ELECTRIC HEXADECAPOLE TRANSITION MOMENT IN ^{152}Sm †

F. S. Stephens, R. M. Diamond, and N. K. Glendenning
Lawrence Radiation Laboratory, University of California, Berkeley, California 94720

and

J. de Boer
Rutgers University, New Brunswick, New Jersey 08903, and Universität München, München, Germany
(Received 30 March 1970)

The Coulomb excitation of the 4^+ rotational state of a deformed nucleus can proceed by both double $E2$ and single $E4$ transitions. The excitation cross section also contains a term corresponding to the interference between the two. The Coulomb excitation of ^{152}Sm with ^{4}He projectiles has been carefully measured and the results were analyzed with a computer program including $E4$ terms. Best fits were obtained for $\langle 0^+ \| \mathfrak{M}(E4) \| 4^+ \rangle = (+0.35 \pm 0.11)e$ b^2.

Coulomb excitation can be one of the most reliable methods for determining electric multipole matrix elements of nuclei. However, one must be careful to include all processes having effects comparable with the one being measured, and the analysis may become very complicated or even ambiguous. The present study of an $E4$ transition moment in ^{152}Sm began as the evaluation of a correction to measurements[1] of $B(E2; 2^+ \to 4^+)$. It became apparent that this particular correction was not very well known and could be rather large, especially when light projectiles were used so that double $E2$ excitation is weak. The accurate determination of the $B(E2; 2^+ \to 4^+)$ value from lifetime measurements[2] made it possible to combine that result with the Coulomb excitation measurements and determine the $E4$ moment.

The experiment consisted of an accurate determination of the intensity of the $4^+ \to 2^+$ gamma-ray transition in ^{152}Sm relative to those of the $2^+ \to 0^+$ transitions in ^{152}Sm and ^{150}Sm following Coulomb excitation with ^{4}He ions. Targets of both natural samarium and enriched ^{152}Sm were measured at each bombarding energy. This method provides accurately known standard peaks (122 and 334 keV) for comparison at both higher and lower energies than the peak of interest (245 keV). Gamma-ray spectra were simultaneously stored as singles events and as coincidences with ^{4}He ions backscattered through an angle of about 160 deg. These two types of measurement are about equally sensitive to the effect of an $E4$ transition moment, but differ markedly in their sensitivity to many other effects. Thus the agreement of the singles and backscatter results greatly reduces the probability that an important effect has been overlooked.

An overall view of the possibilities for measuring $E4$ transition moments using this method is contained in Fig. 1. We have used the following notation:

$$\langle 0^+ \| \mathfrak{M}(E4) \| 4^+ \rangle = \int \rho r^4 Y_{40} d^3 r = [B(E4; 0^+ \to 4^+)]^{1/2},$$

where ρ is the nuclear charge density. The effect of an $E4$ moment on the cross section for populating the 4^+ level of ^{152}Sm in coincidence with backscattered ^{4}He ions ($d\sigma$) is shown, normalized to the cross section with no $E4$ moment ($d\sigma_0$). This behavior changes very little with projectile scattering angle, so that the corresponding curve for the singles measurements differs by only a few percent. The general shape of this curve is caused by the dominance of the direct $E4$ transition, which depends quadratically on the moment. The weaker interference term (linear) causes the asymmetry about zero. Also shown in Fig. 1 is the relationship of the $E4$ moment to the deformation parameter, β_4, which

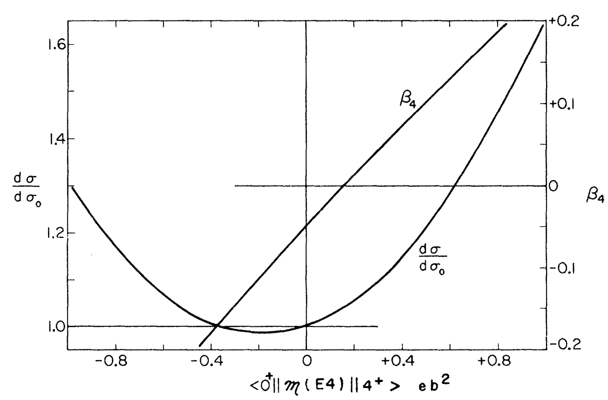

FIG. 1. Relationship between the $E4$ moment and (a) the normalized cross section (backscatter) for populating the 4^+ state of ^{152}Sm with 10.4-MeV ^{4}He ions, and (b) the deformation parameter, β_4, using a radius of $R_0 = 1.2A^{1/3}$ F (see text).

will be defined below. This curve has been constructed by adjusting β_2, for each value of β_4, so that the measured $B(E2; 0^+ \to 2^+)$ value in ^{152}Sm is reproduced. The asymmetry of this curve relative to zero is caused by the positive second-order contribution to the $E4$ moment from β_2. The asymmetries in these two curves make it unlikely that one can measure negative $E4$ moments by this technique, since reasonable values[3] of β_4 (≥ -0.2) do not give rise to sufficiently large negative $E4$ moments to cause measurable deviations in the cross section. This situation renders improbable one of the two possibilities for the moment that would otherwise result from a given cross-section measurement. Small positive values of β_4, however, should produce readily measurable effects in the cross section.

We have bombarded thin (≈ 2 mg/cm^2) self-supporting metallic targets of Sm with 10- to 14-MeV ^{4}He beams from the Lawrence Radiation Laboratory heavy ion linear accelerator. This target thickness ensures that less than 2% of the recoiling nuclei escaped from the target. The beam energy was determined by comparison with a ^{212}Po alpha source.[4] Gamma-ray spectra were measured with a Ge(Li) detector whose relative counting efficiency was determined to an estimated accuracy of 2% using a ^{177m}Lu source.[5] The total conversion coefficients for the transitions were obtained from the tables of Hager and Selzer[6] and should result in uncertainties no greater than 1% in $1+\alpha_T$. The spectra were recorded at a gamma-ray angle of 55 deg relative to the beam direction. The singles measurements were not very sensitive to this angle, but the backscatter coincidence data were. In the latter case we measured the intensity of the 122-keV transition at 45 and 90 deg relative to the beam direction, and obtained an angular-distribution attenuation coefficient, G_2, of 0.93, on the assumption that the relationship between G_2 and G_4 is that given by a magnetic dipole interaction. Since the value of G_2 was near unity for this line, and since the other three transitions of interest have much shorter lifetimes, we assumed no attenuation of the angular distributions in those cases. Finite solid-angle corrections were made using the tables of Black and Gruhle.[7] A small, empirically determined correction was made for the accidental simultaneous arrival in the detector of two 122-keV photons, simulating one of 244 keV. There is also a correction of about 2% in the intensity of the 122-keV line in the natural samarium targets due to photons from ^{147}Sm. A

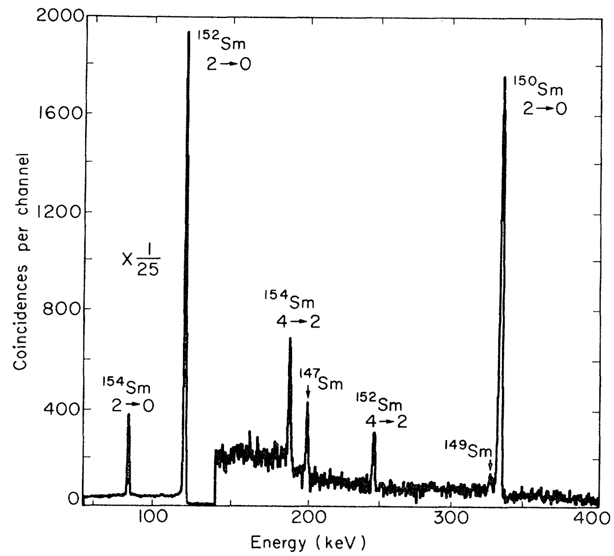

FIG. 2. Gamma-ray spectrum in coincidence with 10.4-MeV ^{4}He projectiles backscattered from a natural samarium target. Accidental coincidences have been subtracted.

typical gamma-ray spectrum in coincidence with backscattered projectiles is shown in Fig. 2.

In calculating the intensity of the 245-keV line, we used a computer program[8] which took account of $E2$, $E3$, and $E4$ excitations. We included the rotational states up to 8^+ in ^{152}Sm, using the $B(E2)$ values given in Diamond et al.[2] The first two of these are $B(E2; 2^+ \to 0^+) = (0.686 \pm 0.014)e^2$ b^2, and $B(E2; 4^+ \to 2^+) = (1.009 \pm 0.033)e^2$ b^2. However, because double $E2$ excitation of the 4^+ level is weak with ^{4}He projectiles, the decay to the 4^+ level from higher levels excited by a single step can be important. The largest contributions of this kind stem from the collective vibrational levels. There is also a small effect on the calculated cross sections of the 4^+ level due to the addition of the vibrational levels. We included in the calculations four vibrational states, whose properties are summarized in Table I. An uncertainty of 25% in the feeding from each vibrational state was assumed. Each $B(E\lambda)$ value[9,10] and branching ratio[11] has been measured. The higher rotational states also feed the 4^+ state, but very weakly. We have also shown in Table I all the contributions to the calculated singles (σ_0) and backscatter ($d\sigma_0$) cross sections of the 4^+ state at 10.38-MeV ^{4}He energy. An important feature is that the effect of the vibrational states relative to the direct population (and $E4$ contributions) is three times smaller in the backscatter spectra than it is in the singles spectra. The

Table I. Calculated population of the 4^+ state.

Level I^π	K	E (MeV)	$B(E\lambda; 0^+ \rightarrow I^\pi)$ (e^2 b$^\lambda$)	$f(4^+)$[a]	$E_\alpha = 10.38$ MeV $f(4^+)\sigma_{Ik}$ (μb)	$f(4^+)d\sigma_{Ik}$ (μb)
2^+	$0(\beta)$	0.811	0.023	0.21	57	2.6
	$2(\gamma)$	1.087	0.083	0.013	5	0.3
3^-	0	1.042	0.14	0.30	29	1.9
	1	1.578	0.078	0.73	9	0.5
6^+	0	0.7067	b	1.00	0.6	0.14
4^+	0	0.3665	b	1.00	328	53.4
Total					429	58.8

[a]Fraction of the decay which goes to the 4^+ level.
[b]Only multiple $E2$ excitation is considered here. The $B(E2)$ values used are given in the text.

omission of other important states of this type should therefore show up as a discrepancy between the singles and coincidence data.

A number of other effects which might influence the calculated cross sections of the 2^+ or 4^+ states in ^{152}Sm were considered, among which were (1) excitation of the giant dipole states; (2) the presence of an appreciable $E6$ transition moment; and (3) static $E2$ and $E4$ moments. None of these gives rise to corrections of appreciable size. In the calculations, rigid-rotor values for $B(E4; 2^+ \rightarrow 4^+)$ were used. If this were 0 instead, then our measured value for $\langle 0^+ \| \mathfrak{M}(E4) \| 4^+ \rangle$ would be increased by about 10%. For ^{150}Sm we used a $B(E2; 2^+ \rightarrow 0^+)$ value[2] of $(0.278 \pm 0.010)e^2$ b^2 and a static moment (prolate) of half the rigid-rotor value. A variation of the static moment from zero to the full rigid-rotor value introduces a change no greater than about ±1% in the cross sections for the 2^+ state. In all cases the agreement between the $2^+ \rightarrow 0^+$ transitions in ^{152}Sm and ^{150}Sm was satisfactory. An effect that has not yet been evaluated is the possibility of quantal corrections to the semiclassical calculations used. These would be expected to lower the calculated cross sections[12] (increase our $E4$ moment) and could be as large as a few percent.

In Fig. 3 we have plotted the ratio of the observed cross section (σ) to those calculated including all feedings (σ_0), against the bombarding energy. The error bars on the data points do not include any of the systematic uncertainties involved in the analysis. The dashed and solid lines show the values for the backscatter and singles data, respectively, corresponding to an $E4$ moment of $+0.35e$ b^2. (The other possible solution, $-0.7e$ b^2, seems improbable.) This is the best fit to the data below 11 MeV, and would not be changed appreciably if we included the 11.1-MeV data and/or all the data from the enriched ^{152}Sm target. The results from the natural samarium target are high at 14 and 12.2 MeV, and possibly at 11.1 MeV also. While we do not fully understand this, it is clear that interference from nuclear inelastic scattering will affect the backscatter results in this direction at sufficiently high energies (almost certainly at 14 MeV). Furthermore, at 14 MeV the singles re-

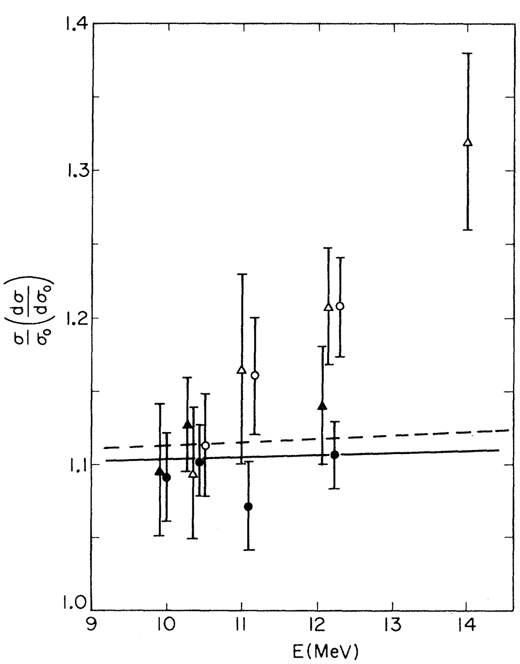

FIG. 3. The measured cross sections for populating the 4^+ level of ^{152}Sm normalized to the appropriate calculated value with no $E4$ moment, plotted against the bombarding energy. The solid points are for enriched ^{152}Sm targets and the open ones for natural samarium targets. The triangles and circles are backscatter and singles results, respectively. The dashed and solid lines are the calculated results for backscatter coincidences and singles, respectively, with $\langle 0^+ \| \mathfrak{M}(E4) \| 4^+ \rangle = +0.35e$ b^2.

sults from the natural samarium target were not evaluated because of the appearance of a shoulder on the 245-keV peak. This shoulder might also be affecting the results from this target at somewhat lower bombarding energies, although no complexity could be detected. Because of the unambiguous consistency of all types of results below 11 MeV, we have chosen to evaluate the $E4$ moment from these data.

The largest single source of uncertainty in the result is due to the $B(E2; 4^+ \rightarrow 2^+)$ value which is known to an accuracy of ±3.3%. This is true largely because it affects both the singles and backscatter results in the same way. The uncertainties in the $B(E2; 2^+ \rightarrow 0^+)$ values are less important here because there are two independent quantities (^{150}Sm and ^{152}Sm). The feeding corrections from the vibrational states cause a large uncertainty in the singles results (3.8%), but only a relatively small one (1.4%) in the backscatter results. Conversely the angular distributions cause much larger uncertainties in the backscatter results (2.9%) than in the singles (0.5%). In both cases the uncertainties due to the peak-area determinations are smaller than ~2%, as are those from other individual sources. The best value for the $E4$ moment, with the known uncertainties taken into account, is $(+0.35 \pm 0.11)e\ \text{b}^2$. This error limit does not include the possible quantal corrections or the possibility of other omitted corrections. We cannot set a real upper limit on these, but it is reassuring that the two types of experiments, whose sensitivity to the various corrections is generally different, yield consistent results. The present experiments have also given information on ^{154}Sm, but a more accurate value for $B(E2; 4^+ \rightarrow 2^+)$ is needed before a meaningful analysis is possible.

If we assume the nucleus to be a rigid, uniformly charged rotor with a sharp surface defined by

$$R = R_0(1 + \beta_2 Y_{20} + \beta_4 Y_{40}),$$

we can evaluate β_2 and β_4 from the measured $E2$ and $E4$ transition moments. Taking the charge radius to be $R_0 = 1.2A^{1/3}$ F, we find $\beta_2 = (+)0.259$ and $\beta_4 = +0.058 \pm 0.032$ in ^{152}Sm. The sign of β_2 has been assumed to be positive in this analysis. These values of β_λ depend on the radius used and change roughly as $R_0^{-\lambda}$. The inclusion of still higher moments would probably affect the deduced deformation parameters slightly. It is interesting to try to compare this shape of the charge field with the shape of the nuclear field measured by Hendrie et al.[13] who found $\beta_2 = +0.246$ and $\beta_4 = +0.048$ for the above value of R_0. These appear to be quite similar, but it is not really clear that this is the proper way to compare these two sets of results. The present value of β_4 is also in reasonable accord with theoretical estimates[3] of nuclear shapes.

We believe the present work shows that it is possible to find experimental conditions where $E4$ transition moments can be reliably determined in Coulomb-excitation measurements. This is true in spite of the fact that many different processes contribute to the observed cross sections and must be taken into account for accurate evaluations. Conversely the presently measured $E4$ moment produces sizable effects that must be included in the precise determination of other matrix elements from Coulomb-excitation studies on ^{152}Sm.

We are indebted to Professor K. Alder for helpful comments, and to Dr. J. R. Leigh, Dr. K. H. Maier, Dr. K. Nakai, Dr. J. L. Quebert, and Dr. D. Ward for their help and numerous discussions. We also thank Mr. T. Gee for preparing the targets, and the operating crew of the heavy ion linear accelerator for their cooperation.

---

†Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]F. S. Stephens, D. Ward, R. M. Diamond, J. de Boer, and R. Covello-Moro, to be published.

[2]R. M. Diamond, F. S. Stephens, R. Nordhagen, and K. Nakai, in *Proceedings of the International Conference on Properties of Nuclear States, Montreal, Canada, 1969,* edited by M. Harvey *et al.* (Presses de l'Université de Montreal, Montreal, Canada, 1969), p. 653, and to be published.

[3]S. G. Nilsson, C. F. Tsang, A. Sobiczewski, Z. Szymanski, S. Wycech, C. Gustafson, I.-L. Lamm, P. Möller, and B. Nilsson, Nucl. Phys. A131, 1 (1969); P. Möller, to be published.

[4]A. Rytz, Helv. Phys. Acta 34, 240 (1961).

[5]F. M. Bernthal, thesis, Lawrence Radiation Laboratory Report No. UCRL-18651, 1969 (unpublished).

[6]R. S. Hager and E. C. Seltzer, California Institute of Technology Technical Report, 1967 (unpublished).

[7]J. L. Black and W. Gruhle, Nucl. Instr. Methods 46, 213 (1967).

[8]A. Winther and J. de Boer, California Institute of Technology Technical Report, 1965 (unpublished), and in *Coulomb Excitation,* edited by A. Winther and J. de Boer (Academic, New York, 1966), p. 303; A. Holm *et al.*, private communication.

[9]F. K. McGowan, R. O. Sayer, P. H. Stelson, R. L. Robinson, and W. T. Milner, Bull. Am. Phys. Soc. 13,

895 (1968).
[10]E. Veje, B. Elbek, B. Herskind, and M. C. Olesen, Nucl. Phys. A109, 489 (1968).
[11]L. Varnell, J. D. Bowman, and J. Treschuk, Nucl. Phys. A127, 270 (1969).
[12]K. Alder and H. K. Pauli, Nucl. Phys. A128, 193 (1969).
[13]D. L. Hendrie, N. K. Glendenning, B. G. Harvey, O. N. Jarvis, H. H. Duhm, J. Saudinos, and J. Mahoney, Phys. Letters 26B, 127 (1968).

TWO ASYMPTOTIC SUM RULES FOR ELECTROPRODUCTION AND PHOTOPRODUCTION*

John M. Cornwall,† Dennis Corrigan, and Richard E. Norton,
University of California, Los Angeles, California 90024
(Received 23 January 1970)

We give a sum rule relating photoproduction to asymptotic electroproduction, and another electroproduction sum rule which tests for the presence of operator Schwinger terms.

In this Letter we report two sum rules, which relate integrals over the scale functions[1] of electroproduction to each other, and to integrals over total photoproduction cross sections. The sum rules are based on Bjorken's idea of scale invariance,[1] the experimental fact[2] that the electroproduction structure functions decrease in the momentum-transfer variable q^2 (for sufficiently large q^2) if the mass of the produced hadronic states is held fixed, and on the assumption that the high-energy form of the imaginary part of the forward Compton amplitude for fixed q^2 has no term characteristic of a Regge pole with $\alpha(0)=0$. Under these circumstances, the sum rule given in Eq. (8a) is valid; and if we assume in addition that there is no Schwinger term in the connected, covariant forward Compton amplitude, the sum rule in Eq. (8b) is also satisfied.

Let $J_\mu(x)$ be the electromagnetic current. The spin-averaged forward Compton amplitude can be written

$$T_{\mu\nu}=i(2p_0)\int d^4x e^{iqx}\langle p|[J_\mu(x)J_\nu(0)]_+|p\rangle=\left(p_\mu-\frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu-\frac{p\cdot q}{q^2}q_\nu\right)T_2+\left(\frac{q_\mu q_\nu}{q^2}-g_{\mu\nu}\right)T_1. \tag{1}$$

The imaginary parts of the T_i are related to the structure functions[1,2] W_i of electroproduction by

$$\operatorname{Im}T_i=2\pi W_i. \tag{2}$$

That $T_{\mu\nu}$ is free of unwanted singularities is assured by imposing conditions on the T_i; for example, T_2 vanishes like q^2 at $q^2=0$. W_1 and W_2 are related to the cross sections for scattering of transverse and longitudinally polarized photons, σ_T and σ_L, by

$$W_1=(4\pi^2\alpha)^{-1}(\nu+q^2/2)\sigma_T, \tag{3a}$$

$$W_2=-(4\pi^2\alpha)^{-1}(\nu+q^2/2)(\nu^2-M^2q^2)^{-1}q^2(\sigma_T+\sigma_L). \tag{3b}$$

Here M is the proton mass, q the virtual photon momentum, $\nu=q\cdot p$, and $\alpha=1/137$. Since σ_L vanishes at $q^2=0$,

$$W_1(q^2,\nu)\xrightarrow[q^2\to 0]{}-\nu^2q^{-2}W_2(q^2,\nu)\to(4\pi^2\alpha)^{-1}\nu\sigma(\nu), \tag{4}$$

where $\sigma(\nu)$ is the total cross section for real photoproduction from a proton. Bjorken has suggested,[1] and experiments apparently verify,[2] that W_1 and νW_2 approach finite functions of the variable $\omega=-q^2/2\nu$ in the limit of large ν and q^2, that is[1]

$$W_1\to F_1(\omega), \tag{5a}$$

$$\nu W_2\to F_2(\omega). \tag{5b}$$

The photoproduction cross section $\sigma(\nu)$ appears[3] to have the Regge asymptotic form for large ν. Let us define, therefore, a truncated cross section $\tilde\sigma(\nu)$ by

$$\tilde\sigma(\nu)=\theta(\nu-\nu_0)\sigma(\nu)-\sum_{\alpha>0}C_\alpha\nu^{\alpha-1};\qquad \nu_0=Mm_\pi+(m_\pi^2/2), \tag{6}$$