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TWO ASYMPTOTIC SUM RULES FOR ELECTROPRODUCTION AND PHOTOPRODUCTION*

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We give a sum rule relating photoproduction to asymptotic electroproduction, and another electroproduction sum rule which tests for the presence of operator Schwinger terms.

In this Letter we report two sum rules, which relate integrals over the scale functions[1] of electroproduction to each other, and to integrals over total photoproduction cross sections. The sum rules are based on Bjorken's idea of scale invariance,[1] the experimental fact[2] that the electroproduction structure functions decrease in the momentum-transfer variable q^2 (for sufficiently large q^2) if the mass of the produced hadronic states is held fixed, and on the assumption that the high-energy form of the imaginary part of the forward Compton amplitude for fixed q^2 has no term characteristic of a Regge pole with $\alpha(0)=0$. Under these circumstances, the sum rule given in Eq. (8a) is valid; and if we assume in addition that there is no Schwinger term in the connected, covariant forward Compton amplitude, the sum rule in Eq. (8b) is also satisfied.

Let $J_\mu(x)$ be the electromagnetic current. The spin-averaged forward Compton amplitude can be written

$$T_{\mu\nu}=i(2p_0)\int d^4x e^{iqx}\langle p|[J_\mu(x)J_\nu(0)]_+|p\rangle=\left(p_\mu-\frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu-\frac{p\cdot q}{q^2}q_\nu\right)T_2+\left(\frac{q_\mu q_\nu}{q^2}-g_{\mu\nu}\right)T_1. \tag{1}$$

The imaginary parts of the T_i are related to the structure functions[1,2] W_i of electroproduction by

$$\operatorname{Im}T_i=2\pi W_i. \tag{2}$$

That $T_{\mu\nu}$ is free of unwanted singularities is assured by imposing conditions on the T_i; for example, T_2 vanishes like q^2 at $q^2=0$. W_1 and W_2 are related to the cross sections for scattering of transverse and longitudinally polarized photons, σ_T and σ_L, by

$$W_1=(4\pi^2\alpha)^{-1}(\nu+q^2/2)\sigma_T, \tag{3a}$$

$$W_2=-(4\pi^2\alpha)^{-1}(\nu+q^2/2)(\nu^2-M^2q^2)^{-1}q^2(\sigma_T+\sigma_L). \tag{3b}$$

Here M is the proton mass, q the virtual photon momentum, $\nu=q\cdot p$, and $\alpha=1/137$. Since σ_L vanishes at $q^2=0$,

$$W_1(q^2,\nu)\xrightarrow[q^2\to 0]{}-\nu^2q^{-2}W_2(q^2,\nu)\to(4\pi^2\alpha)^{-1}\nu\sigma(\nu), \tag{4}$$

where $\sigma(\nu)$ is the total cross section for real photoproduction from a proton. Bjorken has suggested,[1] and experiments apparently verify,[2] that W_1 and νW_2 approach finite functions of the variable $\omega=-q^2/2\nu$ in the limit of large ν and q^2, that is[1]

$$W_1\to F_1(\omega), \tag{5a}$$

$$\nu W_2\to F_2(\omega). \tag{5b}$$

The photoproduction cross section $\sigma(\nu)$ appears[3] to have the Regge asymptotic form for large ν. Let us define, therefore, a truncated cross section $\tilde{\sigma}(\nu)$ by

$$\tilde{\sigma}(\nu)=\theta(\nu-\nu_0)\sigma(\nu)-\sum_{\alpha>0}C_\alpha\nu^{\alpha-1};\qquad \nu_0=Mm_\pi+(m_\pi{}^2/2), \tag{6}$$

where the α are the $t=0$ intercepts of the leading Regge trajectories (e.g., P, P', A_2). If no trajectory crosses $\alpha=0$ at $t=0$, then $\tilde{\sigma}(\nu) < O(\nu^{-1-\epsilon})$ for large ν. Similarly, the behavior of the scale functions F_i in (5) near $\omega=0$ should be governed by the same trajectories as in (6). So we also define

$$\tilde{F}_1(\omega)=\theta(1-\omega)F_1(\omega)-\sum_{\alpha>0} f_{1\alpha}\omega^{-\alpha}, \tag{7a}$$

$$\tilde{F}_2(\omega)=\theta(1-\omega)F_2(\omega)-\sum_{\alpha>0} f_{2\alpha}\omega^{1-\alpha}, \tag{7b}$$

where $\tilde{F}_1(\omega)$ and $\omega^{-1}\tilde{F}_2(\omega)$ vanish faster than ω^ϵ as ω approaches zero.

In terms of the quantities defined above, the two sum rules are

$$1+(2\pi^2\alpha)^{-1}\int_0^\infty \tilde{\sigma}(\nu)d\nu=\int_0^\infty d\omega\omega^{-2}\tilde{F}_2(\omega), \tag{8a}$$

$$\int_0^\infty d\omega\omega^{-2}\tilde{F}_2(\omega)=2\int_0^\infty d\omega\omega^{-1}\tilde{F}_1(\omega). \tag{8b}$$

It is understood that for $0<\nu<\nu_0$, and for $\omega>1$, $\tilde{\sigma}$ and the $\tilde{F}_i$ are given entirely by the asymptotic Regge forms in (6) and (7).

The left-hand side of (8a) is equal to the residue of a fixed pole at $\alpha=0$ in the real part of the forward on-shell Compton amplitude. Damashek and Gilman[4] have analyzed the photoproduction data and determined this residue to lie somewhere between 0.12 and 1.59, with perhaps a preferred value of about 0.8. Unfortunately, the $\tilde{F}_i$ in (8) are not yet well determined. The available data[2] on W_2 in the region of small ω ($\omega \lesssim 0.05$) also have small $|q^2|$ ($\lesssim 1$ GeV^2), and it is doubtful that the scale limit has been attained. We hope that when the large-angle data are analyzed, these sum rules can be checked.

If the basic constituents of the proton all have spin $\frac{1}{2}$, it is plausible that σ_L/σ_T should vanish in the scale limit,[5] and that there should be no operator Schwinger terms. Conversely, there is one interesting experimental circumstance under which the sum rule (8b) can be checked. If it is found that $\sigma_L/\sigma_T \sim F_2(\omega)-2\omega F_1(\omega)\to 0$ in the scale limit, then (8b) is automatically satisfied, and there are no operator Schwinger terms. An early experimental test of the vanishing of σ_L/σ_T is quite feasible.

We sketch the derivation of these sum rules, postponing details to a more complete article. The amplitude T_2 has a Deser, Gilbert, and Sudarshan (DGS) representation[6,7] of the type

$$T_2=-q^2\iint\frac{d\sigma d\beta h(\sigma,\beta)}{q^2+2\beta\nu-\sigma+i\epsilon}$$
$$[0\le\sigma(\beta)\le\sigma\le\infty;\ -1\le\beta\le1], \tag{9}$$

where $h(\sigma,\beta)$ is even in β. Scale invariance requires[8]

$$\int d\sigma h(\sigma,\beta)=0 \tag{10}$$

and

$$F_2(\omega)=-\tfrac{1}{4}\omega\int d\sigma\,\sigma\partial h(\sigma,\omega)/\partial\omega, \tag{11}$$

as can be seen by taking the imaginary part of (9) and using (2) and (5b). We write a subtracted dispersion relation for T_2 in q^2 for fixed ν:

$$T_2(q^2,\nu)=\frac{2q^2}{\nu}\int_{-2\nu}^\infty\frac{dq'^2}{q'^2(q'^2-q^2)}\nu W_2(q'^2,\nu). \tag{12}$$

Eventually, we are interested in the region of large ν at $q^2=0$. For large ν, scale invariance allows us to change the upper limit in (12) to 2ν, as follows: For $q'^2>2\nu$, νW_2 is found to be

$$\nu W_2(q'^2,\nu)=\frac{q'^2}{4}\int_{-2\nu+q'^2}^{2\nu+q'^2}d\sigma h\left(\sigma,\frac{\sigma-q'^2}{2\nu}\right). \tag{13}$$

The fact that both the upper and lower limit of this integral increase linearly with ν (for fixed ω'), together with (10) and (11) imply that the error accrued in making this replacement vanishes as $\nu\to\infty$.[9]

The spin-averaged forward Compton amplitude for real photons can be written in two ways, in terms of T_2, or in terms of the usual dispersion relations in ν:

$$T_{\gamma p}(\nu)=-\frac{\nu^2}{q^2}T_2(\nu,q^2)|_{q^2=0}, \tag{14}$$

$$=-2+\frac{\nu^2}{\pi^2\alpha}\int\frac{d\nu'\sigma(\nu')}{\nu'^2-\nu^2}. \tag{15}$$

For the sake of brevity, make the artificial assumption that there are no Regge trajectories with intercepts above zero, so $\sigma=\tilde{\sigma}$, and $F_i=\tilde{F}_i$. Insert (12) in (14), and go to the large ν limit[10]:

$$T_{\gamma p}(\nu)\xrightarrow[\nu\to\infty]{}\int_{-2\nu}^{2\nu}\frac{dq'^2}{q'^2}\left(\frac{-2\nu}{q'^2}\right)\nu W_2(q'^2,\nu)$$
$$=-2\int_0^1 d\omega\omega^{-2}F_2(\omega). \tag{16}$$

In the second equality, we have used (5b) and the fact that $F_2(\omega)$ is even in ω. On the other hand, the large ν limit of (15) is just

$$-2-\frac{1}{\pi^2\alpha}\int d\nu'\sigma(\nu'). \tag{17}$$

Equating these two expressions leads to the sum rule in (8a) under the artificial conditions stated. It is straightforward, however, to remove this restriction and allow terms in the asymptotic form of $\sigma(\nu)$ which behave like $\nu^{\alpha-1}$ with $\alpha>0$.

The presence of these terms implies that $h(\sigma, \beta)$ has parts that go like $h_\alpha(\sigma)\beta^{1-\alpha}$ for small β; these contributions to νW_2 as given by (13) must be integrated over q'^2 before the limit $\nu \to \infty$ is performed in Eq. (12). The result of this procedure is the sum rule for the truncated cross sections given in Eq. (8a).

The sum rule in (8b) follows from a similar argument, except that we write an unsubtracted dispersion relation in q^2 for $T_1 - \nu^2 q^{-2} T_2$ instead of the relation given in Eq. (12). The assumption that no unwanted subtraction constant is necessary is equivalent to the assumption that there is no Schwinger term in the connected part of the matrix element. Note that, from Eq. (10), T_2 obeys an unsubtracted dispersion relation in q^2. This is equivalent, from earlier work,[7] to the statement that there is no Schwinger term associated with T_2 in the Compton amplitude. However, the sum rule (8a) is not affected if it should turn out that T_2 needs a subtraction.

The derivation given here can be generalized to apply when the Compton amplitudes are expressed in the general form of the DGS representation,[7] rather than in the simple form typified by Eq. (9). The details of this derivation, as well as other applications of those ideas, will be submitted in a more lengthy article.

Note added in proof. – Professor Roman Jackiw has informed us that he, R. Van Royen, and G. B. West have discussed a sum rule similar to our Eq. (8b); their derivation is valid only if $F_2 - 2\omega F_1$ has no Regge poles with $\alpha > 0$.[11]

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[10]While the first integral in (16) contains contributions from small timelike q'^2, the second is expressed entirely in terms of spacelike data. It is only in the limit $\nu \to \infty$ that is possible. By our artificial hypothesis, νW_2 vanishes (for fixed q'^2) faster that ν^{-1} as $\nu \to \infty$, so that the only way that the first integral in (16) can remain finite in this limit is that it is dominated by terms with $|q'^2| \sim \nu$ — i.e., the scaling region. It is an important consequence of the DGS representation that $F_2(\omega)$ is symmetric in ω, so that the timelike parts are expressible in terms of data with spacelike q'^2.

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MULTIPARTICLE COLLISIONS AT HIGH ENERGIES*

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The hadronic cascade of high-energy particles in nuclei is discussed. As the incident energy of the bombarding particles is increased, a larger fraction of the products of the first collision strike the second nucleon. In a few successive collisions of relatively small momentum transfers in which the bombarding mass is increased, it is possible to double the energy available in the center-of-mass system. This can conceivably be a very useful analytical tool at high energies.

In the last ten years the subject of multiple scattering of high-energy particles in nuclei has received a considerable amount of theoretical and experimental study.[1,2] The interest has mainly centered on elastic and coherent processes. In this note we want to consider the possibilities of the study of inelastic processes arising from multiple interactions in a complex nucleus. We are not particularly interested in the usual sort of nucleonic cascade which flares out laterally as it proceeds through the nucleus. As the energy of the incident particle is increased the particles associated with the upper vertex, i.e., those particles going forward in the center of