} + \tfrac{1}{3}mn + 3[C^{(2)}(mDn\bar{D}) - C^{(2)}(mD) - C^{(2)}(n\bar{D})]\},$$

where $C^{(2)}$ refers to the Casimir operator of the total, the mD and the $n\bar{D}$ SU(3) labels.[6] We find for $U(3)^{(m)}$, for the (DDD) baryon, $M([300]) = 3A$, $M([210]) = 0$, $M([111]) = -3A$, and for $(D\bar{D})$ mesons, $M([210]) = \frac{1}{3}A$, $M([111]) = -7/3A$. In order that [111] lie lowest, we must have $A > 0$. With this choice, all magnetic singlets lie lowest within each $(mDn\bar{D})$ system. The magnetic singlet spectrum, for two-body exchange, is shown in Fig. 1. Taking account of $\langle H_{\mathrm{dyon}}{}^0\rangle$, and requiring the $D\bar{D}$ and DDD levels to be degenerate (on a strong-interaction scale), we find $M_D c^2 = \frac{3}{2}A$. The resulting hadron spectrum—"weak case"—is shown in Fig. 1. The lowest "exotic" state appears to be in the meson channel, the $2D2\bar{D}$ system at ≈3 GeV. A few of the magnetic-charge "exotic" states are shown, the lowest appearing to be the $D\bar{D}$ magnetic octet at ≈17 GeV.

In case (b)—"strong" magnetic-exchange force $-H_{\mathrm{dyon}}{}^0$ should play no significant role in determining the hadron spectrum; by contrast, three-body exchange forces should be significant. The three-dyon (baryon) $\mathrm{U}(3)^{(m)}$ spectrum is found to be $M([300]) = 3A + B$, $M([210]) = -\frac{1}{2}B$, $M([111]) = -3A + B$. In order that [111] lie lowest, we must have $A > 0$ as before and $A > B$. Further, we must require that all other $(3nD)$ $\mathrm{U}(3)^{(m)}$ singlets lie higher. The six-dyon system is critical, for we find $M([222]) = -3A - 4B$. To make the three-dyon system lowest in energy, we must have $B < 0$, i.e., three-body exchange forces of strength roughly comparable with and opposite to that of the two-body exchange forces. There is then competition between the two- and three-body forces, tending to different symmetries. The three-body forces $(\sim n^3)$ would dominate except that their range is surely appreciably shorter and that beyond six dyons antisymmetric spatial states enter: This will tend to cut off the three-body effects for large n. If we take $n = 9$ to be the cutoff, $M([900]) = M([333])$, we obtain $B \leq -\frac{1}{3}A$. We extend the three-body exchange forces to the $(mDn\bar{D})$ system analogously as for the two-body case. In this manner the two- and three-body exchange forces combine to yield a satisfactory spectrum for baryons and mesons are split. We ascribe this split to a strong (linear) dependence on the baryonic charge. Taking $B = -\frac{1}{2}A$, and DDD degenerate with $D\bar{D}$, we find the separate meson and baryon spectra as shown in Fig. 1.

We have yet to justify the assertion that there are two distinct versions of the dyon-three-triplet model. In assigning fractional electric and magnetic charge to the dyon, we chose a sign convention opposite to that of Schwinger; this, by itself, does not constitute a true distinction. But we also chose to define the transformation properties of the dyon to be $(\underline{3}, \underline{3}^*)$; alternatively

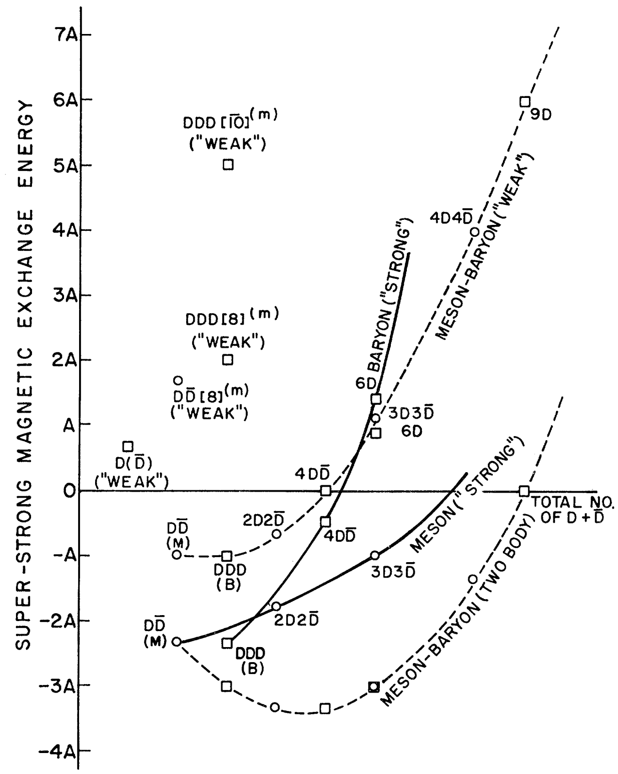

FIG. 1. The hadron spectrum for two cases: (a) "weak" magnetic-exchange interaction (dashed curve) and (b) "strong" magnetic-exchange interaction (solid curve); isolated points are magnetically charged states; all others are magnetically neutral.

we might have defined the dyon to transform as $(\underline{3}, \underline{3})$. These two choices correspond to distinct, and inequivalent, models.

The existence of two versions of the model leads to an interesting consequence linking space-time and unitary spin properties. For each SU(3) group there corresponds a reflection operator, R_e and R_m. One has then four possibilities when operating on a dyon state: $(\underline{3}, \underline{3}^*)$; $(\underline{3}, \underline{3})$; $(\underline{3}^*, \underline{3})$; and $(\underline{3}^*, \underline{3}^*)$. Two pairs may be distinguished as the dyon and antidyon of the two versions of the model. Note that the relative sign of e vs g differs in the two pairs; note also that a parity reflection (P) reverses the relative sign of e vs g. In order that each version be unique and preserved under all symmetries, we must exclude half of the possible states. Just as in the analogous case of the neutrino, this necessitates linking two reflection operators: R_m and P are not separately symmetries, only the product PR_m, which we denote as the "dyon parity operator."[7]

We feel it significant that both "weak" and "strong" cases agree in the prediction that of the two types of "exotic states"—the magnetic monopole states and the exotic quark-model states —the latter should occur first, and in the meson spectrum. The energy at which these "quark-exotic" mesonic channels open sets the scale for the superstrong magnetic-exchange energy. Below this energy the dyon–three-triplet model validates the conventional quark-model approach —explaining its paradoxes and limiting attention to the $(D\bar{D})$ and (DDD) magnetic singlets.

We wish to thank Professor Hugh Robinson for discussions.

---

*Research supported in part by the Army Research Office (Durham) and the National Science Foundation.

[1]J. Schwinger, Science 165, 757 (1969), and 166, 690 (1969).

[2]M. Y. Han and Y. Nambu, Phys. Rev. 139, B1006 (1965).

[3]M. Gell-Mann and Y. Ne'eman, in The Eightfold Way, edited by M. Gell-Mann and Y. Ne'eman (W. A. Benjamin, Inc., New York, 1964); E. C. Fowler and L. C. Biedenharn, Nuovo Cimento 33, 1329 (1964).

[4]See, for example, J. J. J. Kokkedee, The Quark Model (W. A. Benjamin, Inc., New York, 1969).

[5]C. Shull and R. Nathans, Phys. Rev. Letters 19, 384 (1967); W. B. Dress, J. K. Baird, P. D. Miller, and N. F. Ramsey, Phys. Rev. 170, 1200 (1968).

[6]The final result, in fact, depends only on the number of dyons and the final $U(3)^{(m)}$ labels.

[7]Schwinger has made remarks that can be interpreted in this way, in Proceedings of the Third Coral Gables Conference on Symmetry Principles at High Energy, University of Miami, 1966, edited by A. Perlmutter, J. Wojtaszek, G. Sudarshan, and B. Kurşunoğlu (W. H. Freeman & Company, San Francisco, Calif., 1966), p. 247. One can see the necessity for linking P and R_m in a different way. If P is a valid symmetry for the neutron, say, then trivially the $E1$ moment vanishes. But under the dyon parity operation this no longer follows.