density, temperature, and magnetic probe signals. The decay time is equal to that estimated from a hydrodynamic model for the end loss. Hence it might be said that the decay of plasma energy is essentially determined by the end loss.

The plasma is observed not to lose its stability against the following disturbances: (1) Installation of several segments of insulating material at the stagnation circles. If shorting currents, which work to suppress the toroidal drift, flow along the circle, they are blocked by the obstructions. (2) Insertion of two disks of insulators in the vicinity of the rings, whose diameters are the same as that of the discharge tube and the surfaces are normal to the tube axis. The plasma column is cut with them so that the domain of integral  $\int dl/B$  is bounded by the walls of insulator disks (here  $B$  is the magnetic flux density and  $dl$  is the line element along the magnetic line of force).

When the rings are mounted at the alternate midplanes of the mirror bottles, a part of the

plasma surface touches the wall immediately after the pinch  $[Fig. 2(e)]$ . In this case the field is configured not by a series of caulked cusps, but by the alternate caulked cusp and mirror.

From the examinations mentioned above it is concluded that the configuration of a series of caulked-cusp fields is effective for the suppression of toroidal drift of high-beta plasma in the earlier stage of theta pinch, as well as for the stabilization of low-beta plasma in the later stage.

The authors wish to express sincere thanks to Professor K. Takayama for his encouragement, and to Dr. Y. Suzuki, Dr. A. Mori, and Dr. M. Masuzaki for their valuable discussions. The authors are also indebted to Mr. K. Adati, Mr. K Hayase, and Mr. S. Hirabayashi for their helpful assistance in the measurement of plasma parameters.

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## FEEDBACK SUPPRESSION OF COLLISIONLESS, MULTIMODE DRIFT WAVES IN A MIRROR-CONFINED PLASMA\*

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Simultaneous suppression of several unstable drift wave modes in a collisionless plasma has been achieved using several independent feedback systems, each one tuned to a different mode. This is an improvement over broadband systems in which suppression of one mode has generally led to enhancement of other modes.

Feedback stabilization has recently been reported for the suppression of drift waves' and driftlike instabilities<sup>2,3</sup> in collision-dominated plasmas, where only one mode existed or was isolated by careful adjustment of the plasma parameters. For collisionless plasmas, we and  $t_{\text{at}}$  and that many azimuthal others<sup>4-6</sup> typically find that many azimuthal varying modes are present simultaneously and that isolation of one mode is very difficult or impossible. Our problem is, thus, simultaneous suppression of many modes, clearly the more general situation in plasmas. The significant result of this work is that by using filtered-feedback loops, one to a mode, the modes may be independently and simultaneously suppressed. A theory for feedback stabilization of drift waves where the feedback probe acts as an electron sink has been given by Simonen, Chu, and Hendel' and Furth and Rutherford. '

The apparatus on which the experiments were performed is the Berkeley plasma instability experiment device, in which a potassium plasma is produced by contact ionization in a 50-cm-long mirror magnetic field of mirror ratio 2.5. The density used was about  $10^8$  cm<sup>-3</sup>, the ion and electron temperatures were about 0.2 eV, and magnetic fields at the midplane ranged from 0.5 to 1.0 kG. At this temperature and density the unstable waves are considered collisionless in the sense that the ion collision frequencies are at least one order of magnitude lower than the observed frequencies:  $v_{in} \approx 700$ ,  $v_{II} \approx 140$ ,  $v_{Ie}$ Hz, and  $v_{ef} \approx 50$  kHz; the wave frequencies run from 7 to 50 kHz. A dc radial electric field causes the plasma to rotate at speeds comparable with and larger than the diamagnetic drift speed.



FIG. 1. Schematic of feedback arrangement. Each feedback system consists of a filter, phase shifter, amplifier, and suppressor probe. Each system is tuned to a different mode.

The waves to be suppressed by feedback were identified by the following characteristics, all of which fall within the theoretically expected behavior for collisionless drift waves<sup>4,8</sup>: (1) maximum oscillation amplitude near the maximum of ' $(1/n)(dn/dr);$  (2)  $v_{th}^{\phantom{th}I} \ll \omega/k_{\parallel} \ll v_{th}^{\phantom{th}e}$  (velocity ratios 1:30:270); (3)  $k_{\parallel} \ll k_{\perp} (k_{\parallel}/k_{\perp} \simeq 0.06$  for  $m=2$ mode); (4) azimuthal propagation in the electron diamagnetic direction; (5) frequency proportional to  $T/B^s$  (s near 1.0); (6) on leads  $\delta\varphi$  by about 75°; and (7)  $\delta n / n_0 \approx \delta \varphi / KT \approx 0.05$  with variations from this value depending on the azimuthal mode number,  $m$ . We observe these waves in a steady state, presumably the saturated state of the instability; the wave characteristics agree remarkably well with small-amplitude theory, implying a very low level of saturation.

Feedback suppression was set up to be tried with several loops, as shown in Fig. 1. The feedback probes were located at the position of maximum instability amplitude so that the feedback<br>gain required for suppression was minimized.<sup>1,9</sup> mum instability amplitude so that the feedback gain required for suppression was minimized.

Attempts at stabilization when two or more modes are present, using amplifier(s) with bandwidth(s) sufficient to overlap the frequencies of the several modes, have tended to be unsuccessful. We and others<sup>1,2</sup> have found that as one mode is suppressed, one or more of the other modes are usually enhanced, as illustrated in Fig. 2(a); with three modes  $(m = 1, 2, 3)$  present, the feedback phase was varied through 360° for the dominant mode  $(m = 2, f = 15$  kHz), with the



Phase shift applied to 25kHz signal (m=5mode)

## $(h)$

FIG. 2. (a)  $m=1,2,3$  azimuthal modes are alternately suppressed and built up (to 7 times natural amplitude for  $m=1$  mode) when broadband feedback is applied.  $B_{\text{midplane}} = 0.67 \text{ kG}, n = 1 \times 10^8 \text{ cm}^{-3}$ , and cathode temperature =  $2100^{\circ}$ K. The feedback loop gain was about 2000, which gave a feedback probe voltage of 2 V prior to suppression. (b) Comparison of suppression of  $m=3$  mode and enhancement of  $m=2$  mode when broadband feedback was used (dashed lines) and when  $m = 3$ passband feedback was used (solid lines). Broadband feedback greatly enhanced the  $m = 2$  mode.

alternate suppression and enhancement of each mode. In such experiments usually there is little decrease in the total wave energy, so that overall stabilization is not achieved. By limiting the bandwidth of the signal to be amplified in the feedback loop, using the filters shown in Fig. 1, so that each loop tends to operate on only one mode, we have been able to achieve suppression without enhancement.

When only one feedback system was used, representative results are as shown in Fig. 2(b), with and without filtering. Without filtering, the  $m = 3$  mode can be reduced to a few percent of its natural (no feedback) level where, however, the  $m = 2$  mode is increased by a factor of more than 3 so that the total wave energy is up by about a factor of 10. With filtering, it is seen that the  $m = 3$  mode can be reduced to about 30% of its



FIG. 3. (a) Spectrum of natural signal. (b) Amplitude characteristics of three filters tuned to pass the  $m=2$ , 3, 4 mode frequencies. (c)  $m=3$  feedback system operated alone suppressed the  $m=3$  without significantly enhancing any of the other modes. (d)  $m = 4$  feedback system operated alone suppressed the  $m = 4$  mode. (e)  $m=3$  and 4 systems operated simultaneously. (f)  $m=2, 3, 4$  systems operated simultaneously suppressed all the low-frequency drift modes to within a few times the surrounding noise level.

natural level, with only slight enhancement of the  $m = 2$  mode (about 10%), with total wave energy down about  $30\%$ . In other tests, suppression with no enhancement has been obtained.

When three feedback systems are used, representative results are as given in Fig. 3. The natural mode spectrum is shown in (a), dominated by modes  $3$  (22 kHz) and  $4$  (27 kHz). The three filters were tuned as shown in (b) to peak at the  $m = 2, 3, 4$  mode frequencies; the filter  $Q'$ s were

made to be roughly 7, such that the voltage loop gains at neighboring modes were down by at least 5. Using large amplifier gain only in the  $m = 3$  loop, this mode was suppressed, as in (c): with large gain only in the  $m = 4$  loop, the result is similar, as in (d). Little or no enhancement of other modes is observed. With large gains in the  $m = 3$  and  $m = 4$  loops, these modes are both suppressed, as in (e), with some enhancement of the  $m = 2$ , 5 modes. By adding large gain to the  $m = 2$  loop, this mode was reduced from its value in  $(e)$ , as shown in  $(f)$ . This sequence shows that the use of independent narrowband feedback loops has resulted in reducing one to three drift modes to within a few times the local noise value, with little enhancement of the unstabilized modes.

Immediate application of these circuits might be to stabilize a variety of laboratory plasmas, possibly using more compact and sophisticated circuitry. More importantly, because the parameters in this experiment [e.g.,  $\omega_{0}i^2/\omega_{ci}^2 \gg 1$ ,  $a_i(1/n)(dn/dr)$  not small, non-Maxwellian  $f_i(v)$ , collisionless ions, etc.] scale to much larger density, higher temperature plasmas, encouragement is given to the solution of the problem of low-frequency  $(\omega \leq \omega_c)$  multiple mode stabilization in fusion plasmas by means other than utilizing special magnetic field shapes.

The authors wish to thank Dr. H. W. Hendel for the original encouragement in these feedback experiments and M. J. Bales for designing and building the necessary electronics.

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<sup>\*</sup>Work supported by the U. S. Atomic Energy Commission, Contract No. AT(043)-34, P. A. 128.



FIG. 3. (a) Spectrum of natural signal. (b) Amplitude characteristics of three filters tuned to pass the  $m=2$ , 3, 4 mode frequencies. (c)  $m=3$  feedback system operated alone suppressed the  $m=3$  without significantly enhancing any of the other modes. (d)  $m = 4$  feedback system operated alone suppressed the  $m = 4$  mode. (e)  $m=3$  and 4 systems operated simultaneously. (f)  $m=2, 3, 4$  systems operated simultaneously suppressed all the low-frequency drift modes to within a few times the surrounding noise level.