HIGH-MOMENTUM-TRANSFER NEUTRON-LIQUID-HELIUM SCATTERING BOSE CONDENSATION*

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Neutron inelastic scattering with high momentum transfers was used to probe the single-particle motions in liquid ^{4}He at temperatures ranging from 4.2 to 1.27°K. The observed scattered neutron distributions below T_λ are consistent with a two-component model of liquid helium involving condensation into a zero-momentum ground state. A fractional condensate density of (7.4 ± 1.5) % and an average kinetic energy per particle of 14.1 ± 0.6°K for helium at 1.27°K is deduced from a comparison of the experimental results with a theory for high-energy neutron–liquid-He scattering.

A number of inelastic neutron-scattering measurements of liquid ^{4}He II have been made in the past. While most of these experiments were designed to measure collective modes and to determine the phonon-roton dispersion curve, a recent paper by Cowley and Woods[1] reports neutron-scattering measurements at energy and momentum transfers much higher than those associated with rotons or phonons. In this paper, we present results[2] for neutron scattering on liquid ^{4}He II obtained with momentum transfers more than twice as large as in previously published work. At sufficiently high values of energy transfer $\hbar\omega = \hbar^2\kappa^2/2m_{\rm He}$, the neutron-scattering process should be describable as an interaction between the neutron and a single helium particle. Under these circumstances, the neutron probe would be sensitive to the motions of single helium atoms.

Hohenberg and Platzman[3] have suggested the use of high-energy neutron scattering to detect the presence of a zero-momentum condensate in He II. For neutrons scattered from helium atoms in the condensate, the energy transferred from the neutron would be equal to the recoil energy broadened by final-state interactions. For helium atoms not in the condensate, the energy transfer would be the recoil energy broadened by the Doppler shifts of the noncondensed helium particle momenta. Hohenberg and Platzman have estimated that the Doppler broadening from the noncondensed atoms will be several times larger than the broadening expected for interparticle interactions. If this simple model is qualitatively valid, the neutron-scattering cross sections are expected to be comprised of two components: a narrow one for scattering from the condensate atoms and a wider one for scattering from noncondensed atoms. The fractional areas under these two parts of the cross-section curve will be proportional to ρ_0/ρ and $1-\rho_0/\rho$, the fractional condensate and fractional noncondensate helium densities, respectively. Recent theoretical[4-7] estimates for ρ_0/ρ at $T=0$ are between 0.08 and 0.25.

Measurements of inelastic neutron scattering from liquid helium were made with the Battelle-Northwest rotating crystal and chopper time-of-flight spectrometer.[8] The initial, monochromatic, pulsed neutron beam was produced by the (200) or the (220) planes of a spinning aluminum single crystal. Initial energies were 0.1715 and 0.343 eV. Scattered neutron spectra were obtained by time of flight (TOF) at seven scattering angles ranging from 14.5° to 154.3°. Best instrumental resolution occurs for the largest fractional energy transfers. For target atoms which can recoil, this implies large-angle scattering. The cross-section measurements for the 154.3° detector bank were therefore used for detailed analysis of the shape of the He double-differential cross sections. Since high resolution[3] is required to observe the structure expected for a condensate, instrumental resolution was further optimized by using a 4-m flight path with the 154.3° counter bank. The liquid helium scattering sample was contained in the thin-walled tail of a Dewar which was pumped to vary the temperature between 4.2 and 1.25°K along the saturated vapor pressure curve. Scattered neutron spectra were obtained for a number of temperatures in the range 1.25-4.20°K.

Typical TOF spectra of neutrons scattered at 154.3° from liquid helium at 4.20 and 1.27°K are presented in Fig. 1. A flat background has been subtracted and the areas under both TOF peaks have been made equal to permit comparison of shapes. The instrumental resolution is indicated by the bar near the center of the scattering peaks. Counts in the TOF channels have been summed in pairs to provide some smoothing of the statistical fluctuations. A difference in width, at half-

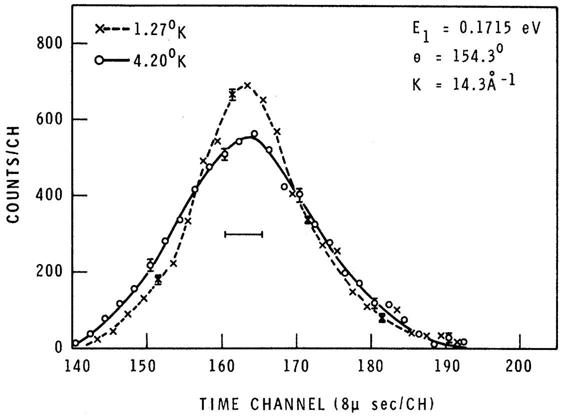

FIG. 1. Scattered neutron spectra for liquid helium at 1.27 and 4.20°K.

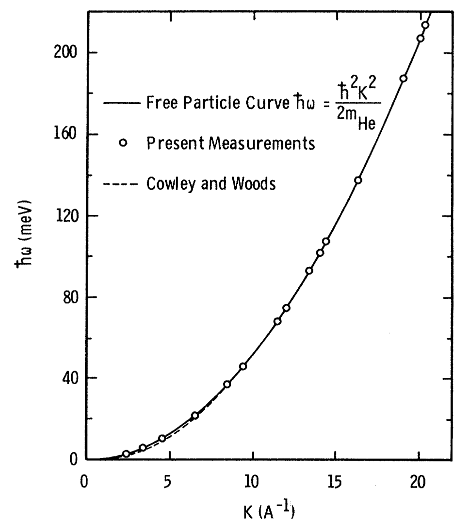

FIG. 2. The free-particle excitation curve for liquid ^{4}He. Regions investigated in previous studies by Cowley and Woods (Ref. 1) and in the present work are indicated.

height, and in the shapes, can be seen in the 1.27 and 4.2°K scattering distributions.

After conversion to double-differential cross sections, the energy transfer at the center of the scattered neutron peaks is found to be close to that expected for freely recoiling ^{4}He atoms. Figure 2 indicates the locations in κ and $\hbar\omega$ of the present measurements on the free-particle excitation curve. This figure also shows the range of κ values covered in the experiments of Cowley and Woods.[1] The highest resolution results in the present measurements are at $\kappa = 14.33$ and 20.26 Å^{-1}. At these momentum transfers, the average measured recoil energies and theoretical recoil energies for free particles are, respectively, 106.55 ± 0.8 and 107.15 meV for $E_1 = 171.5$ meV, and 213.8 ± 1.6 and 214.30 meV for $E_1 = 343$ meV. Present results at all other κ values also lie on the free-particle excitation curve within the standard error on the mean; however, at the lowest κ values the experimental error is large, 35-5% in the range 2-5 Å^{-1}, respectively.

An analysis of some of the measured double-differential cross sections has been carried out to determine if their detailed shapes are consistent with a two-component model like that of Hohenberg and Platzman.[3] It is observed that a somewhat skewed Gaussian gives a good fit to the cross sections at constant θ and for $T > T_\lambda$. Although the scattering law for free particles is a Gaussian in constant κ, when a Gaussian $S(\kappa, \omega)$ is converted to $d^2\sigma/d\Omega dE$ at constant θ the resulting shape is similar to that of the measured helium cross sections for $T > T_\lambda$. While a good fit, based on the standard χ^2 test, can be made to the cross sections above T_λ with a single skewed Gaussian, a poorer fit is obtained for a single skewed Gaussian when applied to the experimental data at $T = 1.27$°K. The cross sections above and below T_λ therefore appear to have distinctly different shapes. A theoretical model which takes into account the formation of a second component, the condensate, at $T < T_\lambda$, and which assumes a Gaussian shape for $S(\kappa, \omega)$, has been developed recently by Puff and Tenn (PT).[9] The cross section from the PT model is

$$\frac{d^2\sigma}{d\Omega dE_2} = \frac{\sigma_b}{8\pi^2\hbar}\left(\frac{E_2}{E_1}\right)^{1/2}\left\{\frac{2\rho_0}{\rho}\left(\frac{\pi}{\Gamma_1(\kappa)}\right)^{1/2}\exp\left[-\Gamma_1^{-1}(\hbar\omega-\hbar\omega_0)^2\right]+2\left[\frac{\rho-\rho_0}{\rho}\right]\left(\frac{\pi}{\Gamma_2(\kappa)}\right)^{1/2}\exp\left[-\Gamma_2^{-1}(\hbar\omega-\hbar\omega_0)^2\right]\right\}. \quad (1)$$

In this expression σ_b is the bound-atom cross section of He, taken equal to 1.13 b, ρ_0/ρ is the fractional condensate density, E_1 and E_2 are the initial and final neutron energies, respectively. Equation (1) has two parts, a Gaussian with width $\Gamma_1^{1/2}$ representing the condensate scattering and another Gaussian with width $\Gamma_2^{1/2}$ representing scattering from uncondensed helium atoms. Both Gaussians are centered on the free-particle recoil energy $\hbar\omega_0$. The condensate peak width, $\Gamma_1^{1/2}$, is a function of κ and has been estimated in Ref. 3. For noncondensed atoms the peak width $\Gamma_2^{1/2} = [(8/3)(\bar{E}_k/N)\hbar\omega_0 - (\rho_0/\rho)\Gamma_1]^{1/2}/[1-\rho_0/\rho]^{1/2}$, where

Table I. Average kinetic energy per atom and fractional condensate density obtained by least-squares fitting PT model to experiment.

He temperature (°K)	ρ_0/ρ (%)	$\overline{E}_k/N$ (°K)
4.20	2.5 ± 1.3	15.6 ± 0.6
1.27	7.4 ± 1.5	14.1 ± 0.6

$\overline{E}_k/N$ is the average kinetic energy per helium atom. The PT model has been fitted to some of the experimental results at 4.2 and 1.27°K. The Battelle-Northwest computer program LIKELY[10] was used to generate the least-squares fit of theory to experiment. The experimental resolution function, $R(\hbar\omega, \theta)$, was convoluted with the theoretical cross sections for the theory-experiment comparison. Table I contains a summary of the average values of ρ_0/ρ and $\overline{E}_k/N$ obtained in this comparison. Results at 4.20°K are averages from two separate experiments and results for 1.27°K are averages from three experiments all with momentum transfers of 14.33 $Å^{-1}$ and initial neutron energy of 171.5 meV. The fractional area under the Gaussian identified with the condensate is $(7.4\pm1.5)\%$ for a helium temperature of 1.27°K. A least-squares fit of the two-parameter PT theory was also used to obtain a condensate fraction at 4.20°K. Although a nonzero condensate fraction is physically unrealistic for $T > T_\lambda$, this does provide a test of the sensitivity of the model-experiment comparison in predicting a nonzero condensate fraction. Errors on ρ_0/ρ or $\overline{E}_k/N$ are obtained from counting statistics only and assume that the theoretical model of Ref. 9 is adequate. Reference 9 includes a comparison between theory and preliminary averaged data using an average resolution function. The values of ρ_0/ρ and $\overline{E}_k/N$ reported here show small differences which reflect our more precise unfolding of the resolution function and error analysis. The errors given in Table I are minimum errors since they include the influence of counting statistics only and do not include the effect of unknown systematic errors such as possible inadequacies in the theoretical model, Eq. (1). The value of ρ_0/ρ at 1.27°K is consistent with the theoretical calculations[4-7] of 8-25% for $T=0$ and with the Cowley-Woods[1] neutron scattering results of $(17\pm10)\%$ at 1.1°K. Recent theoretical treatments[5,7,11] determine the zero-point kinetic energy for liquid helium to be ~14°K. Since the $\overline{E}_k/N$ probably does not change very rapidly between 0 and 4.2°K, the experimental values of 14.1 ± 0.6 and 15.6 ± 0.6°K at helium temperatures of 1.27 and 4.2°K, respectively, are in good agreement with the theoretical predictions.

Additional and more complete analysis of these neutron-scattering measurements is in progress and will be reported elsewhere.

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