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INTERACTION BETWEEN ELECTRONS AND MOVING DISLOCATIONS IN SUPERCONDUCTORS

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The interaction between a moving dislocation and the electrons in a superconductor is treated theoretically. The results are in qualitative agreement with recent measurements of flow stress in superconductors.

In recent years a number of workers have investigated the motion of dislocations in superconductors. It has been found that the flow stress[1-3] and electronic drag coefficient[4-7] both decrease as the material becomes superconducting. The purpose of this paper is to present a general theoretical treatment of the interaction between a moving dislocation and the electrons in a superconductor. Most workers have attempted to analyze their data using a formula suggested by Mason[8] by analogy with the BCS theory of ultrasonic attenuation,[9]

$$\Gamma = 2f(\Delta) = 2[\exp(\Delta/kT) + 1]^{-1}, \tag{1}$$

where Γ is the ratio of the electronic drag on the dislocation in the superconducting state to that in the normal state and Δ is the BCS energy-gap function. We will show, however, that this approximation is valid only for relatively slow dislocation velocities, $v_d \lesssim 10$ to 10^2 cm/sec. However, several recent experiments[10-12] made at stresses greater than or of the order of the macroscopic yield stress have directly measured dislocation velocities in the range from 5×10^2 to 5×10^4 cm/sec. Additionally, several indirect determinations of dislocation velocities in metals undergoing plastic deformation give values in the range from 10^3 to 5×10^4 cm/sec.[13-15] For such velocities, the frequencies associated with the dislocation wave packet can become greater than $2\Delta/\hbar$, and it is necessary to consider not only the scattering of Bogoliubov quasiparticles but also processes in which two quasiparticles are simultaneously created or destroyed with the emission or absorption of a quantum of energy by the dislocation.

The theory is similar to that used to calculate the attenuation of high-frequency phonons in superconductors.[16] The interaction Hamiltonian in the superconducting state is given by

$$H_I = H_I^{(1)} + H_I^{(2)}, \tag{2}$$

where

$$H_I^{(1)} = \sum_{\vec q}\sum_{\vec k,\sigma}\{V_{D,\vec q}\exp(-i\omega_{\vec q}t)\gamma_{\vec k+\vec q,\sigma}^\dagger\gamma_{\vec k,\sigma} + V_{D,-\vec q}\exp(i\omega_{\vec q}t)\gamma_{\vec k,\sigma}^\dagger\gamma_{\vec k+\vec q,\sigma}\}(u_{\vec k}u_{\vec k+\vec q} - v_{\vec k}v_{\vec k+\vec q}) \tag{3}$$

and

$$H_I^{(2)} = \sum_{\vec q}\sum_{\vec k}\{V_{D,\vec q}\exp(-i\omega_{\vec q}t)(\gamma_{\vec k+\vec q,\uparrow}^\dagger\gamma_{-\vec k,\downarrow}^\dagger + \gamma_{-(\vec k+\vec q),\downarrow}\gamma_{\vec k,\uparrow}) + V_{D,-\vec q}\exp(i\omega_{\vec q}t)(\gamma_{\vec k,\uparrow}^\dagger\gamma_{-(\vec k+\vec q),\downarrow}^\dagger + \gamma_{-\vec k,\downarrow}\gamma_{\vec k+\vec q,\uparrow})\}(u_{\vec k+\vec q}v_{\vec k} + u_{\vec k}v_{\vec k+\vec q}). \tag{4}$$

$V_{D,\vec q}$ is the Fourier transform of the dislocation deformation potential, and is given approximately by[13]

$$V_{D,\vec q} = i\left(\frac{2E_F}{3}\right)b\mu\frac{\sin\varphi_{\vec q}}{q}, \tag{5}$$

where E_F is the Fermi energy, b is magnitude of the Burgers vector, $\varphi_{\vec{q}}$ is the angle between $\vec{q}$ and the Burgers vector,

$$\mu = [1 + 2(v_2/v_1)^2], \tag{6}$$

and v_2 and v_1 are the transverse and longitudinal sound-wave velocities. The frequency $\omega_{\vec{q}}$ is given by

$$\omega_{\vec{q}} = qv_d \cos\varphi_{\vec{q}} \tag{7}$$

and we have written Eqs. (3) and (4) such that the sums over $\vec{q}$ extend only over the first and fourth quadrants in the two-dimensional $\vec{q}$ space, so that $\omega_{\vec{q}}$ is always positive. γ^+ and γ are the quasiparticle creation and annihilation operators and $u_{\vec{k}}$ and $v_{\vec{k}}$ are defined by[17,18]

$$u_{\vec{k}} = \{\tfrac{1}{2}(1 + \xi_{\vec{k}}/E_{\vec{k}})\}^{1/2}, \tag{8}$$

$$v_{\vec{k}} = \{\tfrac{1}{2}(1 - \xi_{\vec{k}}/E_{\vec{k}})\}^{1/2}, \tag{9}$$

where $\xi_{\vec{k}} = \hbar^2k^2/2m - E_F$ and $E_{\vec{k}}$ is the energy required to create a quasiparticle of momentum $\vec{k}$,

$$E_{\vec{k}} = (\xi_{\vec{k}}^2 + \Delta^2)^{1/2}. \tag{10}$$

$H_I^{(1)}$ corresponds to processes in which a quasiparticle is scattered from a state of momentum $\vec{k}$ to $\vec{k}+\vec{q}$ with absorption of momentum $\vec{q}$ and energy $\hbar\omega_{\vec{q}}$ from the dislocation (and the reverse process). $H_I^{(2)}$ contributes to the attenuation only when $\omega_{\vec{q}} > 2\Delta/\hbar$ and describes processes in which a quantum of dislocation energy $\hbar\omega_{\vec{q}}$ decays into two quasiparticles of net momentum $\vec{q}$ (and the reverse processes).

The rate of energy dissipation per unit length of dislocation by these processes is readily calculated using first-order perturbation theory. Following the techniques of Privorotskii[16] and others,[17,18] one finds

$$\left(\frac{dW_d}{dt}\right)_s = \left(\frac{dW_d}{dt}\right)^{(1)} + \left(\frac{dW_d}{dt}\right)^{(2)}$$

with

$$\left(\frac{dW_d}{dt}\right)^{(1)} = -\sum_{\vec{q}} 2B_{\vec{q}}\hbar\omega_{\vec{q}}\alpha_1(\hbar\omega_{\vec{q}}), \tag{11}$$

and

$$\left(\frac{dW_d}{dt}\right)^{(2)} = -\sum_{\vec{q}} 2B_{\vec{q}}\hbar\omega_{\vec{q}}\alpha_2(\hbar\omega_{\vec{q}}), \tag{12}$$

where

$$B_{\vec{q}} = \left(\frac{m^2}{2\pi\hbar}\right)\frac{|V_{D\vec{q}}|^2\omega_{\vec{q}}}{q}, \tag{13}$$

$$\alpha_1(\hbar\omega_{\vec{q}}) = \frac{2}{\hbar\omega_{\vec{q}}}\int_\Delta^\infty \frac{dE\{(E+\hbar\omega_{\vec{q}})E - \Delta^2\}\{f(E) - f(E+\hbar\omega_{\vec{q}})\}}{[(E+\hbar\omega_{\vec{q}})^2 - \Delta^2]^{1/2}[E^2 - \Delta^2]^{1/2}}, \tag{14}$$

$$\alpha_2(\hbar\omega_{\vec{q}}) = \frac{1}{\hbar\omega_{\vec{q}}}\int_\Delta^{(\hbar\omega_{\vec{q}}-\Delta)} \frac{dE\{(\hbar\omega_{\vec{q}} - E)E + \Delta^2\}\{f(E-\hbar\omega_{\vec{q}}) - f(E)\}}{[(\hbar\omega_{\vec{q}} - E)^2 - \Delta^2]^{1/2}[E^2 - \Delta^2]^{1/2}}, \tag{15}$$

and f is the distribution function of Eq. (1). α_2 vanishes unless $\hbar\omega_{\vec{q}} > 2\Delta$. The rate of energy dissipation per unit length of dislocation in the normal state is simply[19]

$$\left(\frac{dW_d}{dt}\right)_n \approx -\sum_{\vec{q}} 2B_{\vec{q}}\hbar\omega_{\vec{q}} \tag{16}$$

$$= \frac{Nmv_F v_d^2 b^2 \mu^2 q_D}{96} = B_n v_d^2, \tag{17}$$

where N is the conduction electron density, v_F is the Fermi velocity, q_D is the Debye wave vector, and B_n is the electronic drag coefficient in the normal state. Converting the sum over $\vec{q}$ in Eqs. (11) and (12) to an integral, introducing a Debye cutoff, and performing a change of variables, one finds that the ratio of the energy dissipation or drag coefficient in the superconducting to that in the normal state for a given temperature and dislocation velocity is

$$\Gamma = \Gamma_1 + \Gamma_2 \tag{18}$$

$$= (16/\pi)\int_0^1 dz \int_0^1 dx\, x^2(1-x^2)^{1/2}[\alpha_1(\hbar\omega_m zx) + \alpha_2(\hbar\omega_m zx)], \tag{19}$$

where $\omega_m = q_D v_d$ is the maximum frequency associated with the dislocation wave packet. The equations can be solved exactly only in the cases $\hbar\omega_m \ll \Delta$ and $\hbar\omega_m \gg \Delta$. For $\hbar\omega_m \ll \Delta$, $\alpha_2 = 0$, $\alpha_1 = 2f(\Delta)$, and $\Gamma = \Gamma_1 = 2f(\Delta)$. For $\hbar\omega_m \gg \Delta$, $\alpha_1 \approx 0$, $\alpha_2 = 1$, and $\Gamma = \Gamma_2 = 1$. The maximum value of $\Delta(T = 0°\mathrm{K})$ for typical superconductors varies from about 10^{-5} to 10^{-3} eV and it is clear, as previously stated, that Eq. (1) holds only for $v_d \lesssim 10$ to 10^2 cm/sec. For larger velocities, the complete expressions, given by Eqs. (19), (14), and (15), must be treated. The integrals cannot be solved analytically and were evaluated numerically.[20]

Figure 1 shows Γ plotted versus T/T_c for various values of $\hbar\omega_m/\Delta_0$, where $\omega_m = q_D v_d$ and Δ_0 is the energy gap at $T = 0$. The superconducting drag coefficient for a particular dislocation velocity and temperature is given by the product of the appropriate value of Γ in Fig. (1) and the normal-state drag coefficient, B_n, given in Eq. (17). To illustrate the typical shapes of the two components of the electron drag ratio, Γ_1 and Γ_2 are plotted separately for $\hbar\omega_m/\Delta_0 = 5$. It is clear that even for a moderate value like $\hbar\omega_m/\Delta_0 = 1$, the difference between the complete theory and the low-velocity approximation is appreciable. As one goes to higher velocities, it is seen that Γ does not fall to zero at any temperature but levels off at a finite and quite sizable value. The resulting curves are reminiscent of the amplitude dependence observed in ultrasonic attenuation experiments[4,5]; however, it is more likely that this effect arises from dislocation breakaway from pinning points,[4] in view of the relatively small dislocation velocities (10 to 10^2 cm/sec) expected for the frequencies and stresses utilized in these measurements. Nevertheless, the ultrasonic problem merits further study, particularly since the drag coefficient is a function of dislocation velocity in the superconducting state and existing theories do not consider this possibility.[21]

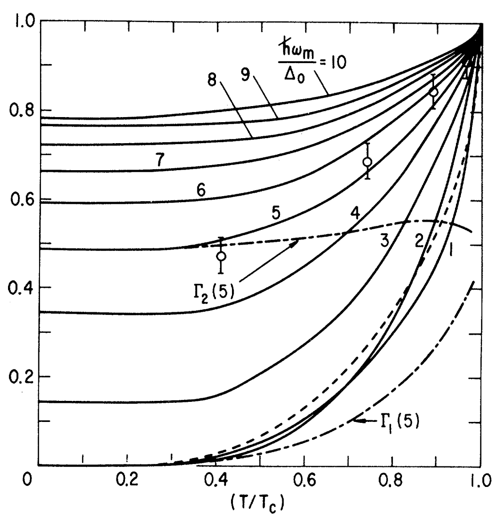

FIG. 1. Temperature dependence of the ratio of the total electronic dislocation drag in the superconducting state to that in the normal state. The dashed curve is the low-velocity approximation of Eq. (1). For $\hbar\omega_m/\Delta_0 = 5$, the two components of the drag ratio are shown separately by the dash-dot curves. Γ_1 arises from quasiparticle scattering and Γ_2 from processes involving the creation or annihilation of two quasiparticles.

Recently Alers, Buck, and Tittmann[1] have studied the flow stress of In at a number of temperatures below T_c. They find that a plot of the difference between the flow stress in the normal state, σ_N, and that in the superconducting state, σ_S, against $(T/T_c)^2$ is approximately linear. Assuming that the change in stress arises solely from the change in electronic drag and that v_d is constant, $\sigma_N - \sigma_S = (Bv_d/b)(1-\Gamma)$. The latter assumption follows from the view, detailed elsewhere,[14] that the instantaneous dislocation velocity is determined by the potential energy gained when dislocations pass short-range obstacles to their motion. The instantaneous velocity is the velocity of importance in present considerations and may be much larger than the average velocities normally measured.[14] For small changes in applied stress, the change in this velocity should be negligible.

All temperature dependence is then contained in the factor $1-\Gamma$ which is plotted against $(T/T_c)^2$ for several values of $\hbar\omega_m/\Delta_0$ in Fig. 2. It is seen that the curves are reasonably linear for $\hbar\omega_m/\Delta_0 \sim 3$ to 10 over the temperature range of the experiment. The energy gap at $T = 0°$K for In is about 5.13×10^{-4} eV, which gives dislocation velocities $v_d = (1.76,\ 2.93,\ \text{and } 4.11)\times10^4$ cm/sec for $\hbar\omega_m/\Delta_0 = 3$, 5, and 7, respectively. The normal-state drag coefficients required to produce reasonable agreement for the data of Alers,

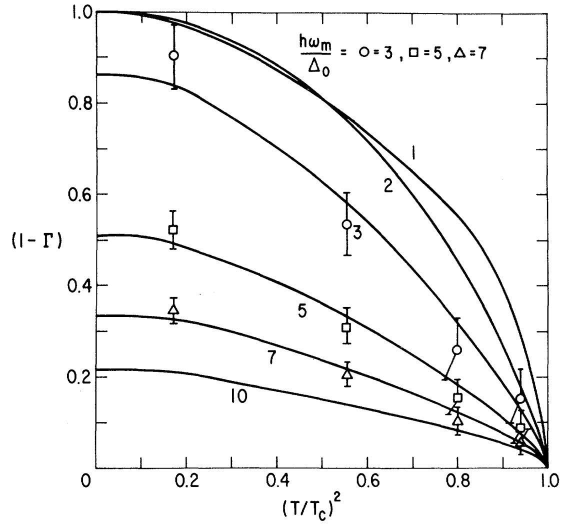

FIG. 2. Dependence of the normalized difference in stress between the normal and superconducting states, $1-\Gamma$, on $(T/T_c)^2$ for various values of $\hbar\omega_m/\Delta_0$. The experimental points shown here and in Fig. 1 are obtained from the data of Alers, Buck, and Tittmann (Ref. 1) as described in the text.

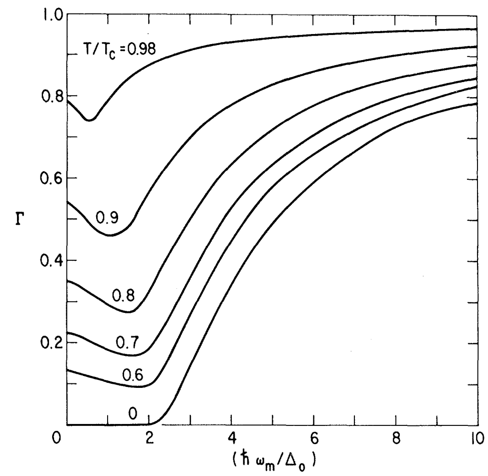

FIG. 3. Dependence of the electronic drag ratio on the parameter $\hbar\omega_m/\Delta_0$ at various temperatures.

Buck, and Tittmann with this theory for these velocities are then $B_n = (2.31,\ 3.35,\ \text{and}\ 3.62) \times 10^{-6}$ dyn sec/cm^2 for $\hbar\omega_m/\Delta_0 = 3$, 5, and 7, respectively. The experimental points shown in Fig. 2 are the values of $1-\Gamma$ determined from the experimental values of $\sigma_N - \sigma_S$ using these values of B_n and v_d. Additionally, we have shown the experimental values of Γ for $\hbar\omega_m/\Delta_0 = 5$ in Fig. 1. While the observed temperature dependence is in reasonable agreement with the theory, the deduced values of the normal state drag coefficient are smaller than those found from Eq. (17) by roughly a factor of 3 to 5. One might speculate, therefore, that other components of the stress, besides that due to electronic drag, are changed in the transition to the superconducting state. Nevertheless, it seems likely that most of the temperature dependence of the stress change is associated with electronic drag.

A more sensitive feature of the theory is indicated by Fig. 3 where we have plotted Γ vs $\hbar\omega_m/\Delta_0$ at various values of T/T_c. The variation of Γ with increasing dislocation velocity is marked, and with the exception of the case $T/T_c = 0$, the curves show a minimum. This feature of the theory could be tested by measuring dislocation velocity as a function of applied stress using stress pulse techniques[10-12] in a superconductor for which velocity is proportional to the stress in the normal state.

Alers, Buck, and Tittmann[2] found no detectable change in $\sigma_N - \sigma_S$ for Pb and In on varying the strain rate by a factor of 50. However, measured dislocation velocities in a number of high-purity close-packed metals,[10-12] including Pb,[11] are found to be proportional to the applied stress. A change in strain rate in such materials which is not accompanied by a proportionate change in stress must, therefore, be largely accommodated by a change in mobile dislocation density rather than in dislocation velocity.

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EVIDENCE FOR FLUCTUATION EFFECTS ABOVE T_c IN ISOTOPICALLY AND METALLURGICALLY PURE BULK TYPE-I SUPERCONDUCTORS*

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Magnetic measurements of superconducting transitions of very pure isotopically separated bulk samples of Ga and Zn reveal a large "paraconductive" effect. This is contrary to the prediction for such systems by Aslamazov and Larkin based on corrections to first order in the fluctuations in the system. It is suggested that this discrepancy may be explained by increased electron pair lifetimes due to the absence of isotopic and other scattering of phonons allowing higher order processes to be significant.

According to the theory of Aslamazov and Larkin (AL),[1] the electrical conductivity of a superconducting material in the normal state increases as the temperature approaches T_c because of fluctuations in the system (paraconductivity). This effect can be large for $T-T_c \sim 10^{-3}$ K in thin films and whiskers, but for pure bulk type-I superconductors, appreciable paraconductivity should be confined to the presently unobservable temperature range $T-T_c \sim 10^{-15}$ K. The theory predicts both a magnitude and a temperature dependence for the effect. Measurements on a variety of thin films[2] have shown rather good agreement with the theory for most substances measured. There were, however, substantial deviations from the theoretical magnitudes for measurements on low-resistivity Al films.[2,3] This Letter presents results of measurements on pure bulk type-I superconductors which disagree with the present prediction of the theory, and proposes a mechanism by which these results, those on similar systems, and the Al film results may be understood.

In magnetic measurements of the isotope effect in superconducting Zn[4] and Ga,[5] it was observed that some of the samples exhibited broad superconducting to normal (S-N) transitions in small magnetic fields. We have found that this paraconductivity is only observed in metallurgically and isotopically pure samples and in small magnetic fields (≤ 1 Oe). We observed measurable paraconductivity as much as 60 mK above the transition. Experimental details of the measurements are found in a previous paper.[4]

Figure 1 is a composite of S-N transitions of samples of Ga of relatively high metallurgical purity. The ordinates are the outputs of a mutual inductance detection system that was nulled when the samples were normal. Traces I, II, and III are transitions of Ga^{71} (99.61% Ga^{71}, 0.39% Ga^{69}) in 0.00, 0.52, and 1.04 Oe, respectively. Trace IV is the transition of natural Ga (60.0%