Table I. S_{eff} at $T=0$ and $x_0=0$ for various values of U. The critical potential, below which moment formation does not occur, is $\pi\Delta/A^2$.

U	S_{eff} (parquet)	S_{eff} (nonparquet)
$1.5U_c$	0.113	0.100
$2U_c$	0.162	0.145
$3U_c$	0.211	0.192
$10U_c$	0.270	0.253
∞	0.299	0.281

$S=\frac{1}{2}$ expected for the singly occupied localized s state treated in this model. In addition, the density of states, while closer to that expected for $U \gg \Delta$ than the density of states of the nonparquet equations, still has much too strong a background in the region between the two peaks. Hence the parquet-diagram technique of taking into account the influence of paramagnon exchange on local moment formation, while improving some of the features of the original model, is not completely satisfactory quantitatively. However, we have not taken into consideration the $\bar{G}$ terms of Eq. (4) nor the effects of a finite frequency width for the paramagnons. These problems are presently under study.

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†Present address.

[1]H. Suhl, Phys. Rev. Letters 19, 442, 735 (E) (1967).

[2]P. W. Anderson, Phys. Rev. 124, 41 (1961).

[3]P. A. Wolff, Phys. Rev. 124, 1030 (1961).

[4]M. J. Levine and H. Suhl, Phys. Rev. 171, 567 (1968); M. J. Levine, T. V. Ramakrishnan, and R. A. Weiner, Phys. Rev. Letters 20, 1370 (1968).

[5]D. R. Hamann, Phys. Rev. 186, 549 (1969). Several serious failings of this model without paramagnon exchange as a satisfactory explanation of the Kondo effect are pointed out in this reference.

[6]We shall use the terminology of the Anderson model in this Letter.

[7]M. T. Béal-Monod and D. L. Mills, Phys. Rev. Letters 24, 225 (1970).

[8]A. A. Abrikosov, Physics 2, 5 (1965); B. Roulet, J. Gavoret, and P. Nozières, Phys. Rev. 178, 1072 (1969).

[9]We use the antisymmetrized formulation of many-body diagrams, as in A. A. Abrikosov, L. P. Gor'kov, and I.-E. Dzyaloshinski, in *Methods of Quantum Field Theory in Statistical Physics*, edited by R. Silverman (Prentice-Hall, Englewood Cliffs, N. J., 1963).

[10]We use the imaginary frequencies of finite-temperature many-body theory, $i\omega_\nu = 2\nu\pi i/\beta$ for bosons, $(2\nu+1)\pi i/\beta$ for fermions, $\beta = 1/kT$, so that a Kronecker delta represents an infinitely narrow mode. This approximation is clearly best at high temperatures where the frequencies are widely spaced.

CHARGE EXCHANGE PART OF THE EFFECTIVE TWO-BODY INTERACTION*

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From an essentially model-independent analysis of the ratio of cross sections for the reactions ^{7}Li$(p,n_0)^7$Be$_{\text{g.s.}}$, ^{7}Li$(p,n_1)^7$Be(0.43 MeV), and ^{6}Li$(p,n_0)^6$Be$_{\text{g.s.}}$, it is concluded that $V_{\sigma\tau}/V_\tau = 0.66 \pm 0.08$ and is independent of bombarding energy from 10-20 MeV.

Recently, there have been several attempts to obtain quantitative estimates of the effective two-body force required by a microscopic description of nuclear reactions.[1,2] In particular, the charge-exchange part of this effective force has received a great deal of attention.[3-6] Unfortunately all attempts to obtain quantitative extimates of the spin and charge-exchange force ($V_{\sigma\tau}$) are for light nuclei ($A \lesssim 27$) where neither the nuclear structure information nor optical parameters are particularly well known. However, from a comparison of the ^{7}Li$(p,n_0)^7$Be$_{\text{g.s.}}$ and ^{7}Li$(p,n_1)^7$Be(0.43 MeV) cross sections one can obtain the ratio of the charge and spin-exchange to the charge-exchange part of the effective two-body force. In this case the data can be analyzed using a microscopic description of the reaction where, to first order (monopole approximation), the nuclear structure information is experimentally determined from beta decay. Since the final states have the same isospin and are sufficiently close in energy (0.43 MeV), there is almost no uncertainty introduced into the theoretical analysis of the ratio of cross sections due to optical parameter uncertainties. Hence, the deduced results are essentially model independent.

The ^{7}Li$(p,n_0)^7$Be$_{\text{g.s.}}$, ^{7}Li$(p,n_1)^7$Be(0.43 MeV), and ^{6}Li$(p,n_0)^6$Be$_{\text{g.s.}}$ cross sections were measured for bombarding energies between 9.8 and 13.9 MeV using standard time-of-flight techniques.[7] At proton energies above 14 MeV, the ground-state and first-excited–state neutrons from ^{7}Li could not be resolved; therefore the sum of these cross sections along with the ^{6}Li$(p,n_0)^6$Be$_{\text{g.s.}}$ cross sections were measured up to 19.6 MeV bombarding energy. In Fig. 1 angular distribution data for the lowest (9.8 MeV) and highest bombarding energy (19.6 MeV) are shown. One notes that the difference in angular distributions in Fig. 1(a) is characteristic of the large difference in momentum transfer for this low bombarding energy. There is some indication of a small broad resonance in the n_1 cross section around 9 MeV based on lower energy measurements[8,9] and hence the 9.8-MeV data probably contain a substantial contribution (~25%) from compound-nucleus decay. The ratio of the ^{7}Li$(p,n_1)^7$Be(0.43 MeV) cross section to the ^{7}Li$(p,n_0)^7$Be$_{\text{g.s.}}$ cross section is shown in Fig. 2(a). The ratio of the ^{7}Li$(p,n_1)^7$Be(0.43) cross section to the ^{6}Li$(p,n_0)^6$Be$_{\text{g.s.}}$ cross section, corrected for the difference in phase space, is shown in Fig. 2(b). The ratio of the ^{7}Li(p,n) cross sections (ground state plus first-excited state) to the ^{6}Li$(p,n_0)^6$Be$_{\text{g.s.}}$ cross section is shown in Fig. 2(c) where a phase-space correction has been applied due to the difference in Q values.

Several microscopic treatments of direct in-

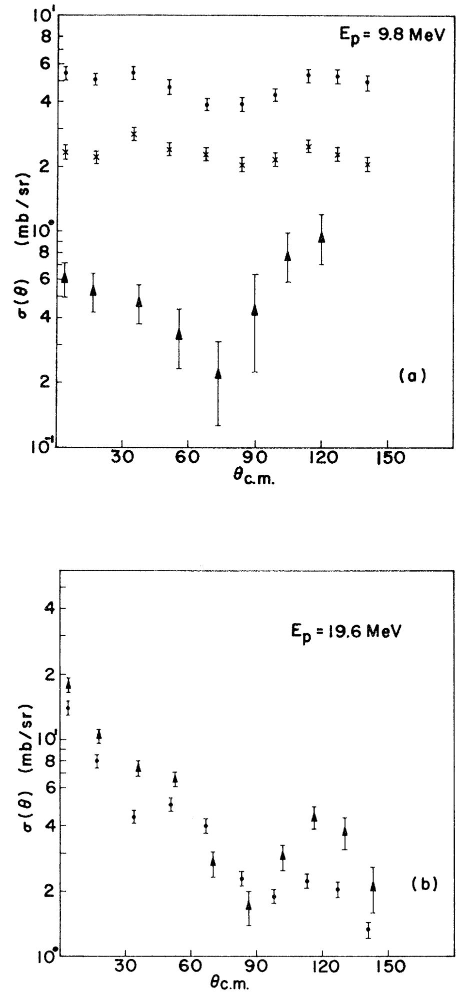

FIG. 1. Angular distributions for the Li(p,n) reactions. (a) The solid circles are for the reaction ^{7}Li$(p,n_0)^7$Be$_{\text{g.s.}}$, the crosses for the reaction ^{7}Li$(p,n_1)^7$Be (0.43 MeV), and the triangles for the reaction ^{6}Li$(p,n_0)^6$Be$_{\text{g.s.}}$. (b) The ^{7}Li$(p,n_0+n_1)^7$Be (g.s.+0.43 MeV) cross sections are shown as solid circles while the ^{6}Li$(p,n_0)^6$Be$_{\text{g.s.}}$ cross sections (multiplied by 10) are shown as triangles.

elastic scattering are available[10-12] and here we only present a summary of the main results applied to the (p,n) reaction. In L-S coupling the differential cross section is given by the expression

$$\frac{d\sigma}{d\Omega}=\left(\frac{2m}{4\pi\hbar^2}\right)^2\frac{k_f}{k_i}\frac{1}{2}\frac{1}{(2J_i+1)}\sum_{II'LM}(2I+1)\left|\sum_{l_1l_2}D_{l_1l_2}(II'L)F^{l_1l_2}(k_f')\right|^2, \qquad (1)$$

where

$$D_{l_1l_2}(II'L)=4\langle l_2\|Y_L\|l_1\rangle C(T_iT_f1;\,T_i,\,1-T_i,\,1)S(J_iJ_fI(LI');\,T_iT_f\tau;\,l_1l_2)[\delta_{I'1}V_{\sigma\tau}+\delta_{I'0}V_\tau]. \qquad (2)$$

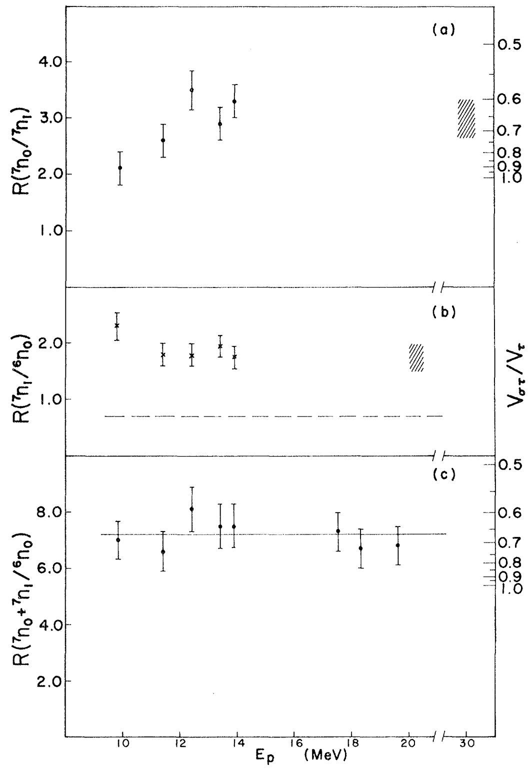

FIG. 2. The ratios of Li(p,n) cross sections are shown as a function of bombarding energy. Ratios involving ^{6}Li cross sections have been corrected for the differences in phase space. The deduced ratio of force constants is shown on the right-hand scale. (a) The ratio of the ^{7}Li$(p,n_0)^7$Be$_{g.s.}$ to ^{7}Li$(p,n_1)^7$Be (0.43 MeV) cross sections. (b) The ratio of the ^{7}Li$(p,n_1)^7$Be (0.43 MeV) to ^{6}Li$(p,n_0)^6$Be$_{g.s.}$ cross sections. The dashed line at 0.7 indicates the expected ratio. (c) The ratio of ^{7}Li$(p,n_0+n_1)^7$Be (g.s. + 0.43 MeV) to ^{6}Li$(p,n_0)^6$Be$_{g.s.}$ cross sections. The solid line indicates the average value.

The spectroscopic information is contained in S, and we have assumed that the two-body charge-exchange force can be written as

$$V_{ij}=(\vec{\tau}_i\cdot\vec{\tau}_j)[V_\tau+(\vec{\sigma}_i\cdot\vec{\sigma}_j)V_{\sigma\tau}]f(r_{ij}). \qquad (3)$$

In the monopole approximation ($L=0$) the (p,n) cross section can be further simplified to the form

$$\frac{d\sigma}{d\Omega}=[(N-Z)V_\tau^2\delta_{I'0}+\langle\sigma\rangle^2V_{\sigma\tau}^2\delta_{I'1}]\sigma_0(\theta), \qquad (4)$$

where the nuclear structure information is contained in $\langle\sigma\rangle^2$ and the overlap of wave functions (radial integrals) and distorted-wave effects are lumped into $\sigma_0(\theta)$. The form of Eq. (4) is more general than implied by our derivation. The derivation does assume a central effective two-nucleon interaction. Tensor charge-exchange forces are almost certainly present.[5,13-15] However, they have been shown[13] not to be important for the (p,n) reaction in light nuclei at low energies unless the central transition is abnormally small, such as in the reaction ^{14}C$(p,n_0)^{14}$N$_{g.s.}$. The presence of different ranges for our forces in Eq. (3) would imply a normalization of V_τ with respect to $V_{\sigma\tau}$ proportional to the cube of the appropriate range and produces a slight deterioration of the equivalence of the angular distributions implied by Eq. (4). The inclusion of space exchange[16] also does not affect the form of Eq. (4) and implies a renormalization of the relative force constants only if the ranges are different.

To analyze our data we require nuclear structure information. From the measured ft values from beta decay we can obtain the relevant nuclear structure information $\langle\sigma\rangle^2$ as follows[17,18]:

$$ft=\frac{6120}{D_F\delta_{I'0}+1.4\langle\sigma\rangle^2D_{GT}\delta_{I'1}}, \qquad (5)$$

where $D_F=D_{GT}=1$ in our case and the 1.4 represents the square of the ratio of coupling constants. The ft values from the literature,[19] the deduced $\langle\sigma\rangle^2$, and the implied $\langle\sigma\rangle^2$ for the (p,n) reaction are listed in Table I. One could have calculated $\langle\sigma\rangle^2$ directly from the wave functions. Using the coefficients of fractional parentage in L-S coupling[20] we have calculated the appropriate $\langle\sigma\rangle^2$ which are also listed for comparison in Table I.

From Table I and Eq. (4) we obtain the ratio of the ^{7}Li$(p,n_0)^7$Be$_{g.s.}$ cross section to the ^{7}Li$(p,n_1)^7$Be(0.43 MeV) cross section, $R(^7n_0/n_1)$, as

$$R(^7n_0/n_1)=\frac{V_\tau^2+1.47V_{\sigma\tau}^2}{1.27V_{\sigma\tau}^2}. \qquad (6)$$

Table I. Nuclear structure information.

Beta decay: Observed transition	ft	$\langle\sigma\rangle^2$	(p,n) reaction: Observed reaction	$\langle\sigma\rangle^2$	$\langle\sigma\rangle_{LS}^2$
^{7}Be$_{g.s.}\to{}^7$Li$_{g.s.}$	2000	1.47	^{7}Li$(p,n_0)^7$Be$_{g.s.}$	1.47	1.67
^{7}Be$_{g.s.}\to{}^7$Li(0.48)	3450	1.27	^{7}Li$(p,n_1)^7$Be(0.43)	1.27	1.33
^{6}He$_{g.s.}\to{}^6$Li$_{g.s.}$	802	5.45	^{6}Li$(p,n_0)^6$Be$_{g.s.}$	1.82	2.00

In Fig. 2(a) the values of the ratio $V_{\sigma\tau}/V_\tau$ are shown in the right-hand scale corresponding to the cross-section ratios indicated on the left-hand scale. With the exception of the point at 9.8 MeV (which is suspect due to the possibility of compound nucleus contributions) the ratio of $V_{\sigma\tau}/V_\tau$ lies between 0.58 and 0.75.

One also has a very simple prediction for the ratio of cross sections for the first excited state of ^{7}Be as compared with the ground state of ^{6}Be as

$$R({}^7n_1/{}^6n_0)=0.7K. \tag{7}$$

The expectation that this ratio should be independent of bombarding energy is well borne out by experiment for energies greater than 10 MeV [see Fig. 2(b)]. The cross-hatched area represents an estimate of the ratio at 20 MeV obtained by extrapolating the $^7n_1$ cross sections measured at higher energies ($23 \leq E_p \leq 53$ MeV) by Locard et al.[5] The K is required because our neglect of optical parameters is no longer justified in this case since in addition to the large difference in Q value we have different isospins in the initial and final states and hence quite possibly different optical parameters. There is another difficulty in that $\langle\sigma\rangle^2$ was obtained from the $^6\mathrm{He}\to{}^6\mathrm{Li}$ beta decay, but ^{6}Be is unstable against particle emission and hence the overlap of the wave functions will not be as large as estimated from beta decay. Our observed value of $K=2.6$ is almost a factor of 2 larger than one might have estimated on the basis of optical parameters alone but is indeed energy independent.

We can now obtain the ratio of the $^7\mathrm{Li}(p,n)^7\mathrm{Be}$ (g.s. +0.43 MeV) cross section to the first-excited-state cross section through the use of Eq. (7):

$$R\left(\frac{{}^7n_0+{}^7n_1}{{}^7n_1}\right)=R\left(\frac{{}^7n_0+{}^7n_1}{{}^6n_0}\right)\frac{1}{0.7K}=\frac{V_\tau^2+2.74V_{\sigma\tau}^2}{1.27V_{\sigma\tau}^2}. \tag{8}$$

In Fig. 2(c) the ratio $({}^7n_0+{}^7n_1)/{}^6n_0$ is plotted (left-hand scale) versus bombarding energy from 10 to 20 MeV. It is quite clear from these data that the measured ratio of cross sections is essentially energy independent. To estimate the sensitivity of the observed ratio of cross sections to the ratio of the force constants we use the empirically determined value of K to obtain the right-hand scale of Fig. 2(c). As an additional check on our procedure we have plotted the ratio of the zero-degree cross sections obtained by Clough et al.[6] at 30 MeV for the reactions $^7\mathrm{Li}(p,n_0)^7\mathrm{Be}_{\mathrm{g.s.}}$ and $^7\mathrm{Li}(p,n_1)^7\mathrm{Be}$(0.43 MeV). This is shown as the cross-hatched area in Fig. 2(a). Although it is only a measurement at a single angle, at higher energies normalizing to the forward maximum of a (p,n) angular distribution gives the same answer as normalizing to the integrated cross section. We can only conclude that this measurement is compatible with our deduced ratio of force constants.

In order to estimate the magnitude of higher multipole contributions to the cross sections we must now rely on our model calculations. Using L-S wave functions and calculating only the direct terms implied by Eqs. (1), (2), and (3) we find the $L=2$ contribution to the cross section is less than 1% of the monopole contribution. This, however, is a lower limit. If one takes into account the enhancement of the $L=2$ multipole due to space-exchange and tensor-force contributions we obtain a more realistic estimate of 5%. Assuming extreme conditions, the ratio of cross sections would be altered by not more than 9%, which is of the same order as the statistical uncertainties that we quote for the cross-section data. Thus, we conclude that our use of the monopole approximation is justified.

From the analysis of the $^7\mathrm{Li}(p,n)^7\mathrm{Be}$ (g.s. +0.43 MeV) and $^6\mathrm{Li}(p,n_0)^6\mathrm{Be}_{\mathrm{g.s.}}$ cross sections we have established that the ratio of the spin and charge-exchange to charge-exchange force constants of the effective two-body force is essentially energy independent for bombarding energies from 10 to 20 MeV. From a comparison of the $^7\mathrm{Li}(p,n_0)^7\mathrm{Be}_{\mathrm{g.s.}}$ and $^7\mathrm{Li}(p,n_1)^7\mathrm{Be}$(0.43 MeV) cross sections we obtain the value for the ratio of the force constants $V_{\sigma\tau}/V_\tau=0.66\pm0.08$. Both the energy dependence and the magnitude of the ratio are in excellent agreement with the values of the effective interaction[2,21] obtained from the Kallio-Kolltveit two-body force. It looks hopeful that the (p,n) reaction, at least in a phenomenological sense, may now be used as a spectroscopic tool for light nuclei for strong transitions.

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[1]For an excellent review of this approach see G. R. Satchler, Nucl. Phys. A95, 1 (1967).

[2]F. Petrovich, H. McManus, V. A. Madsen, and J. Atkinson, Phys. Rev. Letters 22, 895 (1969).

[3]J. D. Anderson, S. D. Bloom, C. Wong, W. F. Hornyak, and V. A. Madsen, Phys. Rev. 177, 1416 (1969).

[4]G. C. Ball and J. Cerny, Phys. Rev. 177, 1466 (1969).
[5]S. M. Austin, P. J. Locard, W. Benenson, and G. M. Crawley, Phys. Rev. 176, 1227 (1968); P. J. Locard, S. M. Austin, and W. Benenson, Phys. Rev. Letters 19, 1141 (1967).
[6]A. S. Clough, C. J. Batty, B. E. Bonner, C. Tschalär, L. E. Williams, and E. Friedman, Nucl. Phys. A137, 222 (1969).
[7]B. D. Walker, J. D. Anderson, J. W. McClure, and C. Wong, Nucl. Instr. Methods 29, 333 (1964).
[8]R. R. Borchers and C. H. Poppe, Nucl. Phys. 129, 2679 (1963).
[9]J. D. Anderson and C. Wong, unpublished.
[10]N. K. Glendenning, Phys. Rev. 144, 829 (1966).
[11]G. R. Satchler, Nucl. Phys. 77, 481 (1966).
[12]V. A. Madsen, Nucl. Phys. 80, 177 (1966).
[13]F. A. Schmittroth, thesis, Oregon State University, 1968 (unpublished).
[14]C. Wong, J. D. Anderson, J. McClure, B. Pohl, V. A. Madsen, and F. Schmittroth, Phys. Rev. 160, 769 (1967), and J. D. Anderson, V. A. Madsen, F. A. Schmittroth, and C. Wong, to be published.
[15]E. Rost and P. D. Kunz, Bull. Am. Phys. Soc. 14, 71 (1969), and to be published.
[16]J. Atkinson and V. A. Madsen, Phys. Rev. 1, 1377 (1970).
[17]S. D. Bloom, *Isobaric Spin in Nuclear Physics*, edited by J. D. Fox and D. Robson (Academic, New York, 1966), p. 895.
[18]E. J. Konopinski, *Theory of Beta Radioactivity* (Oxford Univ., Oxford, England, 1966), p. 403.
[19]T. Lauritsen and F. Ajzenberg-Selove, Nucl Phys. 78, 1 (1966).
[20]H. A. Jahn and H. van Wieringer, Proc. Roy. Soc. (London), Ser. A 209, 502 (1951).
[21]F. Petrovich, private communication.

PRODUCTION OF SINGLE NEGATIVE PIONS FROM DEUTERIUM WITH POLARIZED PHOTONS*†

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The asymmetry $A=(d\sigma_\perp - d\sigma_\parallel)/(d\sigma_\perp + d\sigma_\parallel)$ of the differential cross section for the reaction $\gamma d \to \pi^- pp$ has been studied with linearly polarized photons of 3.0 GeV at squared four-momentum-transfers between 0.15 and 2.0 $(\mathrm{GeV}/c)^2$. The asymmetry was found to be positive at $-t$ values below 0.3 $(\mathrm{GeV}/c)^2$, dipping to negative values between 0.4 and 0.6 $(\mathrm{GeV}/c)^2$, and then rising again to positive values above 0.7 $(\mathrm{GeV}/c)^2$.

Experiments on single $\pi^\pm$ production from nucleons with unpolarized photons[1] have shown that over a large range of energy and momentum transfer the π^-/π^+ cross-section ratio $R = d\sigma(\gamma n \to \pi^- p)/d\sigma(\gamma p \to \pi^+ n)$ is appreciably smaller than one, indicating a strong interference between the isovector and isoscalar photon amplitudes. In a t-channel exchange picture, this implies the interference between exchange amplitudes of opposite G parity. The interference can occur only between exchanges of the same spin and parity, for instance between exchange amplitudes of ρ $(G=+1)$ and A_2 $(G=-1)$, which have natural spin and parity, $P(-1)^J=+1$, or between B $(G=+1)$ and π $(G=-1)$, which have unnatural spin and parity, $P(-1)^J=-1$.

Experiments with linearly polarized photons can separate the natural from the unnatural spin and parity exchanges, since at high energy and small momentum transfers photons that are linearly polarized perpendicular (parallel) to the pion production plane contribute only to the natural (unnatural) spin and parity exchange mode.[2] Data from such experiments provide a stringent test of various theoretical models for the photoproduction of pions.[3] Combined π^+ and π^- data, where available,[4,5] permit one to determine the magnitude of the $G=\pm1$ interference term separately in each of the spin and parity exchange modes and may aid in the identification of the contributing amplitudes.[6] Previous polarized-beam experiments on π^- production covered small $-t$ values up to 0.6 $(\mathrm{GeV}/c)^2$; this experiment extends the range of four-momentum transfers in π^- production up to $-t=2.0$ $(\mathrm{GeV}/c)^2$.

We have studied the reaction $\gamma d \to \pi^- pp$ with linearly polarized photons of energy 3.0 GeV at squared four-momentum-transfers, $-t$, between 0.15 and 2.0 $(\mathrm{GeV}/c)^2$. Coincidence yields were measured between the pion and one of the recoil protons using photons polarized both perpendicular ($\perp$) and parallel ($\parallel$) to the pion production plane. From these measurements, differential cross-section ratios $(d\sigma_\perp/d\sigma_\parallel)_{\gamma d \to \pi^- pp}$, and asymmetries $A^- = [(d\sigma_\perp - d\sigma_\parallel)/(d\sigma_\perp + d\sigma_\parallel)]_{\gamma d \to \pi^- pp}$, were determined.

Electrons of 6.0 GeV from the Cambridge Electron Accelerator (CEA) incident on a suitably oriented diamond monocrystal produced a bremsstrahlung beam with the characteristic polarized-