[1] I. Rudnick, private communication.
[2] C. H. Anderson and E. S. Sabisky, Phys. Rev. Letters 18, 236 (1967).
[3] I. Rudnick, R. S. Kagiwada, J. C. Fraser, and E. Guyon, Phys. Rev. Letters 20, 430 (1968), give a formula for the thickness which they obtained from the measurements of W. D. McCornick *et al.*, Phys. Rev. 168, 249 (1968), and R. Bowers, Phil. Mag. 44, 485 (1953). This formula has since been used by several authors as a general guide for the film thickness.
[4] C. H. Anderson and E. S. Sabisky, Phys. Rev. Letters 21, 987 (1968); E. S. Sabisky and C. H. Anderson, Phys. Rev. B (to be published).
[5] In a very recent private communication, I. Rudnick informs us that the major loss mechanism of the phonons is probably due to attenuation in the liquid film and not transmission into the gas.
[6] W. H. Strehlow and E. L. Cook, Phys. Rev. 188, 1256 (1969).
[7] J. Wilks, *The Properties of Liquid and Solid Helium* (Oxford Univ., Oxford, England, 1967).
[8] D. Hemming, Bull. Am. Phys. Soc. 8, 91 (1963).
[9] E. M. Lifshitz, Zh. Eksperim. i Teor. Fiz. 29, 94 (1955) [Soviet Phys. JETP 2, 73 (1956)].

PAIR TUNNELING AS A PROBE OF FLUCTUATIONS IN SUPERCONDUCTORS*

D. J. Scalapino
Department of Physics, University of California, Santa Barbara, California 93106
(Received 10 March 1970)

We suggest a method of using the average pair field of one superconductor to probe the fluctuation pair field of a second metal at temperatures above the latter's superconducting transition point. The two metals should be fabricated as a strongly coupled tunnel junction and the dc I-V characteristic measured as a function of both the bias voltage V and the amplitude of a magnetic field H parallel to the junction interface. The resultant I-V characteristic is predicted to give a direct measurement of the frequency and wave-vector dependence of the Δ susceptibility characteristic of the superconducting transition.

In most second-order phase transitions the order parameter can be directly coupled to an external field and the response of the order parameter to this coupling determines the characteristic susceptibility associated with the onset of order. The frequency and wave-vector dependence of this characteristic susceptibility provide the most direct probe of the fluctuations associated with the phase transition. Two well-known examples of this are magnetic transitions, where the magnetization is coupled to a magnetic field giving the magnetic susceptibility, and the liquid-gas transition, in which the density is coupled to the pressure giving the compressibility. In a superconducting system, the analogous characteristic susceptibility involves coupling to the pair field Δ which is off-diagonal in electron number space. For this reason one cannot couple to Δ with a classical field, and the question of whether, in principle, the Δ susceptibility of a superconductor can be directly measured has been raised.[1] The electrical conductivity[2] and magnetic susceptibility,[3] which have been observed near the superconducting transition temperature, involve convolutions of Δ susceptibilities. Recently, however, Ferrell[4] has suggested that the Δ susceptibility could be obtained by measuring the frequency-dependent conductivity of a Josephson junction in which one side of the junction is near its transition temperature while the other is well below its transition temperature. In this case, the average pair field of the higher transition temperature superconductor provides the necessary off-diagonal coupling.

Here we explore this idea of using the average pair-field amplitude of one superconductor to probe the fluctuation pair field of a second metal at temperatures above its superconducting transition temperature. We show that a measurement of the dc I-V characteristic can provide a direct determination of frequency and wave-vector dependence of the Δ susceptibility. The frequency is set by the bias voltage and the wave vector is determined by the application of a magnetic field parallel to the junction interface. Whether this proposed experiment will in fact provide a useful probe of the Δ fluctuations depends critically upon the ability to fabricate strongly coupled tunnel junctions. Some estimates of the parameters which enter this type of measurement are discussed in the conclusion.

Although a direct microscopic calculation starting from the tunneling Hamiltonian can be carried out, the following phenomenological approach introduced by Ferrell[4] provides more insight into the underlying physics. In particular it focuses on the similarity of the superconducting probe to the more familiar situation which involves a classical coupling field.

It is known that the low-temperature coupling energy of a Josephson junction consisting of two different superconductors is given by[5]

$$E_1 = \frac{1}{R_N} \frac{|\Delta'||\Delta|}{|\Delta'|+|\Delta|} K\left(\frac{|\Delta'|-|\Delta|}{|\Delta'|+|\Delta|}\right)\cos(\varphi'-\varphi). \quad (1)$$

Here $|\Delta|$ is the gap and φ the phase of one side, and the primed quantities are associated with the other side. R_N is the reduced junction resistance defined such that in the normal state the current is $e^2V(\hbar R_N)^{-1}$, and K is a complete elliptic integral. For $\Delta' \gg \Delta$, K reduces to a logarithm and the energy density per unit surface area is

$$E_1/A = C|\Delta|e^{i\varphi} + \text{c.c.}, \quad (2)$$

with the coupling strength C given by

$$C = (R_N A)^{-1} \ln|4\Delta'/\Delta| e^{-i\varphi'}. \quad (3)$$

If a bias voltage V is applied across the junction, this coupling energy oscillates at a frequency $2eV$. In addition, if a small magnetic field H is applied parallel to the surface of the junction, the energy density varies along the surface in a direction (x) perpendicular to the direction of the applied field. Here we will be interested in a junction where the unprimed film has a thickness d much less than its penetration depth while the primed material is a thick film having a penetration depth λ'. In this case, the interaction energy density in the presence of the dc biases V and H becomes

$$E_1 A = Ce^{-i(\omega t - qx)}|\Delta|e^{i\varphi} + \text{c.c.}, \quad (4)$$

with the frequency and wave vector set by V and H according to[6]

$$\omega = 2eV/\hbar, \quad q = (2eH/\hbar c)(\lambda' + \tfrac{1}{2}d). \quad (5)$$

Now suppose the temperature is just above the transition temperature of the unprimed side, T_c, but well below that of the primed side, T_c'. In this case the pair field of the higher transition temperature side Δ' has a well-defined average value and negligible fluctuations. Furthermore, φ' can be taken as a fixed reference for the relative phase. On the other hand, the average value of Δ vanishes and $|\Delta|e^{i\varphi'}$ should be replaced by the fluctuating pair-field operator $\Delta(x) = |g|\psi_\uparrow(x) \times\psi_\uparrow(x)$ of the lower-T_c metal. In this way, the coupling energy E_1 can be interpreted as an effective interaction Hamiltonian

$$H_1 = \bar{C}e^{-i\omega t}\int d^2x\, e^{iqx}\Delta(x) + \text{H.c.}, \quad (6)$$

where $\ln|4\Delta'/\Delta|$ has been replaced by $\ln|4T_c'/T_c|$ so that

$$\bar{C} = (R_N A)^{-1} \ln|4T_c'/T_c| e^{-i\varphi'}. \quad (7)$$

The associated pair-transfer current operator is simply

$$I_1 = (2ie/\hbar)\bar{C}e^{-i\omega t}\int d^2x\, e^{iqx}\Delta(x) + \text{H.c.} \quad (8)$$

The part of the current arising from fluctuations of $\Delta(x)$ is given by

$$\langle I_1\rangle = (4e/\hbar)\,\text{Im}\bar{C}e^{-i\omega t}\int d^2x\, e^{iqx}\langle\Delta(x,t)\rangle. \quad (9)$$

Using standard linear response theory to determine $\langle\Delta(x,t)\rangle$ due to the coupling interaction H_1, one obtains

$$I_1(V,H) = \langle I_1\rangle = (4e|\bar{C}|^2A/d\hbar)\,\text{Im}\chi(\omega,q), \quad (10)$$

where $\chi(\omega,q)$ is the Fourier transform of the characteristic Δ susceptibility,

$$\chi(x,t) = -i\langle[\Delta(x,t), \Delta^+(0,0)]\rangle\theta(t). \quad (11)$$

Here ω and q are determined by the dc bias fields V and H according to Eq. (5). In writing Eq. (10) it was assumed that the thickness d was small compared with the coherence length $\xi(T)$ so that fluctuations perpendicular to the junction surface can be neglected.

The susceptibility χ has been calculated in various approximations. The behavior of the current can be estimated using the mean-field form[7]

$$\chi^{-1}(\omega,q) = N(0)\left[\ln(T/T_c) + \psi\left(\frac{1}{2} + \frac{Dq^2 - i\hbar\omega}{4\pi kt}\right) - \psi(\tfrac{1}{2})\right]. \quad (12)$$

Here $N(0)$ is the single-spin density of states, D the diffusion constant, and ψ the digamma function. For small values of $\epsilon = (T-T_c)/T_c$, (12) reduces to

$$\chi^{-1}(\omega,q) = N(0)\epsilon(1 - i\omega/\Gamma_0 + \xi^2(T)q^2), \quad (13)$$

where Γ_0 is the relaxation time and $\xi(T)$ the coherence length:

$$\Gamma_0 = \frac{8}{\pi}\frac{kT_c}{\hbar}\epsilon, \quad \xi(T) = 0.74\xi_0\epsilon^{-1/2} \text{ or } = 0.85(\xi_0 l)^{1/2}\epsilon^{-1/2}; \quad (14)$$

ξ_0 is the BCS coherence length $0.18\hbar V_F/k_B T_c$ and l is the electron mean free path. The upper and lower forms for $\xi(T)$ given in (14) correspond to the clean ($l \gg \xi_0$) and the dirty ($\xi_0 \gg l$) limits, respectively.

In the presence of an external magnetic field, the susceptibility χ is modified. However, within the semiclassical approximation, χ takes on its $H=0$ value and the effect of the magnetic field is accounted for by the phase factor e^{iqx} already introduced in Eqs. (4) and (5). The semiclassical approximation is an appropriate starting point when

$$(2eH/\hbar c)d^2 < 1. \tag{15}$$

It follows from (5) to (13) that the size of the field H needed to probe the q dependence of χ is set by the condition that the relative pair phase shift by 2π over a coherence length:

$$(2e/\hbar c)(\lambda' + d/2)\xi(T)H \sim 1. \tag{16}$$

Combining this with (15) it follows that the semiclassical approximation can be used as long as $[\lambda'\xi(T)]^{1/2} > d$.

Using Eq. (13) for χ, the fluctuation contribution to the dc current is

$$I_1(V,H) = \frac{4eA|\overline{C}|^2}{dN(0)\epsilon} \frac{\omega/\Gamma_0}{[1+\xi^2(T)q^2]^2 + (\omega/\Gamma_0)^2}. \tag{17}$$

To understand the size of this current it is useful to normalize it to the current $I_N(V) = e^2V(\hbar R_N)^{-1}$ which would flow if both metals were normal. Regrouping the remaining terms, this ratio can be written as

$$\frac{I_1(V,H)}{I_N(V)} = \frac{\ln^2|4T_c'/T_c|}{\ln|4\Delta'/\Delta|} \frac{E_1}{U\epsilon^2}|(1+\xi^2(T)q^2)^2 + (\omega/\Gamma_0)^2|^{-1}. \tag{18}$$

Here E_1 is the Josephson coupling energy (1) when both sides are well below their transition temperatures and U is the condensation energy of a volume Ad of the unprimed side, $\frac{1}{2}N(0)\Delta^2 Ad$. The coupling energy E_1 directly obtained by multiplying the observed Josephson current I_1 by $\hbar/2e$.

Recent progress in junction fabrication techniques suggests that an Al-Pb junction having a Josephson current density of order 10^3 A/cm^2 could be constructed.[8] If $d \sim 500$ Å then the ratio E_1/U is of order 10^{-4}. In this case when the temperature is 1% (~20 mdeg) above the transition temperature of the Al, the fluctuation contribution to the current in zero magnetic field is equal to the normal-state current out to a voltage determined by

$$\hbar\Gamma_0/2e \cong 0.2\epsilon \text{ mV}. \tag{19}$$

The wave-vector dependence can be probed by a magnetic field such that $\xi q \sim 1$ which gives, for H in gauss,

$$H \cong 30\epsilon^{1/2} \text{ or } \cong 30(\epsilon_0/l)^{1/2}\epsilon^{1/2}, \tag{20}$$

for a clean and a dirty superconductor, respectively. For the present system, if diffuse scattering occurs at the film surfaces it will give an upper limit of d for l, even in a clean Al film. For the Al-Pb system with $\epsilon \sim 10^{-2}$, the characteristic voltage (19) is of order microvolts and, taking $l = d$, the characteristic magnetic field is of order 10 G.

It will of course be necessary to separate this part of the pair fluctuation current from other contributions to the I-V characteristic. One problem in this respect is of course the quasiparticle tunneling. The fluctuations introduce structure into the quasiparticle density of states but this is most significant at voltages above Pb gap. In addition, however, noise fluctuations associated with quasiparticle tunneling are known to modify the I-V characteristics of symmetric junctions at the low voltages of interest. Here, the fact that Pb is well below its transition temperature suppresses these quasiparticle effects and it therefore appears possible that the type of experiment we have suggested can provide a useful probe of the Δ susceptibility of a superconductor. Another aspect, currently under investigation, is the modification of the response associated with the interaction between fluctuations.[9]

The author wishes to thank D. N. Langenberg, J. M. Rowell, and B. N. Taylor for information on the present state of tunnel-junction fabrication.

---

*Research supported by the U. S. Army Research Office (Durham).

[1]V. Ambegaokar, in Proceedings of the International Conference on the Science of Superconductivity, Stanford, Calif., August 1969 (to be published).

[2]R. E. Glover, Phys. Letters 25A, 542 (1967); R. A. Ferrell and H. Schmidt, Phys. Letters 25A, 544 (1967); L. G. Aslamazov and A. I. Larkin, Phys. Letters 26A, 238 (1968).

[3]H. Schmidt, Z. Physik 216, 336 (1968); J. P. Gollub, M. R. Beasley, R. S. Newbower, and M. Tinkham, Phys. Rev. Letters 22, 1288 (1969).

[4]R. A. Ferrell, in Proceedings of the International Conference on the Science of Superconductivity, Stan-

ford University, Stanford, Calif., August 1969 (to be published), and Center for Theoretical Physics, University of Maryland, Technical Report No. 981 (unpublished), and Low Temp. Phys. 1, 423 (1969).

[5]P. W. Anderson, in *Progress in Low Temperature Physics*, edited by C. J. Gorter (North-Holland, Amsterdam, 1947), Vol. V, Chap. 1.

[6]This expression for q when one side of the junction is thin compared with a penetration depth follows in a straightforward manner from the discussion given in Ref. 5.

[7]See for example, C. Caroli and K. Maki, Phys. Rev. 159, 306 (1967).

[8]W. H. Schroen, J. Appl. Phys. 39, 2671 (1968); K. Schwidtal and R. D. Finnegan, J. Appl. Phys. 40, 2123 (1969).

[9]K. Maki, Progr. Theoret. Phys. (Kyoto) 39, 897 (1968).

INTERACTION BETWEEN ELECTRONS AND MOVING DISLOCATIONS IN SUPERCONDUCTORS

G. P. Huffman and N. Louat
Edgar C. Bain Laboratory for Fundamental Research, United States Steel Corporation Research Center, Monroeville, Pennsylvania 15146
(Received 20 October 1969)

The interaction between a moving dislocation and the electrons in a superconductor is treated theoretically. The results are in qualitative agreement with recent measurements of flow stress in superconductors.

In recent years a number of workers have investigated the motion of dislocations in superconductors. It has been found that the flow stress[1-3] and electronic drag coefficient[4-7] both decrease as the material becomes superconducting. The purpose of this paper is to present a general theoretical treatment of the interaction between a moving dislocation and the electrons in a superconductor. Most workers have attempted to analyze their data using a formula suggested by Mason[8] by analogy with the BCS theory of ultrasonic attenuation,[9]

$$\Gamma = 2f(\Delta) = 2[\exp(\Delta/kT)+1]^{-1}, \tag{1}$$

where Γ is the ratio of the electronic drag on the dislocation in the superconducting state to that in the normal state and Δ is the BCS energy-gap function. We will show, however, that this approximation is valid only for relatively slow dislocation velocities, $v_d \lesssim 10$ to 10^2 cm/sec. However, several recent experiments[10-12] made at stresses greater than or of the order of the macroscopic yield stress have directly measured dislocation velocities in the range from 5×10^2 to 5×10^4 cm/sec. Additionally, several indirect determinations of dislocation velocities in metals undergoing plastic deformation give values in the range from 10^3 to 5×10^4 cm/sec.[13-15] For such velocities, the frequencies associated with the dislocation wave packet can become greater than $2\Delta/\hbar$, and it is necessary to consider not only the scattering of Bogoliubov quasiparticles but also processes in which two quasiparticles are simultaneously created or destroyed with the emission or absorption of a quantum of energy by the dislocation.

The theory is similar to that used to calculate the attenuation of high-frequency phonons in superconductors.[16] The interaction Hamiltonian in the superconducting state is given by

$$H_I = H_I^{(1)} + H_I^{(2)}, \tag{2}$$

where

$$H_I^{(1)} = \sum_{\vec{q}}\sum_{\vec{k},\sigma}\{V_{D,\vec{q}}\exp(-i\omega_{\vec{q}}t)\gamma_{\vec{k}+\vec{q},\sigma}{}^\dagger\gamma_{\vec{k},\sigma} + V_{D,-\vec{q}}\exp(i\omega_{\vec{q}}t)\gamma_{\vec{k},\sigma}{}^\dagger\gamma_{\vec{k}+\vec{q},\sigma}\}(u_{\vec{k}}u_{\vec{k}+\vec{q}} - v_{\vec{k}}v_{\vec{k}+\vec{q}}) \tag{3}$$

and

$$H_I^{(2)} = \sum_{\vec{q}}\sum_{\vec{k}}\{V_{D,\vec{q}}\exp(-i\omega_{\vec{q}}t)(\gamma_{\vec{k}+\vec{q},\uparrow}{}^\dagger\gamma_{-\vec{k},\downarrow}{}^\dagger + \gamma_{-(\vec{k}+\vec{q}),\downarrow}\gamma_{\vec{k},\uparrow}) + V_{D,-\vec{q}}\exp(i\omega_{\vec{q}}t)(\gamma_{\vec{k},\uparrow}{}^\dagger\gamma_{-(\vec{k}+\vec{q}),\downarrow} + \gamma_{-\vec{k},\downarrow}\gamma_{\vec{k}+\vec{q},\uparrow})\}(u_{\vec{k}+\vec{q}}v_{\vec{k}} + u_{\vec{k}}v_{\vec{k}+\vec{q}}). \tag{4}$$

$V_{D,\vec{q}}$ is the Fourier transform of the dislocation deformation potential, and is given approximately by[13]

$$V_{D,\vec{q}} = i\left(\frac{2E_F}{3}\right)b\mu\frac{\sin\varphi_{\vec{q}}}{q}, \tag{5}$$