nates solely from a Ruderman-Kittel-Yosida-type long-range spin polarization about the impurity and is entirely unrelated to the "Kondo" phenomena. However, in that case it becomes difficult to explain the rapid degrading of the beat as the temperature is raised above T_K.

We are grateful to Mr. B. Paton for continued fruitful discussion, Dr. L. Windmiller for communicating results prior to publication, and Mr. W. Fisher for preparing the alloy samples.

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THEORETICAL ESTIMATES FOR THE DECAY RATES OF THE SPIN FLUCTUATIONS IN $RbMnF_3$†

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(Received 12 December 1969)

We have calculated the decay rates of the spin fluctuations in the hydrodynamic region above T_N for the isotropic antiferromagnetic $RbMnF_3$. Our results agree in magnitude, temperature dependence, and wave-vector dependence with the experimental values reported by Lau et al.

In a recent Letter, Lau et al.[1] have reported measurements of the inelastic scattering of neutrons from $RbMnF_3$ in the vicinity of the Néel point. Among the data presented are values of the decay rates of the spin fluctuations $\vec{S}_{\vec{q}+\vec{K}_0}(t)$ in the hydrodynamic region above the ordering temperature. [Here $\vec{S}_{\vec{K}_0}$ is the staggered spin operator and $q/K_0 \ll 1$.] In this Letter we report calculations of the decay rates which are in substantial agreement with the experimental values in their temperature dependence, wave-vector dependence, and overall magnitude.

Our calculated values are obtained from a self-consistent theory for the damping constants first introduced for isotropic ferromagnets by Bennett and Martin[2] and subsequently extended by Kawasaki to isotropic antiferromagnets[3] and by us to planar ferromagnets.[4] Two desirable features of the self-consistent theory are that it yields results in agreement with the predictions of the dynamic scaling laws[5] and that it involves as parameters the measurable static properties of the system (lattice parameters, exchange constants, susceptibilities, and correlation lengths). Because of this we are able to incorporate considerable empirical information into the theoretical calculation by using experimental values for these parameters wherever they appear.

The basic equations for the isotropic antiferromagnet follow from the analysis given in Ref. 3. The decay rates are obtained from the solutions of coupled nonlinear integral equations involving spin fluctuations with wave vectors in the vicinity of the center of the Brillouin zone and near the point $\vec{K}_0$, the reciprocal lattice vector of the magnetic superlattice. Writing these rates as $\varphi(\vec{q})$ and $\varphi(\vec{q}+\vec{K}_0)$, we have

$$\varphi(\vec{q}) = \frac{4}{\chi_0\beta_0 N}\sum_{\vec{p}}{}' \frac{[J(\vec{q}-\vec{p}-\vec{K}_0)-J(\vec{K}_0+\vec{p})]^2}{\varphi(\vec{q}-\vec{p}-\vec{K}_0)+\varphi(\vec{p}+\vec{K}_0)}\chi(\vec{K}_0+\vec{p})\chi(\vec{q}-\vec{p}-\vec{K}_0), \tag{1}$$

$$\varphi(\vec{q}+\vec{K}_0) = \frac{32\chi_0}{\chi(\vec{q}+\vec{K}_0)\beta_0 N}\sum_{\vec{p}}{}' \frac{[J(0)]^2\chi(\vec{K}_0+\vec{p})}{\varphi(\vec{q}-\vec{p})+\varphi(\vec{K}_0+\vec{p})}. \tag{2}$$

Here $\chi(q)$ is the wave-vector-dependent susceptibility per spin in units of $g^2\mu_B^2$, $\chi_0=\chi(0)$ is the uniform-field susceptibility at the Néel point, $J(\vec{q})$ is the Fourier transform of the exchange interaction, $\beta_0=1/kT_N$, and N is the number of spins in the lattice. The prime on the summation is to indicate that $|\vec{p}|$ is restricted to the interval $0\leq p\leq k_m$, where k_m is chosen large enough to include all the hydrodynamic modes but sufficiently small to exclude the microscopic fluctuations.[6] As will be shown below our results are relatively insensitive to variations in k_m for $k_m \gtrsim 3k_c$, k_c being the inverse range parameter. We use the Ornstein-Zernike form for $\chi(\vec{q}+\vec{K}_0)$,

$$\chi(\vec{q}+\vec{K}_0)=\frac{\chi_s k_c^2}{(k_c^2+q^2)}, \tag{3}$$

where χ_s is the staggered susceptibility.[7] The product $\chi_s k_c^2$, which appears only in the equation for $\varphi(\vec{q})$, is set equal to $(6Ja^2)^{-1}$, the value it has in the molecular field approximation for a simple cubic lattice with lattice parameter a and nearest-neighbor interactions.

We have solved Eqs. (1) and (2) using the values $J=3.4°K$,[8] $a=4.2$ Å,[8] $T_N=83°K$,[1] and $\chi_0=86\times10^{-6}$ emu/g.[9] In order to display the behavior predicted by the dynamic scaling laws we write the solutions in the form

$$\varphi(\vec{q}+\vec{K}_0)=k_c^{3/2}\sum_{n=0}^{7} b_n(q/k_c)^{2n}, \tag{4}$$

$$\varphi(\vec{q})=Ck_c^{-1/2}q^2; \tag{5}$$

the coefficients b_n and c have been calculated for various k_m.[10] Values of b_0 and b_1/b_0 obtained with different cutoff parameters are displayed in Table I. It is to be noted that increasing k_m from $4k_c$ to $5k_c$ leads to only a 3% increase in b_0 and a smaller change in b_1/b_0 indicating asymptotic behavior at this point.[11]

In Ref. 1 $\varphi(\vec{K}_0)$ is reported to have the value $10.8k_c^{1.4\pm0.2}$ meV, where k_c is measured in inverse angstroms. In order to facilitate comparison between our results and the experimental values, we have replotted the experimental values for $\varphi(\vec{K}_0)$ shown in Fig. 2 of Ref. 1 against $k_c^{3/2}$ and fitted the points with a straight line. In this way we obtain the result $\varphi(\vec{K}_0)(\text{expt})=13.3k_c^{3/2}$ which compares favorably with our values for $k_m=3k_c$, $13.0k_c^{3/2}$, and $k_m=4k_c$, $13.9k_c^{3/2}$. We also obtain the value 0.91 for the ratio b_1/b_0 whereas experimentally $b_1/b_0=1.11$. A qualitative indication of the extent of the agreement for larger values of $\vec{q}$ can be inferred from a plot of $\varphi(\vec{q}+\vec{K}_0)/\varphi(\vec{K}_0)$ and $\omega_{k_c}(\vec{q})/\omega_{k_c}(0)$ against $(q/k_c)^2$.

Table I. Values of b_0 and b_1/b_0.

k_m/k_c [a]	b_0 [b]	b_1/b_0
1	7.2	0.92
2	11.2	0.91
3	13.0	0.91
4	13.9	0.91
5	14.3	0.92

[a] k_m is the cutoff in the integration over p in Eqs. (1) and (2).
[b] $\varphi(\vec{K}_0)=b_0k_c^{3/2}$.

Here $\omega_{k_c}(\vec{q})$ is the characteristic frequency which is defined by an integral over the energy distribution of scattered neutrons.[1] When this distribution is Lorentzian $\omega_{k_c}(\vec{q})$ can be interpreted as a decay rate. From Fig. 1 it is evident that the two curves lie reasonably close to one another over the range $0\leq(q/k_c)^2\leq 9$.

For C we obtain the value 4.0 meV Å$^{3/2}$ ($k_m=3k_c$). Thus when $k_c=0.02$ Å^{-1} ($T-T_N\approx 1°K$) the spin-diffusion constant, $Ck_c^{-1/2}$, has the value 29 meV Å^2. This is to be compared with the room-temperature result, 8 meV Å^2, reported by Windsor, Briggs, and Kestigian.[12] We have also examined the dependence of $\varphi(\vec{K}_0)$ and $\varphi(\vec{q})$ on J, χ_0, T_N, and a. We find the approximate behavior

$$\varphi(\vec{K}_0)\sim J(\chi_0T_N)^{1/2}(k_ca)^{3/2}, \tag{6}$$

$$\varphi(\vec{q})\sim J^{-3/2}\chi_0^{-2}T_N^{1/2}(k_ca)^{-1/2}q^2, \tag{7}$$

for small variations about the values given above.

Our final point concerns the approximations

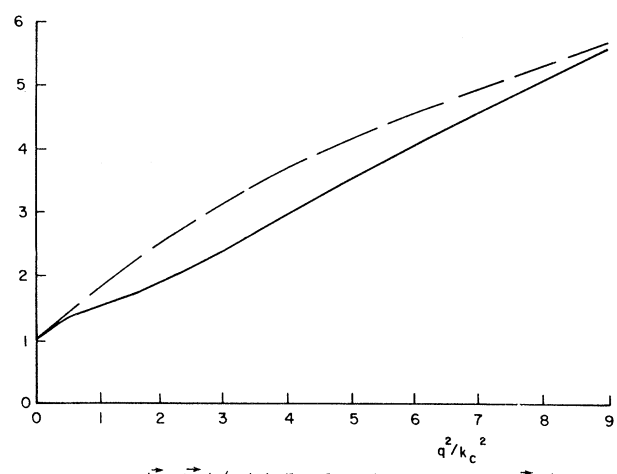

FIG. 1. $\varphi(\vec{q}+\vec{K}_0)/\varphi(0)$ (broken line) and $\omega_{k_c}(\vec{q})/\omega_{k_c}(0)$ (solid line) vs $(q/k_c)^2$. The values of $\omega_{k_c}(q)$ are obtained from Ref. 1. $\varphi(\vec{q}+\vec{K}_0)$ is calculated with $k_m=3k_c$.

implicit in the self-consistent theory. The most critical of these involves the factorization of the equal-time four-spin correlation function into a product of two-spin functions. By invoking the Widom-Kadanoff scaling laws, Kawasaki has shown that the sum over the coupled and the sum over the decoupled functions have the same asymptotic dependence on k_c and q as long as the wave vectors are restricted in magnitude in the sense implied in Eqs. (1) and (2).[13] What may be a further indication of the accuracy of the approximation comes from a calculation carried out at infinite temperature. In this limit the relative error incurred in the decoupling is on the order of $(k_m a)^3/6\pi^2$. The assumption of exponential decay for the long-time behavior of the spin fluctuations, which is implicit in Eqs. (1) and (2), is less critical. We expect it to be valid for $q \lesssim k_c$; for larger values of q some error is introduced. However the fluctuations with $q \gg k_c$ make comparatively small contributions to the integrals.[14] In summary, we have shown that a theory which treats self-consistently the lowest order processes for the damping of the spin fluctuations (decay into two modes) can account for the temperature dependence, wave-vector dependence, and overall magnitude of the decay rates reported for $RbMnF_3$.

We would like to thank the authors of Ref. 1 for providing us with a copy of their work prior to publication. We would also like to thank Professor C. C. Lin and Mr. J. P. Mahoney for measurements of the susceptibility.

†Work supported in part by the National Science Foundation.

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