rant direction, and negative for 2nd and 4th, the average value of $\lambda(x,y)$ is 0 [number of Feynman diagrams with $\overline{\lambda}=0 \gg$ number of Feynman diagrams with $\overline{\lambda}\neq 0$]. Similarly $\overline{\mu}=\overline{\nu}$. Therefore, the product of propagators in a given section is approximated in the limit of $q\to\infty$ [$d\to 0$] by const $\times \exp[\overline{\mu}\iint_d dxdy\ \mathcal{L}(\varphi)]$.

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NEW CONSTRAINTS IN HIGH-ENERGY ELECTRON-POSITRON ANNIHILATION INTO HADRONS*

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(Received 27 February 1970)

We consider relations between the total cross section $\sigma_T(e^+ + e^- \to$ hadrons) and the differential cross section for $e^+ + e^- \to H +$ anything, where H is a hadron. We obtain new restrictions on inelastic form factors in the timelike region. One of our results is to show that field algebra is incompatible with scale invariance *à la* Bjorken and present experimental data.

It has been known[1] for a long time that high-energy hadron production in electron-positron collisions, in the single-photon exchange approximation, contains direct information about the constitution of the hadronic current in the region of timelike momentum transfers. The asymptotic behavior of the total cross section $\sigma_T(e^+ + e^- \to$ hadrons) allows us to distinguish[2] among the different kinds of current algebra and to know whether or not there is a finite hadronic contribution to the electric change, etc.

A second method[3,4] of studying the structure of the electromagnetic current has also been discussed: The differential cross section $d\sigma(e^+ + e^- \to H +$ anything) with respect to the energy of the hadron H can probe the electromagnetic current for the timelike momentum transfer if scale invariance *à la* Bjorken[5] is valid.

In this paper we show how the properties of the electromagnetic current can be explored in greater detail by examining the relationship between $\sigma_T(e^+ + e^- \to$ hadrons) and the processes $e^+ + e^- \to H +$ anything. New restrictions are deduced which the different versions of current algebra must fulfill. One of our results is to show that the field algebra, introduced by Knoll, Lee, Weinberg, and Zumino[6] is inconsistent with scale invariance *à la* Bjorken[5] and present experimental data.[7] This result, obtained in the timelike region, can also be obtained, under a weaker form, in the spacelike region using a new sum rule recently given by Jackiw, Van Royen, and West.[8] Here our discussion will be general, and we will consider Bjorken asymptotics[3-5] only as a particular case.

Let us consider the kinematics first. The second-rank tensor

$$P_{\mu\nu} = N\sum_n (2\pi)^3\delta^{(4)}(q-P-P_n)\langle 0|j_\mu(0)|n, H(P)\text{out}\rangle\langle \text{out}H(P), n|j_\nu(0)|0\rangle$$

$$\equiv \overline{W}_1^H(q^2,\nu)\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)+\frac{\overline{W}_2^H(q^2,\nu)}{m_H^2}\left(P_\mu-\frac{\nu}{q^2}q_\mu\right)\left(P_\nu-\frac{\nu}{q^2}q_\nu\right), \tag{1}$$

where j_μ is the electromagnetic current, P and q are the momenta of the hadron H and the virtual photon, respectively, $\nu = P\cdot q$, and $N=2P_0$ for the boson H ($N=P_0/m_H$ for the fermion H), is directly related[3,4] to the differential cross section $d\sigma(e^+ + e^- \to H +$ anything). Throughout we imply a spin sum if H has a spin. On the other hand, the spectral function of the photon propagator $\Pi(q^2)$ is defined by[9]

$$\Pi_{\mu\nu} = \sum_z (2\pi)^3\delta^{(4)}(q-P_z)\langle 0|j_\mu(0)|z\rangle\langle z|j_\nu(0)|0\rangle = (-g_{\mu\nu}q^2+q_\mu q_\nu)\Pi(q^2). \tag{2}$$

Let the maximum multiplicity of the particle H in the state z, for a given mass squared q^2, be $n_H(q^2)$; then energy conservation requires

$$q_0 \geq m_H n_H(q^2). \tag{3}$$

Considering the positive definiteness of every contribution of intermediate states, we can obtain the inequalities[10] in the center-of-mass system (where $\vec{q}=0$, $\nu=P_0q_0$, and $P^0{}_0=0$)

$$\Pi(q^2) \geq -\frac{1}{3q^2 n_H(q^2)}\int \frac{d^3P}{2P_0}P^\mu{}_\mu \geq -\frac{2\pi}{3q^2 n_H(q^2)}\int_a^b d\cos\theta \int \frac{|\vec{P}|P_0 dP_0}{2P_0}P^3{}_3, \tag{4}$$

where θ is the angle between the z axis and $\vec{P}$; a and b are arbitrary numbers which satisfy the condition $1 \geq b > a \geq -1$. After performing the integration over the angle θ, we express the inequalities (4) in terms of the inelastic form factors of H, $\overline{W}_1{}^H(q^2,\nu)$ and $\overline{W}_2{}^H(q^2,\nu)$, as follows:

$$\Pi(q^2) \geq \frac{2\pi}{q^2 n_H(q^2)}\int_{m_H(q^2)^{1/2}}^{q^2/2}\frac{d\nu}{(q^2)^{1/2}}\left(\frac{\nu^2}{q^2}-m_H{}^2\right)^{1/2}\left[x\overline{W}_1{}^H(q^2,\nu)+\frac{y}{3}\left(\frac{\nu^2}{q^2 m_H{}^2}-1\right)\overline{W}_2{}^H(q^2,\nu)\right], \tag{5}$$

where $x=(\cos b-\cos a)/6$ and $y=(\cos^3 b-\cos^3 a)/6$ are arbitrary numbers such that $\frac{1}{3}\geq x,y>0$. Notice that to derive (5), we have only considered the case where H is $\pi^\pm, K^\pm, K^0, \overline{K}^0, p, n, \Sigma^\pm, \Xi^\pm$ such that $m_H \leq P_0 \leq q_0/2$. The factor $1/n_H(q^2)$ appears in (4) and (5) because the summation in (2) can be decomposed in the following way:

$$\sum_z |z\rangle\langle z| = \sum_{l=0}^{n_H(q^2)} \sum_{\substack{\text{anything}\\ \text{but } H}} |lH, \text{anything but } H\rangle\langle lH, \text{anything but } H|.$$

In this complete summation, when $l \geq 2$, a statistical factor $(l!)^{-1}$ takes into account the fact that we have l identical particles. In contrast, we have, in (1),

$$\sum_n |n, H(P)\rangle\langle H(P), n| = \sum_{l=1}^{n_H(q^2)} \sum_{\substack{\text{anything}\\ \text{but } H}} |H(P), (l-1)H, \text{anything but } H\rangle\langle H(P), (l-1)H, \text{anything but } H|.$$

Therefore in (1), for each term, with lH particles, only a statistical factor $[(l-1)!]^{-1}$ appears. Defining $\omega = q^2/P\cdot q$, we can rewrite (5) as

$$\Pi(q^2) \geq \frac{2\pi}{n_H(q^2)}\int_2^{(q^2)^{1/2}/m_H}\frac{d\omega}{\omega^4}\left(1-\frac{m_H{}^2\omega^2}{q^2}\right)^{1/2}\left[x\omega\overline{W}_1{}^H\left(q^2,\frac{q^2}{\omega}\right)+\frac{y}{3}\left(1-\frac{m_H{}^2\omega^2}{q^2}\right)\frac{\nu}{m_H{}^2}\overline{W}_2{}^H\left(q^2,\frac{q^2}{\omega}\right)\right]. \tag{6}$$

This is the main relation from which we will derive many consequences, using the trivial kinematical constraint (3) and the asymptotic property of $\Pi(q^2)$ in different models.

The spectral function $\Pi(q^2)$, defined by Eq. (2), is related to the total cross section by

$$\sigma_T(e^+ + e^- \to \text{hadrons}) = \pi(4\pi\alpha)^2\Pi(q^2)/q^2. \tag{7}$$

Let us suppose the asymptotic behavior of the total cross section to be

$$\sigma_T(q^2) \to O((q^2)^{-m}). \tag{8}$$

Various models have predicted different values[2] of the parameter m: (1) $m=1$ for quark model[1,11]; (2) $m>1$ for "compound" field algebra (cfa) (finite contribution to the electric charge); (3) $m>2$ for "divergent" field algebra[6] (dfa) (finite Schwinger terms); (4) $m>3$ for "finite" field algebra (ffa) (free-field behavior for the current as $q^2\to\infty$). Now that we know the asymptotic property of the left-hand side in (5), we can discuss the right-hand side in the Bjorken limit,[5] i.e., $q^2\to\infty$, ω fixed.

(A) If the limit exists,[3,4] namely

$$\overline{W}_1{}^H\left(q^2,\frac{q^2}{\omega}\right) \to \overline{F}_1{}^H(\omega) \text{ finite}, \quad \frac{\nu}{m_H{}^2}\overline{W}_2{}^H\left(q^2,\frac{q^2}{\omega}\right) \to \overline{F}_2{}^H(\omega) \text{ finite}, \tag{9}$$

then the inequalities (6) become, when $q^2\to\infty$,

$$O\left(\frac{1}{(q^2)^{m-1}}\right) \geq \frac{2\pi}{n_H(q^2)}\int_2^{\beta(q^2)^{1/2}/m_H}\frac{d\omega}{\omega^4}\left(1-\frac{m_H{}^2\omega^2}{q^2}\right)^{1/2}\left[x\omega\overline{F}_1{}^H(\omega)+\frac{y}{3}\left(1-\frac{m_H{}^2\omega^2}{q^2}\right)\overline{F}_2{}^H(\omega)\right], \tag{10}$$

with

$$x\omega\overline{F}^H(\omega)+\frac{y}{3}\left(1-\frac{m_H{}^2\omega^2}{q^2}\right)\overline{F}_2{}^H(\omega) \geq 0. \tag{11}$$

To avoid a kinematical zero in the integrand at the upper bound, $\omega=(q^2)^{1/2}/m_H$, we have for convenience taken in (1) an arbitrary new upper bound $\beta(q^2)^{1/2}/m_H$ $(0<\beta<1)$ without violating the inequality. This will be useful in case (b) (see below).

(a) If the integral in (10) exists when $q^2\to\infty$, then

$$O((q^2)^{-(m-1)}) \geqslant \text{const}/n_H(q^2) \text{ or } n_H(q^2) \geqslant O((q^2)^{m-1}), \tag{12}$$

unless

$$x\omega\bar{F}_1{}^H(\omega)+(y'/3)\bar{F}_2{}^H(\omega)\equiv 0. \tag{13}$$

But, because x and y' $[\equiv y(1-m_H{}^2\omega^2/q^2)]$ are arbitrary numbers, (13) implies $\bar{F}^H(\omega)=\bar{F}_2{}^H(\omega)\equiv 0$. Apart from this special case, the restriction (3) then implies a constraint on the parameter m:

$$(q^2)^{1/2}/m_H \geqslant n_H(q^2) \geqslant O((q^2)^{m-1}) \text{ or } m\leqslant\tfrac{3}{2}. \tag{14}$$

Quark algebra and cfa are compatible with this constraint but dfa and ffa violate it.

(b) If the integral in (10) does not exist, then stronger conditions can be obtained by similar considerations. If the functions $\bar{F}_i{}^H$ are no more divergent than $\omega\bar{F}_1{}^H(\omega)$, $\bar{F}_2{}^H(\omega)\sim\omega^{r+3}$ $(r\geqslant 0)$, then we have:

(1) $n_H(q^2)\geqslant O((q^2)^{r/2}) \Rightarrow r\leqslant 1$ for quark model.

(2) $n_H(q^2)> O((q^2)^{r/2}) \Rightarrow r< 1$ for cfa.

(3) $n_H(q^2)> O((q^2)^{r/2}q^2) \Rightarrow$ contradiction for dfa.

(4) $n_H(q^2)> O((q^2)^{r/2}q^4) \Rightarrow$ contradiction for ffa.

Taking the results of (a) and (b) together, we see that scale invariance *à la* Bjorken[5] is incompatible with the divergent and finite field algebras unless $\bar{F}_1{}^H(\omega)=\bar{F}_2{}^H(\omega)=0$. Since we know that $\bar{F}_I{}^P(\omega)$ can be defined from an analytical continuation[3,4] of the high-energy electroproduction structure functions $F_I(\omega)$, and since the experiments[7] show that at least $F_2(\omega)\neq 0$, we can also exclude the case where $\bar{F}_1{}^H(\omega)=\bar{F}_2{}^H(\omega)=0$.

(B) If the Bjorken limit[5] does not exist and if the form factors diverge at most like $\omega\bar{W}_1{}^H$, $\nu\bar{W}_2{}^H/m_H{}^2$ $\sim(q^2)^S\omega^{r+3}$, then

$$O((q^2)^{-(m-1)})\geqslant[1/n_H(q^2)]O((q^2)^{S+(r/2)\theta(r)}), \tag{15}$$

where $\theta(r)=1$ $(r>0)$ or 0 $(r\leqslant 0)$. Therefore,

(1) $n_H(q^2)\geqslant O((q^2)^{S+(r/2)\theta(r)})\Rightarrow S+\frac{1}{2}r\theta(r)\leqslant\frac{1}{2}$ for quark model;

(2) $n_H(q^2)> O((q^2)^{S+(r/2)\theta(r)})\Rightarrow S+\frac{1}{2}r\theta(r)<\frac{1}{2}$ for cfa;

(3) $n_H(q^2)> O((q^2)^{S+(r/2)\theta(r)}q^2)\Rightarrow S+\frac{1}{2}r\theta(r)<-\frac{1}{2}$ for dfa;

(4) $n_H(q^2)> O((q^2)^{S+(r/2)\theta(r)}q^4)\Rightarrow S+\frac{1}{2}r\theta(r)<-\frac{3}{2}$ for ffa.

For dfa and ffa we have $S<-\frac{1}{2}$ and $S<-\frac{3}{2}$, respectively. Therefore dfa and cfa are consistent with the above inequalities only if $\bar{W}_1{}^H(q^2,\nu)$ and $\nu\bar{W}_2{}^H(q^2,\nu)/m_H{}^2$ are identically zero in the Bjorken limit[5] [this case, i.e., $\bar{F}_1{}^H(\omega)=\bar{F}_2{}^H(\omega)=0$, is discussed in (A) above].

It is interesting to note that one can obtain the same conclusion from the following considerations in the spacelike region. Recently Jackiw, Van Royen, and West[8] have derived the sum rule

$$\frac{P_0}{m}\int d^3x\, e^{-i\vec{q}\vec{x}}\langle P|[j_0(0,\vec{x}),j_i(0)]|P\rangle=2q_i\int_{-2}^{0}\frac{d\omega}{\omega^2}[\omega F_1(\omega)+F_2(\omega)], \tag{16}$$

with

$$\omega F_1(\omega)+F_2(\omega)\geqslant 0 \quad (-2\leqslant\omega\leqslant 0). \tag{17}$$

The left-hand side of this equation is zero in any model which has no q-number Schwinger terms. For any such model, if this sum rule is valid [namely if the Bjorken limit exists and $\omega F_1(\omega)+F_2(\omega)$ $\to O(\omega^{l+\epsilon})$ as $\omega\to 0$],[8] then $\omega F_1(\omega)+F_2(\omega)=0$. On the other hand, Callan and Gross[12] have shown that in the case of field algebra $F_1(\omega)=0$. Therefore, combining the results of Refs. 8 and 12, we have $F_1(\omega)$ $=F_2(\omega)=0$. Notice that our argument in the timelike region is more general, because it allows for a possible breaking of scale invariance and for the possibility that the integral in the right-hand side of Eq. (10) may diverge.

In conclusion, our analysis shows that for all models which predict that $\sigma_T(q^2) \sim O(1/(q^2)^m)$, $m > \frac{3}{2}$ (e.g., algebra of fields), we must have

$$\lim_{\substack{q^2 \to \infty \\ \omega \text{ fixed}}} \overline{W}_1^H(q^2, \nu) = \lim_{\substack{q^2 \to \infty \\ \omega \text{ fixed}}} \frac{\nu}{m_H^2} \overline{W}_2^H(q^2, \nu) = 0. \tag{18}$$

Furthermore, if $\overline{F}_i^p(\omega)$ can be defined from an analytical continuation[3,4] of the high-energy electroproduction structure functions $F_i(\omega)$, then any model with $m > \frac{3}{2}$ is inconsistent with the experimental data.[7]

The authors would like to thank Professor S. Brodsky, Dr. M. Chanowitz, Professor T. Kinoshita, and Dr. P. Roy for stimulating discussions.

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*Work supported in part by the Office of Naval Research.

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