EFFECT OF EXCHANGE ON MAGNETOSTATIC MODES

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When surface spin pinning is included in the thin-film magnetostatic-mode eigenvalue problem, exchange must be included even when the exchange energy is negligible. The magnetization is changed drastically in general, but the intensities and frequencies are essentially the same as for the pure magnetostatic modes (exchange constant $D=0$ and no explicit pinning mechanism).

Since the magnetization of the pure magnetostatic modes (exchange constant $D=0$ and no explicit pinning[1]) is a single sinusoidal function of z', where the z' axis is normal to the plane of the film, it might be expected that pinning the magnetostatic modes (negligible exchange energy) would give rise to large intensities, as in the case of exchange modes (negligible microwave demagnetization energy).[2] However, it will be shown that this is not the case. The intensities and frequencies of the magnetostatic modes are essentially independent of explicit surface spin pinning. Thus the intensities of the magnetostatic modes as a function of $k_{z'}$ cannot be used to obtain information about the pinning. Such information will have to be obtained from the exchange modes or mixed exchange-magnetostatic modes.

Imposing a pinning condition such as $\vec{m}=0$ on the magnetostatic-mode problem with $D=0$ overdetermines the boundary conditions at $z'=\pm\frac{1}{2}S$, where S is the film thickness, and no solution exists. In this case it is necessary to include the exchange interaction even though Dk^2 is negligible. Mathematically, the exchange term is a singular perturbation.

The system equations are

$$d\vec{M}/dt=|\gamma|\vec{M}\times[\hat{z}H_i+\vec{h}+4\pi\Lambda\nabla^2\vec{M}], \qquad (1a)$$

$$\nabla\cdot(\vec{h}+4\pi\vec{m})=0, \quad \nabla\times\vec{h}=0, \qquad (1b)$$

where $\vec{M}=\hat{z}M_s+\vec{m}$ is the magnetization, γ is the gyromagnetic ratio, $\vec{H}_i$ is the internal (demagnetized) field, $\vec{h}$ is the microwave demagnetization field, $\Lambda=D/4\pi M_s$, and M_s is the saturation magnetization. The solutions to (1) are linear combinations of terms containing three wave vectors,[3] in contrast to the exchange case or the pure magnetostatic case for which only one wave vector is required. For a given $\vec{k}_f\equiv\hat{x}'k_{x'}+\hat{y}'k_{y'}$, the three values of $k_{z'}^2$ are easily found by substituting m_x, $m_y\sim\cos k_{z'}z'$ into (1). The result is that the three values of $k_{z'}^2$ are simply the roots of the well-known dispersion relation $k^2\Omega^2=k^2(\Omega_H+\Lambda k^2)(\Omega_H+\Lambda k^2+\sin^2\theta_k)$, where $\Omega\equiv\omega/4\pi|\gamma|M_s$, $\Omega_H\equiv H_i/4\pi M_s$, and $\sin^2\theta_k=(k_x^2+k_y^2)/k^2$. In perpendicular resonance, i.e., for $k_{z'}=k_z$ and $k_f^2\equiv k_x^2+k_y^2$, the three roots of this equation in the limit of small Λk^2 are k_{ms}, k_E, and ik_{ngl}, where

$$k_{ms}^2\cong k_f^2(\Omega_H^2+\Omega_H-\Omega^2)(\Omega^2-\Omega_H^2)^{-1}, \qquad (2a)$$

$$k_E=\pm[(\Omega-\Omega_H)/\Lambda]^{1/2},$$

$$ik_{ngl}=\pm i[(\Omega+\Omega_H)/\Lambda]^{1/2}. \qquad (2b)$$

Since each of these k_z's is a solution to (1) and the system is invariant to $z\to -z$ in perpendicular resonance, the general solution for the magnetic potential ψ (where $\vec{h}=\nabla\psi$) inside the film is the sum of three waves:

$$\psi\sim A_{ms}\cos(k_{ms}z-\tfrac{1}{2}\pi\eta)+A_E\cos(k_Ez-\tfrac{1}{2}\pi\eta)$$
$$+A_{ngl}(\cosh k_{ngl}z)^{1-\eta}(\sinh k_{ngl}z)^{\eta},$$

where $\eta=0$ or 1 for even or odd modes, respectively. At the surfaces at $z=\pm\frac{1}{2}S$, $\psi=d\psi/dz=m_x=m_y=0$. These boundary conditions can be written as a set of three homogeneous equations for the A's by eliminating E between the ψ and $d\psi/dz$ equations, where $\psi\sim E\exp(-k_f|z|)$ outside the sample. The secular equation is easily solved to give $k_{ms}\tan(\frac{1}{2}k_{ms}S-\frac{1}{2}\pi\eta)\cong k_f$. This result, which determines the quantized values of k_{ms}, is the same as that[4] for pure magnetostatic modes. A second solution, for the exchange modes, is not of interest here. Solving for the A's shows that the A_{ngl} term is negligible even at $z=\pm\frac{1}{2}S$ and that $m_x, m_y\sim\cos k_{ms}z+B\cos k_Ez$, where $B=\cos\frac{1}{2}k_{ms}S/\cos\frac{1}{2}k_ES$, as illustrated in Fig. 1(a).

These results have an intuitive explanation, which is useful in predicting the results for other cases such as parallel resonance or finite-film resonance. Solving (2a) for Ω gives $\Omega\cong[\Omega_H(\Omega_H+\sin^2\theta_k)]^{1/2}\equiv\Omega_{ms}$, which is just the frequency[4] of a pure magnetostatic mode. Since $\Omega=\Omega_H+\Lambda k^2$ for an exchange mode, (2b) shows that k_E is the wave vector for an exchange wave which is degenerate with the magnetostatic wave. Note that the Zeeman frequency is positive and the ex-

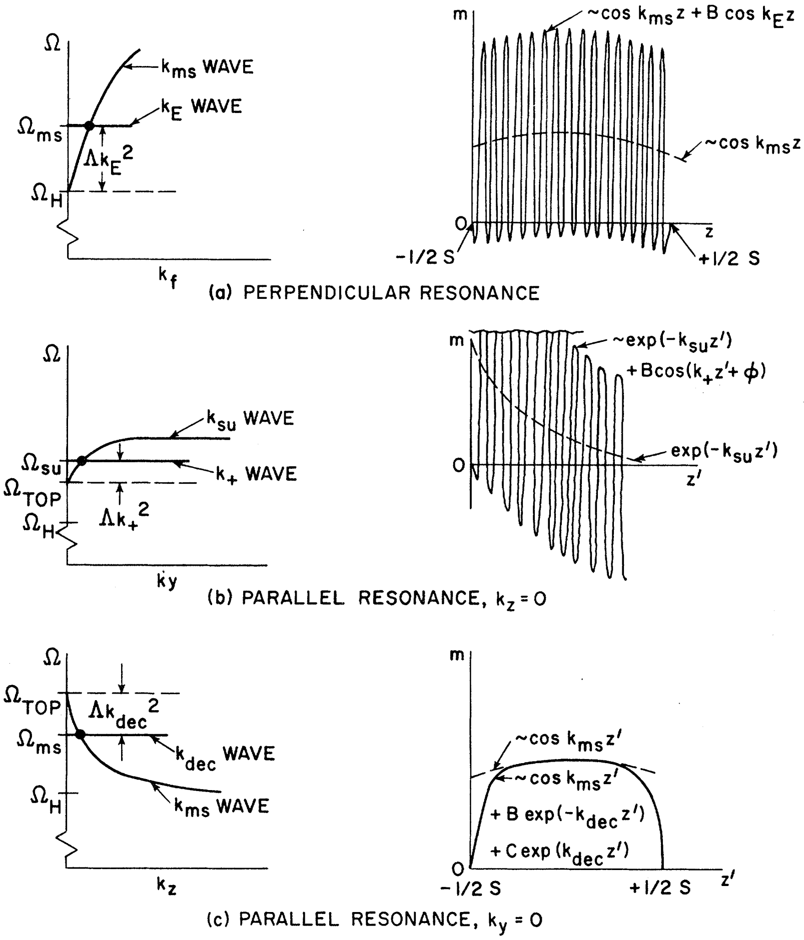

FIG. 1. Illustration of the effects of pinning on magnetostatic modes. The various frequencies are illustrated in the left side of the figure, and the variations of the microwave magnetization across the film thickness are illustrated in the right side, where the dashed lines are for pure magnetostatic modes and the solid lines are for magnetostatic modes with pinned surface spins.

change frequency is positive for oscillating waves [negative $\alpha \equiv (d^2m_x/dz'^2)/m_x$] or negative for decaying waves (positive α). The frequency of the k_E wave is $\Omega_E = +\Omega_{ms}$, and that of the ik_{ngl} wave is $\Omega_{ngl} = -\Omega_{ms}$. The k_{ms} and k_E waves can be admixed freely to satisfy the surface pinning conditions ($m_x = m_y = 0$ in the present example) since the frequencies are the same. The decaying ik_{ngl} wave is far off frequency ($\Omega = -\Omega_{ms} \neq +\Omega_{ms}$) since α is positive, and its amplitude is negligible.

Since $\Lambda(\pi/S)^2 \ll 1$ for magnetostatic modes, the k_E wave must have many oscillations, i.e., $k_z \gg \pi S$, in order to make its frequency $\Omega_H + \Lambda k_E^2$ equal to the magnetostatic-wave frequency Ω_{ms}. See Fig. 1(a). Thus the intensity is controlled by the term $\cos k_{ms} z$ since $\cos k_E z$ integrates to zero approximately. In other words, the intensities of the pinned magnetostatic modes are the same as those of the pure magnetostatic modes (having no explicit pinning).

The corresponding results for parallel resonance, i.e., for $k_{z'} = k_x$, $k_{y'} = k_y$, and $k_{x'} = k_z$, can be inferred from the results above. For $k_z = 0$, the values of k_x obtained from the solution of the dispersion relation are $k_x = ik_{su}$, k_+, and k_-, where $k_{su} = \pm k_y$ and $\Lambda k_\pm^2 = -(\Omega_H + \frac{1}{2}) \pm (\frac{1}{4} + \Omega^2)^{1/2}$. The frequency Ω_{su} of the ik_s wave is simply the frequency of the Damon and Eshbach magnetostatic surface wave.[5] The frequencies of the $k_\pm$ waves are $\Omega_\pm = \pm\Omega_{su}$. The decaying k_- wave is off frequency, and its amplitude is negligible. The oscillating k_+ wave has the same frequency as the surface wave, and a linear combination of these two waves is chosen to satisfy the surface pinning conditions. See Fig. 1(b).

For the bulk modes with $k_y = 0$ in parallel resonance, solving the dispersion relation for k_x

gives $\Lambda k_{(\pm)}{}^2 = -\Omega_H - \frac{1}{2} \pm [(\Omega_H + \frac{1}{2}) - (\Omega_{\rm top}{}^2 - \Omega^2)]^{1/2}$, where $\Omega_{\rm top}{}^2 \equiv \Omega_H{}^2 + \Omega_H$, and $k_{\rm ms}$, which is just the Damon and Eshbach[5] bulk-mode wave vector having $\Omega = \Omega_{\rm ms}$. The other two frequencies are $\Omega_{(\pm)} = \pm\Omega_{\rm ms}$. The decaying $k_{(-)}$ wave is off frequency, and its amplitude is negligible. A linear combination of the degenerate $k_{\rm ms}$ and $k_{(+)}$ waves is chosen to satisfy the surface pinning conditions. See Fig. 1(c). The second ($k_{(+)} \equiv ik_{\rm dec}$) wave is decaying, in contrast to the results of the previous two cases, because the frequency must be lowered from $\Omega_{\rm top}$ to $\Omega_{\rm ms}$ and a positive α (decaying wave) lowers the frequency. This $ik_{\rm dec}$ wave changes $\vec{m}$ only very near the surfaces, where it rounds off $\vec{m}$ to zero at $z' = \pm\frac{1}{2}S$.

Several conclusions of Wolfram and De Wames,[6] based on incorrect generalizations of computer solutions for m_x and ω for several specific values of $k_f S$, Ω_H, etc., contradict the present results. The k_E or k_+ wave, not the ik_{ngl} or k_- wave, is the important one for satisfying the boundary conditions, and the amount of the k_E or k_+ wave in $\vec{m}$ is large away from the crossovers in general, as illustrated in Figs. 1(a) and 1(b). In their semi-infinite-medium calculation of the lifetimes of surface modes, their solution satisfies the surface conditions only at certain isolated instances of time since a traveling wave $m_x \sim \exp(ik_x x)$ [with implicit $\exp(i\omega t)$ time dependence] cannot satisfy their pinning condition $dm_x/dx = 0$ at $x = 0$ for all times. It is misleading to consider $\vec{m}$ as an admixture of bulk and surface waves simply because one $k_{z'}$ is imaginary and another is real. For example, in Fig. 1(c) the $ik_{\rm dec}$ wave rounds off $\vec{m}$ to zero at the surface.

Several experimental results can be explained in terms of the theory. Sparks and co-workers[7] observed that the higher-branch (larger values of $k_{z'}$) magnetostatic modes in a 12.4-μm yttrium-iron-garnet film had very small intensities. Single-sine-wave pinned modes would have large intensities[7,8] in contrast to the small observed intensities. The two-wave solutions discussed above should have small intensities, in agreement with the experimental results. The theoretical results also explain the fact that surface modes have been observed[9] under conditions for which the surface spins are expected to be pinned[8]; it might have been expected that making $\vec{m} = 0$ at the surface would have essentially eliminated the surface modes, but the two-mode results above indicate that this is not the case.

It is reasonable to expect that the results for other geometries should be similar. Thus, the explanation of the magnetostatic modes observed[10] in spherical samples in terms of the theoretical results[10] developed for no explicit pinning should be valid even though the surface spins are likely to be pinned and the pure magnetostatic modes have large amplitude at the surfaces in general.

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[1]The surface spins are said to be pinned (or unpinned) if the microwave magnetization $\vec{m} = 0$ (or $d\vec{m}/dz' = 0$) at $z' = \pm\frac{1}{2}S$, where S is the film thickness. In general $a\vec{m} + b\,d\vec{m}/dz' = 0$ at $z' = \pm\frac{1}{2}S$, where a and b vary from mode to mode. If some mechanism other than the usual electromagnetic field continuity conditions at the surface plane holds a and b fixed, there is said to be an explicit pinning mechanism.

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