In summary, we have made a diagrammatic analysis of the method of CBF to fourth order in energy. We find that the zeroth-order CBF perturbation theory using a Jastrow correlating factor accounts for three orders exactly, and partially sums terms to all higher orders, in a general pattern not yet identified. The simpliest second-order perturbation correction in the CBF completes the fourth order, and makes further partial summations of higher order diagrams. Terms which are particularly susceptible to summation by the CBF are ring and ladder diagrams. We are currently attempting to generalize the above results as well as extending this analysis to Fermi system.

Details of this analysis are lengthy, and will be reported elsewhere.

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[5]Jackson and Feenberg, Ref. 2.

[6]Campbell and Feenberg, Ref. 2.

[7]The perturbative correction from such a diagram was considered by Davison and Feenberg for liquid He^4 (see Ref. 2).

PASSAGE OF AN INTENSE RELATIVISTIC ELECTRON BEAM THROUGH A CUSPED MAGNETIC FIELD

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We present results of the injection of a high-intensity, high-current, relativistic electron beam into a cusped magnetic field. 10^{14} electrons are passed through the cusp. Prelimary observations indicate that these electrons produce a "cloud" of relativistic electrons gyrating in a magnetic field and having a small ratio of translational to rotational energies.

In recent years there has been interest in producing a dense "cloud" of relativistic electrons gyrating in a magnetic field with a small ratio of translational to rotational energies. It has been suggested that such a cloud of electrons may be useful for nuclear fusion[1] and particle accelerators.[2]

By injecting a relativistic electron beam perpendicular to a magnetic field B, such a cloud of relativistic electrons can be produced. The electrons orbit in circular trajectories with the Larmor radius $\rho_{Le} = V_\perp/\omega_{ce}$, where $V_\perp$ is the perpendicular component of the electron velocity (with respect to the magnetic field) and $\omega_{ce} = eB/\gamma mc$.

A different approach was suggested[3] in 1963 for the production of a rotational cloud of relativistic electrons. Nonadiabatic orbit calculations showed that charged particles injected into a cusped magnetic field along its axis will pass the magnetic trap provided that

$$\rho_{Le}(V) = V/\omega_{ce} \gtrsim r, \qquad (1)$$

where V is the velocity and r the radial distance of the injected particles from the axis of the cusp. A very thin annular beam of charged particles (e.g., electrons) injected with a velocity V_0 parallel and symmetrical to the cusp axis will emerge with velocity components

$$V_\parallel = V_0\{1-[r_0/\rho_{Le}(V_0)]^2\}^{1/2} \qquad (2)$$

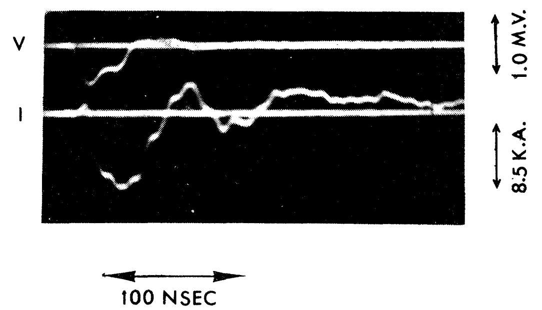

FIG. 1. Typical diode voltage and current traces.

and

$$V_\perp = V_0 r_0/\rho_{Le}(V_0), \tag{3}$$

where $V_\parallel$ and $V_\perp$ are the translational and rotational velocity components of the electrons, respectively, and r_0 is the radius of the annular beam. If the magnetic field outside the cusp is longitudinal and homogeneous, the emerging beam will have the same radius, r_0, and a single electron will describe a helix with a pitch

$$h = 2\pi\rho_{Le}(V_0)\{1-[r_0/\rho_{Le}(V_0)]^2\}^{1/2}. \tag{4}$$

The closer r_0 comes to ρ_{Le}, the tighter the helix will be. It has also been shown[4] that the injection of a low-intensity electron beam into a cusped field is in agreement with these calculations.

We report here some preliminary results of an experiment in which an intense beam of relativistic electrons has been injected into a cusped magnetic field. The relativistic beam is produced by the Cornell relativistic electron beam accelerator described elsewhere.[5]

A voltage pulse of ~5×10^5 V is applied to a field-emission diode[6] producing a beam current of ~10 kA. The beam leaving the diode has an annular shape with an outer radius $r_{out}=0.65$ cm and inner radius $r_{in}=0.35$ cm. The voltage pulse on the diode lasts for approximately 50 nsec. Figure 1 gives typical voltage and current traces.

The diode and the drift tube through which the beam propagates are evacuated to a common base pressure of -2×10^{-4} Torr. A quasi-dc magnetic field with an intensity that can reach 10 kG is applied to the diode and the drift-tube region. Figure 2(a) shows the magnetic coil assembly and the drift tube through which the beam is propagating.

Figure 2(b) shows a time-integrated-light photograph taken when the relativistic electron beam passes through the longitudinal, homogeneous magnetized medium. Calorimeter measurements of the beam energy taken 60 cm "downstream"

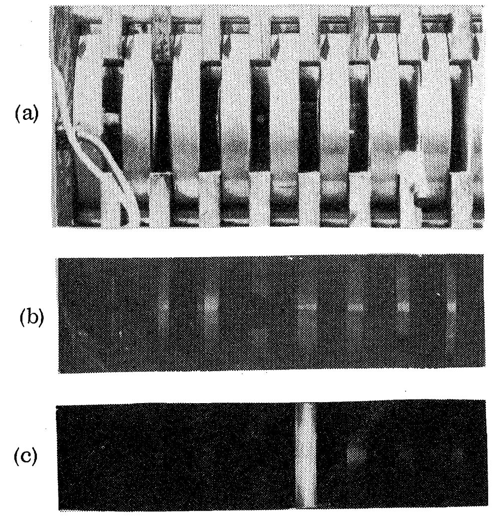

FIG. 2. (a) Magnetic field coil and drift-tube assembly. (b) Electron beam propagating in homogeneous magnetic field. (c) Electron beam propagating through a cusped magnetic field. (The diode is on the left side of these photographs.)

showed that ~95% of the calculated input energy, $\int VIdt$, into the diode was being propagated. The energy in the relativistic beam was found to be ~100 J.

A cusped magnetic field was produced by reversing the direction of the current in half of the magnetic field coil assembly. Figure 2(c) shows a time-integrated-light photograph of the drift-tube region taken while the beam passed through the cusped field.

In order to verify Eq. (3), the magnetic field was raised from 3 to 10 kG and hence ρ_{Le} changed from being greater to being smaller than r_{out}. Figure 3 shows the dependence of the energy of the beam transmitted through the cusp versus magnetic field intensity. The intensity of the magnetic field at which a drop in the energy transmitted is observed is equal to ~5 kG; this

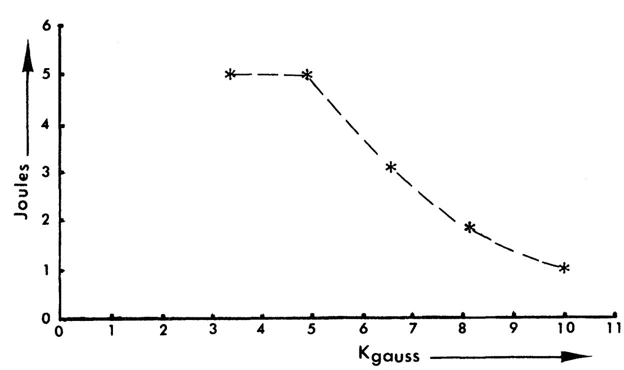

FIG. 3. Energy of electron beam transmitted through the cusp versus magnetic field intensity.

corresponds to $\rho_{Le} \approx 0.6$ cm. In that case the Larmor radius is equal to r_{out}, the outer radius of the beam. The damage pattern on a Lucite plate and photography of light emitted from a thin scintillator plate, employed as a beam target, show that the annular character of the beam has not changed after passing through the cusp. It is interesting to note that from the energy detected by the calorimeter one can determine that $\sim 10^{14}$ electrons successfully passed the cusp region.

Although we do not have conclusive evidence that the electrons describe a helical trajectory after leaving the cusp, a qualitative indication may be obtained from Fig. 2(c). The beam enters the cusp with an energy of 100 J (at the left of the photograph) and emerges with an energy of 5 J (at the right). But the light recorded is more intense at the right than at the left. We can thus conclude that the beam emerged from the cusp with a reduced velocity component parallel to the cusp axis compared with the parallel velocity before the cusp. As no serious energy losses are involved in passage through a magnetic field, the assumption that the decrease in translational energy is followed by an increase in rotational energy is justified.

The seemingly low efficiency in the number of particles transmitted can be a result of the following:

(1) Ripples in the beam created in the transition region. It has been shown[7] that in order to minimize these ripples the length of the cusp L should fulfill the relation

$$L < \lambda = 2\pi V_0/\omega_{ce} = 2\pi \rho_{Le}(V_0), \tag{5}$$

where λ is the wavelength of the ripples. In the experiment described above, relation (5) was not fulfilled and L was of the order of a few times λ. However, it is possible to reduce L in order to evaluate this interpretation.

(2) Lack of self-consistency in the previous calculations[3] used for interpreting the experiments. Space-charge potentials, such as those to be expected in the vicinity of the cusp, are neglected. This assumption is satisfactory for high-energy, low-current beams. In these experiments the current is sufficiently high to make this approximation inadequate. However the low efficiency of particle transmission could also be explained by the buildup of a potential hill in the vicinity of the cusp. Such an effect may be reduced by increasing the particle energy.

Helpful discussions with Professor N. Rostoker and Professor S. Linke are gratefully acknowledged.

Note added in proof.—In a recent similar experiment, x rays and magnetic-probe measurements confirmed that the electrons, while passing the cusped magnetic field, were converting their directional energies to transverse energies. A more detailed result will be presented in a forthcoming paper.

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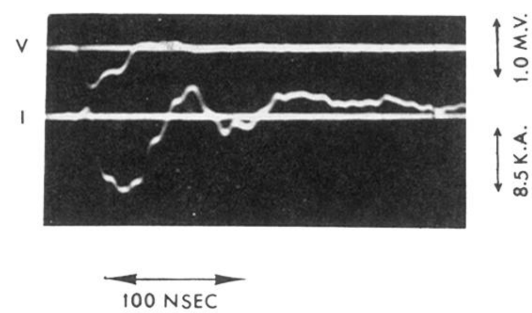

FIG. 1. Typical diode voltage and current traces.

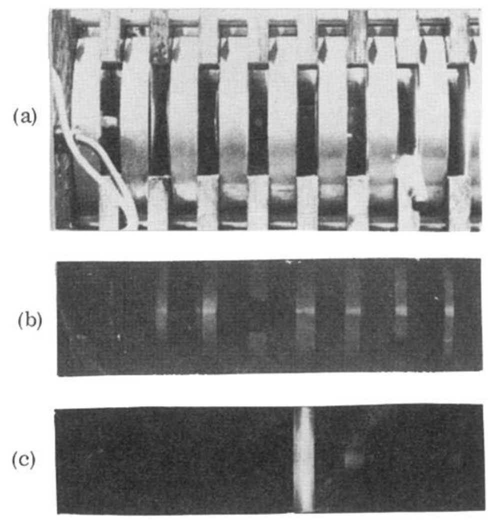

FIG. 2. (a) Magnetic field coil and drift-tube assembly. (b) Electron beam propagating in homogeneous magnetic field. (c) Electron beam propagating through a cusped magnetic field. (The diode is on the left side of these photographs.)