OPTICAL POTENTIAL FROM DYNAMIC ANALYSIS OF ELASTIC ELECTRON-NUCLEUS SCATTERING*

C. Toepffer
Institut für Theoretische Physik, Universität Frankfurt am Main, 6 Frankfurt am Main, Germany

and

Dieter Drechsel
Physics Department, University of Virginia, Charlottesville, Virginia 22903
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Dispersion effects in the analysis of high-energy electron-nucleus scattering lower the cross section obtained from a partial-wave analysis in its minima. At an electron energy $E=750$ MeV and momentum transfer $q=3.9$ fm^{-1} the correction amounts to about 70% for Ca^{40}. These dispersion effects may be described by an optical potential whose real part is determined by the usual static charge distribution, while the small imaginary part peaks at the nuclear surface with a maximum value of about 20 keV.

The classic experiments[1] of elastic electron scattering at momentum transfer $q \lesssim 1$ fm^{-1} have been fitted with a two-parameter Fermi-type charge distribution $\rho_0(r)$ and give information on the radius and the surface thickness of the nucleus. More recent experiments[2,3] at 400 and 750 MeV with momentum transfers up to 3.4 fm^{-1} have shown, however, that even a three-parameter Fermi distribution does not give a satisfactory fit at higher momentum transfer. Instead the charge distribution $\rho_0(r)$ has to be modulated by a small oscillatory function $\Delta\rho(r)$ in order to reproduce the experimental cross section.[3] The experimental data for the scattering of 250-MeV electrons on Ca^{40} have also been explained by Elton and Swift,[4] who derived the static charge distribution from single-particle wave functions. Analyzing the 750-MeV experiment with the same charge distribution, Wright, Tuan, and Huber found that the remaining discrepancy may be accounted for by short-range correlations between the nucleons.[5] Thus in the experiments under consideration the electron is sensitive to rather small details of the charge distribution.

During the past few years, however, several authors have stressed the possible importance of dispersion effects in electron scattering at high momentum transfer.[6-9] This effect is described by diagrams in which two or more virtual photons are exchanged between the electron and the nucleus, leaving the nucleus virtually excited in the intermediate state. For numerical reasons previous calculations have been restricted to lower electron energies, and only a few discrete nuclear levels have been considered. It is the aim of this work to remove these limitations and to calculate the dispersion effects for the experiments at 750 MeV, assuming that the excitation spectrum of the nucleus is described by quasielastic scattering.

If the electron energy is greater than that of all contributing nuclear resonances, off-mass-shell effects may be neglected and the diagonal S-matrix element to second order is[10]

$$S_{cc} = \exp(2i\delta_c^{\mathrm{pot}})\{1-2\pi^2\textstyle\sum_{c'}{}'|\langle c'|H_{\mathrm{int}}|c\rangle|^2\}, \quad (1)$$

where the prime on the sum indicates that the static part of the potential has already been taken into account to all orders by a phase-shift analysis. Since the electroexcitation spectrum at high momentum transfer is dominated by the quasielastic peak, i.e., by scattering on the individual protons, we replace the excited states by plane-wave states for the protons. Since the summation in Eq. (1) runs over all intermediate states, and the energy loss has been neglected, we ignore all distortion effects, in particular the fine structure of the giant resonance at the low-energy side of the quasielastic peak due to long-range correlations, and the shift at the high-energy side due to short-range correlations. Under these assumptions the S-matrix element for partial wave κ may be expressed in the following form:

$$S_{\kappa\kappa} = \exp(2i\delta_c^{\mathrm{pot}})(1-2\Delta_\kappa), \quad (2)$$

with

$$\Delta_\kappa = \pi^2\textstyle\sum_{c'}|\langle c'|H_{\mathrm{int}}|\kappa\rangle|^2 = \alpha^2 Z\int d\tau[\rho_0(r)/r^2]\sum_{\lambda\kappa'}I_{\kappa'\lambda}(r)^2 - \alpha^2 Z[\int d\tau\,\rho_0(r)I_{\kappa 0}(r)]^2, \quad (3)$$

and

$$I_{\kappa'\lambda}(r) = \tfrac{1}{2}[1+(-)^{l+l'+\lambda}]\left(\frac{2j'+1}{2\lambda+1}\right)^{1/2}\begin{pmatrix} j' & \lambda & j \\ \frac{1}{2} & 0 & -\frac{1}{2}\end{pmatrix} r^2\int dr_e r_e^2(f_{\kappa'}f_\kappa + g_{\kappa'}g_\kappa)r_<{}^\lambda r_>{}^{-\lambda-1}. \quad (4)$$

The second term in Eq. (3) subtracts the contribution of the monopole part of the static potential, which has been taken into account in the partial-wave analysis. In our approximation the dispersion terms only depend on the static charge distribution; they are of order α relative to the first-order matrix element and proportional to Z, since we have summed incoherently over the protons.

Since in carrying out the numerical integration of the wave function double precision is used (i.e., 16 valid decimals are retained), the accuracy of the phase shifts is mainly limited by the step width of the numerical integration and by the accuracy of the regular and irregular Coulomb wave functions, which are matched to the numerical solutions at a radius of about 8 fm. The latter problem may be solved by recursion techniques,[7] in particular by downward recursion for the regular Coulomb wave functions.[11] With about nine exact digits for the phase shifts, the cross section will be given exactly down to the region of about 10^{-38} cm^2 sr^{-1}. This has also been checked by reproducing the Born approximation in the limit $Z \to 0$. In order to evaluate the second-order term, the Dirac equation is then integrated to a radius of about 40 fm at 750 MeV; from there on asymptotic expansions of the electron wave functions can be used. Furthermore, it turns out that by replacing the distorted waves by plane waves, i.e., spherical Bessel functions, the correction terms change by only a few percent, even for a nucleus like Ca^{40}. For an actual calculation we use a shell-model charge distribution, which has been derived from single-particle wave functions in a Woods-Saxon potential with spin-orbit coupling and Coulomb terms. The potential is characterized by the parameters used by Elton and Swift,[4] which gave a best fit to 250-MeV elastic scattering. Folding this charge distribution with a Gaussian proton form factor with $a = 0.65$ fm we obtain the cross section shown in Fig. 1 (solid line). Dispersion effects lower the cross section in the minima (dashed line). The relative effect, shown in the upper part of Fig. 1, amounts to 30% at $\theta = 50°$ and 70% at $\theta = 62°$. This is in qualitative agreement with previous results at lower electron energies which take into account only a few intermediate states.[7-9] On the other hand the present experimental errors at the higher momentum transfer are still much larger than the dispersion effect, which therefore cannot yet be "seen" in such an experiment even if we knew the "true" static charge distribution to start with. Nevertheless one may ask whether it is possible to simulate the dispersion effect by a change in the static density parameters. Equivalently, what is the difference between the charge distribution calculated in a first-order calculation and the "true" charge distribution calculated with dispersion effects included? Therefore we have computed cross sections with and without dispersion effect using a parabolic-Fermi-shape density distribution[2] with the parameters which gave a best fit at 250 MeV. The results are given in the lower part of Fig. 2, which also shows the region where the cross section drops below 10^{-38} cm^2 sr^{-1} and the calculation becomes unreliable. Since the dispersion effect (solid line in the upper part of Fig. 2) shows up typically at the minima of the cross section, we were not able to reproduce the same cross section by a change of the parameters of the Fermi-type distribution. As far as the order of magnitude is concerned, the variation $z \to z + 1.5\%$ and $w \to w + 10\%$ gave a relative effect comparable with the dispersion effect on the average (dashed line in upper part of Fig. 2). Since the Fermi distribution has to be modulated by a small oscillatory function, one might also vary the additional parameters introduced by that function.

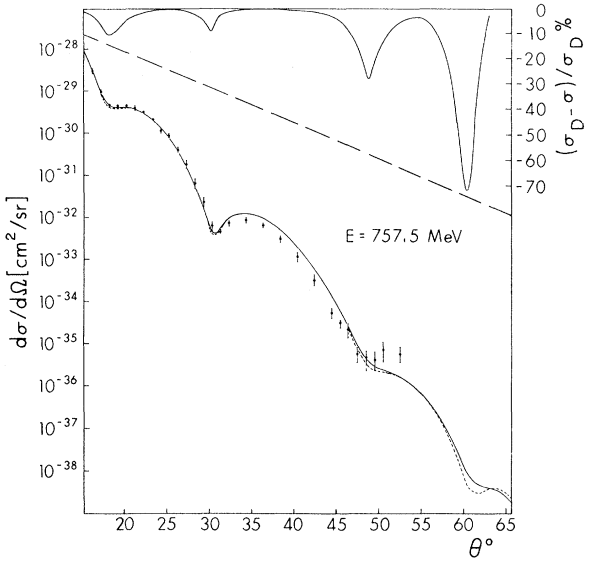

FIG. 1. Cross section for elastic scattering of 750-MeV electrons on Ca^{40} as a function of scattering angle θ calculated from the shell-model charge distribution of Ref. 4 with (dashed line) and without (solid line) dispersion effects. The upper part of the figure shows the relative effect of the dispersion terms, $(\sigma_D - \sigma)/\sigma_D$. The experimental data are from Ref. 3.

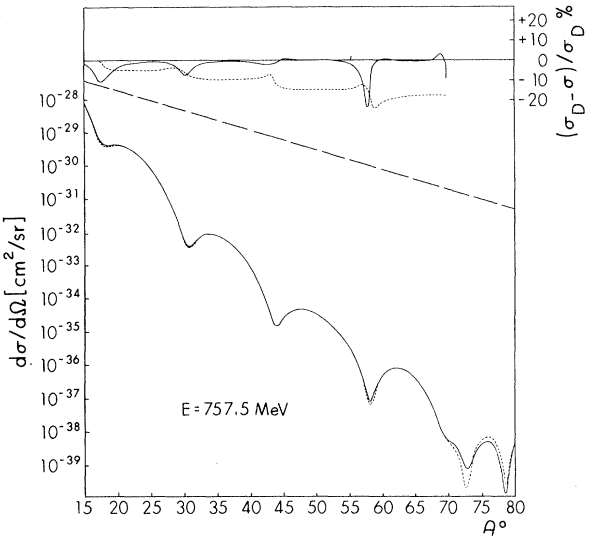

FIG. 2. Cross section for elastic scattering of 750-MeV electrons on Ca^{40} as a function of scattering angle θ calculated from a parabolic-Fermi-shape charge distribution with $c = 3.6685$, $z = 0.5839$, and $w = 0.1017$ fm (Ref. 3). In the lower part the solid line shows the result without, the dashed line with, dispersion corrections. The upper part of the figure gives the relative effect of the dispersion (solid line) and the relative effect on the cross section obtained by varying $z \to z$ +1.5% and $w \to w$ +10% (dashed line).

We think, however, that a change of the density parameters to account for dispersion effects is not meaningful for the following reason: The off-resonance behavior of the electron at high energies leads to a real second-order correction Δ_κ or equivalently to the introduction of a small additional imaginary scattering phase, while the changes in the real part of the phase may be neglected. This means that the second-order effect shows up predominantly in a depletion of the flux in the elastic channel. Indeed, Eq. (1) guarantees the unitarity of the S matrix up to second-order terms. While changes in the real part of the phase may be accounted for by variations in the static charge distribution, the occurrence of an imaginary phase leads to the existence of an imaginary part in the potential. Therefore, it is inconsistent to fit the dispersion effects by a change of the real charge distribution alone.

The correct way to take into account the dispersion effect is therefore to introduce an optical potential $U = V + iW$, whose real part V is the usual electrostatic potential. The imaginary part W will have the following properties: (1) It will be negative, and since it depends on the transition potentials for the virtual excitations, $|W|$ will have a long-range decreasing behavior outside of the nucleus. (2) Since the correction terms Δ_κ are about three orders of magnitude smaller than the potential scattering phases δ_κ, the imaginary part will be much smaller than the real part of the potential. Using the Born-approximation formula

$$\Delta_\kappa = -\int d(kr)\,(kr)^2[j_l^2(kr) + j_{\bar{l}}^2(kr)]\,W(r)/E, \quad (5)$$

we were able to explain the dispersion effects with an imaginary potential $W(r)$, which increases as r for small radii and decreases as r^{-3} outside of the nucleus. For Ca^{40} this imaginary potential has a maximum value of about 20 keV at the nuclear radius and is small compared with the real potential V, whose maximum value is $|V(0)| \approx 10$ MeV at the origin.

From the point of view of the optical model it is immediately clear that the real part of the additional phase, which is due to higher order effects, can be neglected in the high-energy case: Replacing $V \to V + iW$, the wave numbers of the electron become complex, $k \to k_0 + i\gamma$. Since $(E - V)^2 = (chk)^2 + m^2c^4$, we obtain for the real part $k_0 = k[1 + O(\gamma^2/k^2) + O(m^2/k^2)]$, and the correction terms can be neglected since $|W(r)| \ll mc^2 \ll |V(r)| \ll E$. The same is also true for the phases.

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MIRROR SYMMETRY IN THE β DECAY OF THE $A = 20$ AND 25 SYSTEMS*

D. H. Wilkinson
Brookhaven National Laboratory, Upton, New York 11973, and Nuclear Physics Laboratory, Oxford, England

and

D. E. Alburger
Brookhaven National Laboratory, Upton, New York 11973
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The half-life of F^{20} has been determined as 11.03 ± 0.06 sec and β branches have been measured in the decay of Na^{25}. By combining with information on Na^{20} and Si^{25} it is deduced that for the $A = 20$ mirror decays to the Ne^{20} 2^+ first-excited state, $(ft)^+/(ft)^- = 0.933 \pm 0.032$ or 1.062 ± 0.037, while in the $A = 25$ system $(ft)^+/(ft)^- = 1.187 \pm 0.076$ for β decay to all states below 3 MeV.

The simplest expectation for mirror β decays is that they should have identical ft values. It has, however, been known for some time that the ft value for N^{12} decay to the ground state of C^{12} is some 10% greater than that for the mirror decay of B^{12}. Careful analysis[1] suggests that the discrepancy may well be a significant one, not explicable in terms of electromagnetic, second-forbidden, isospin-mixing, and binding-energy corrections, and that its resolution may lie in the reality of second-class currents, specifically the induced tensor interaction,[2,3] i.e., that the β interaction does not respect G parity.[2] Before accepting this fundamental conclusion one must be sure both that the known corrections have been properly evaluated and that there are not others of a "structural" nature that may fluctuate from case to case.

It is therefore important to investigate other cases of mirror decay to see whether or not they fall systematically into line with $A = 12$. If the discrepancy in ft values was due solely to an induced tensor term its magnitude, for light nuclei and $W_0^{\pm} \gg 1$, should be proportional to $W_0^{+} + W_0^{-}$ and approximately state independent.[3,4] We have carried out measurements on the systems $A = 20$ and 25 that enable us to extend the mirror test to these cases. We find that $A = 25$ shows a large departure from mirror symmetry in the same sense as for $A = 12$; the evidence on $A = 20$ is conflicting but suggests that the departure from mirror symmetry could have the opposite sense. These results emphasize the caution that must be used in interpreting the failure of mirror symmetry in terms of second-class currents.

The β^- decay of F^{20} is 99.92% to the first excited state of Ne^{20} at 1.63 MeV.[5] The β^+ decay of Na^{20} has been extensively investigated[6] by measurement of its β rays and the subsequent γ decay and α decay of Ne^{20}. The Na^{20} half-life is reported[6] as 408 ± 6 msec and the β branch to the first excited state of Ne^{20} as 90.0%. However, in view of technical problems in the β-ray measurements we have preferred to make our analysis in terms of the superallowed transition of Na^{20} to its analog in Ne^{20} at 10.270 ± 0.009 MeV.[7] For the Fermi part of this transition we have taken[8] $ft = 3060 \pm 20$ sec; for the Gamow-Teller part we have based ourselves on local systematics and taken the range $ft = 6.3 \times 10^4$-∞ sec; this leads to $ft = 2990 \pm 70$ sec for the two together. For the Na^{20} mass excess we have taken[6,9] 6.87 ± 0.04 MeV; we then use 7.35 ± 0.35 as the factor by which decay of Na^{20} to all α-unstable states of Ne^{20} exceeds