quantum electrodynamic (QED) theory, for the muon g-2 value[17] and for the hydrogen and deuterium Lamb shift.[18] Thus, apart from a possible discrepancy in the electron g-2 value,[16] the only remaining disagreement between theory and experiment in low-energy QED is the present one for positronium. This situation is all the more perplexing since, in contrast to hydrogen and the muon g-2 value, positronium presents a purely QED problem to this order. The calculations do not involve hadronic corrections or possible unexpected leptonic contributions. We therefore suggest that further computations of contributions to order $m\alpha^6 \ln\alpha^{-1}$ be carried out independently.

A detailed presentation of our calculation is in preparation.

One of us (T.F.) would like to thank Professor J. Prentki and Professor W. Thirring for the hospitality extended to him in the Theoretical Study Division of CERN.

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†Work supported in part by the National Science Foundation.

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PAULI EXCITATION OF ATOMS IN COLLISION

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(Received 9 February 1970)

Experimental cross sections are reported for the ionization of K shells of Al in collision with N, O, and Ar (in the energy range 100-300 keV); of Ne with C, Ne, and Al (100-300 keV); and of Al with Ne (0.1-3.2 MeV). They are some 10^3 to 10^5 times larger than the cross sections one predicts for Coulomb excitation of K-shell electrons by swift charged particles. The cross sections are determined by the exchange forces set up in the overlapping electron clouds of atoms in collision. This process is referred to as Pauli excitation.

Recent progress in the study of the removal of inner-shell electrons of atoms by swift charged particles through measurements of characteristic x-ray yields have brought detailed understanding of the underlying Coulomb excitation process. One finds agreement with the ionization cross sections derived in the Born approximation[1] if the deflection of the incoming particle in the field of the target nucleus is taken into account, and if one incorporates the binding of the target electrons to the exciting particle.[2,3]

This note reports data on cross sections for the ionization of K shells by swift heavy atoms.[4] In comparison with the Coulomb-excitation cross sections, the cross sections are enormous—in fact, some 10^3 to 10^5 times larger. We attribute the cross sections to the exchange forces set up during collision by the Pauli exclusion principle in the overlapping electron clouds of the interpenetrating atoms. It forces the electrons into transient quasimolecular configurations leaving the atoms, on separation, in excited states. We refer to this process as the Pauli excitation of atoms in collision.

The dominant role played by the Pauli exclusion principle in atomic collisions has been recognized for many years.[5] Measurements of the electron-energy spectra emitted in atomic collisions[6] have given direct evidence that K-shell vacancies can be produced in such encounters. These measurements, however, do not give access to inner-shell ionization cross sections.

We determined the cross sections of the K-shell ionization of Al by energetic ions of N, O, Ne, and Ar, and the K-shell ionization cross section of Ne in collision with C, Ne, and Al. The measurements were made in the energy range 100-350 keV for N, O, and Ar. The Ne data extend to 3.2 MeV. The only other data of this kind pertain to ions impinging on a carbon target.[7,8]

The method and apparatus for determining inner-shell excitation cross sections from the characteristic x-ray yield Y were described previously.[2] A mass- and energy-analyzed ion beam impinges on a thick target. The emitted x rays are registered and the energy analyzed in a flow-mode proportional counter. The observed x rays are identified as characteristic x rays through an energy measurement accurate to ±5%. The experimental conditions were such that the absorption of the emitted x rays in the target was negligible. In terms of the stopping cross section $S(E)$ of the target for the incoming ion of energy E and the K-shell fluorescence yield ω_K, the K-shell excitation cross section is given by $\sigma_K(E)=\omega_K^{-1}S(E)dY(E)/dE$.

The stopping cross sections for the heavy ions are obtained from the work of Lindhard, Scharff, and Schiøtt[9] combined with corrections for the fluctuations of the electronic stopping cross sections.[10,11] The fluorescence yields are taken to be constant and independent of the incident projectile.[12] On balance, the uncertainty in the absolute magnitude of the resulting excitation cross sections is ±<50%. The relative uncertainty of our cross sections is ±<25%.

The data of the K-shell excitation cross sections $\sigma_K(E)$ in our limited energy range can be summarized adequately by the empirical relation $\sigma_K(E)=P(E/E_1)^n$. The range of the ion energies $E_1 \leqslant E \leqslant E_2$ and the constants P and n for the ion-target pairs are listed in Table I.

The predicted dependence of the K-shell ionization cross section of neon on the charge state of the incident projectile[13] has been confirmed in experiments on gas targets.[14] In our solid targets we expect and find no such dependence, because of the screening in dense targets.

We compare these results with the cross sections expected for Coulomb excitation in the reduced plot shown in Fig. 1. Included are the recent measurements of C on C.[7,8] The solid line represents the theoretical prediction, in the Born approximation, of Coulomb excitation of K shells by charged particles. The data given for $Z_1 \ll Z_2$ (subscript 1 refers to the incident projectile, and 2 to the target) follow this curve closely if compared in terms of the reduced velocity parameter $\eta=v_1^2/v_0^2Z_2^2$ and the reduced ionization potential $\theta=\omega_{2K}/Z_2^2$, where v_1/v_0 is the projectile velocity in atomic units and ω_{2K} is the ionization potential of the target K shells. They are corrected, through the function $9E_{10}$, for the deflection of the projectile in the field of the target nucleus and, through the function ϵ, for the binding of the target electrons to the projectile.[2,15] The graph shows that when $Z_1 \approx Z_2$ the cross sections for the K-shell ionization (1) are many orders of magnitude larger than the Coulomb-excitation cross sections, (2) do not fall

Table I. Summary of experimental ionization cross sections for the K shells of Al and Ne in collision with various atoms. The absolute magnitude of the cross sections are uncertain by ±50%; the relative cross sections are uncertain by ±<25%.

Projectile	Target	Z_1/Z_2	Measured x ray	$\sigma_K(E)=P(E/E_1)^n$, $E_1 \leqslant E \leqslant E_2$: $\sigma_K(E_1)=P$ (b)	n	E_1 (keV)	E_2 (keV)
$^7N_{14}$	$^{13}Al_{27}$	0.54	Al (K)	0.8	3.5	175	300
$^8O_{16}$	$^{13}Al_{27}$	0.61	Al (K)	1.2	3.5	175	300
$^{10}Ne_{20}$	$^{13}Al_{27}$	0.77	Al (K)	1.7	3.0	100	3200
$^{18}A_{40}$	$^{13}Al_{27}$	1.38	Al (K)	3.1×10^1	6.5	175	350
$^{10}Ne_{20}$	$^{13}Al_{27}$	0.77	Ne (K)	6.0×10^2	2.7	100	400
$^{10}Ne_{20}$	$^{10}Ne_{20}$	1.00	Ne (K)	1.0×10^5	3.7	100	200
$^{10}Ne_{20}$	$^6C_{12}$	1.67	Ne (K)	2.3×10^1	1.9	125	300

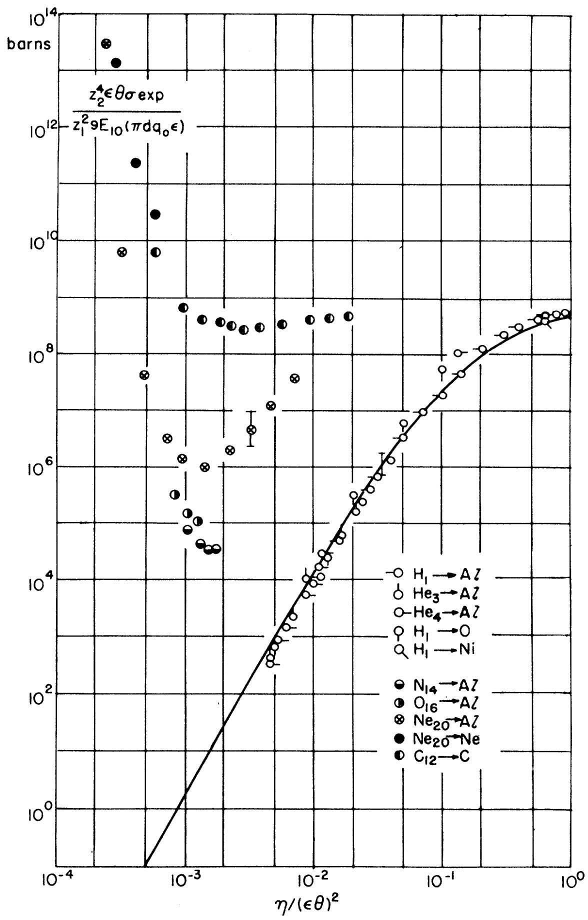

FIG. 1. K-shell ionization cross sections of various targets as excited by the ions indicated in the figure. The reduced variables are discussed in the text. The solid line represents the theoretical K-shell Coulomb ionization cross section in the Born approximation. The open circles mark data for $Z_1/Z_2 \ll 1$. They are taken from Refs. 3 and 15, and J. M. Khan, D. L. Potter, and R. D. Worley, Phys. Rev. 139, A1735 (1965); R. R. Hart, F. W. Reuter, III, H. P. Smith, Jr., and J. M. Khan, Phys. Rev. 179, 4 (1969); and G. A. Bissinger, J. M. Joyce, E. J. Ludwig, W. McEver, and S. Shafroth, Phys. Rev. A 1, 841 (1970). The data for the heavier ions are those listed in Table I and, for C on C, from Ref. 8.

on a common curve, and (3) appear to converge at high energies towards the Coulomb-excitation curve. These observations lead one to conclude that, when $Z_1 \approx Z_2$, Coulomb excitation is not the determining mechanism for K-shell excitation at low energies. We attribute the cross sections to the Pauli excitation of atoms in collision.

The principal distinction between Coulomb excitation and Pauli excitation of K shells can be understood in simple terms. At low energies the target electrons are Coulomb excited only if the projectile penetrates as a bare particle deeply into the target K shell because the probability for this excitation is peaked at an internuclear distance $v_1/\omega_{2K} \ll a_{2K}$.[16] The projectile approaches the target K shell as a bare particle if $Z_1 \ll Z_2$ because then its K-shell radius a_{1K} is much larger than the radius of the target K-shell a_{2K}. By contrast, when $Z_1 \approx Z_2$ the approaching projectile carries a K shell of comparable size. Pauli excitation commences when the electron clouds overlap, i.e., at an internuclear distance of order $2a_{2K}$ where Coulomb excitation is unimportant. This situation is not restricted to K shells, of course.

Pauli excitation as defined here provides a conceptual framework for the understanding of many phenomena that occur in atomic collisions.[17,18] In the low-velocity quasiadiabatic limit, Pauli excitation can be described in terms of the Landau-Zener model of single level crossings each of which occurs over a narrow range of internuclear distances.[19] For example, the K-shell ionization cross section of carbon in collision with carbon fits this two-parameter model well.[8] However, the resulting K-shell level-crossing radii are much larger than those expected from a static level-crossing scheme.[13] Our data lead to similar conclusions.

Pauli excitation in general proceeds through many level crossings that are interdependent and occur over an extended velocity-dependent range of internuclear distances. In the limit of high level-crossing densities the excitation of particle-hole pairs propagates in energy space as in a random walk to the ionization edge. This dynamic limit has been analyzed in terms of a statistical model by Mittleman and Wilets.[20] The resulting cross sections depend on two parameters, an interaction radius and a diffusion constant for the propagation of excitation in energy space. Comparing our data with this model we find reasonable trends for the parameters.

Certain aspects of Pauli excitation can be tested explicitly. We discuss two. A simple argument based on the energy shared in Pauli excitation shows that the collision partner with the lower atomic number should have a much larger probability of leaving the collision with a hole in its K shell (and hence to produce an x ray) than the partner with the higher atomic number. Figure 2 compares the cross sections as a function of Z_1/Z_2, at the energies at which the distance of closest approach, $d = Z_1 Z_2 e^2 (M_1 + M_2)/2M_2 E_1$, is equal to $5\times 10^{-2}(a_{1K} + a_{2K})$. The evi-

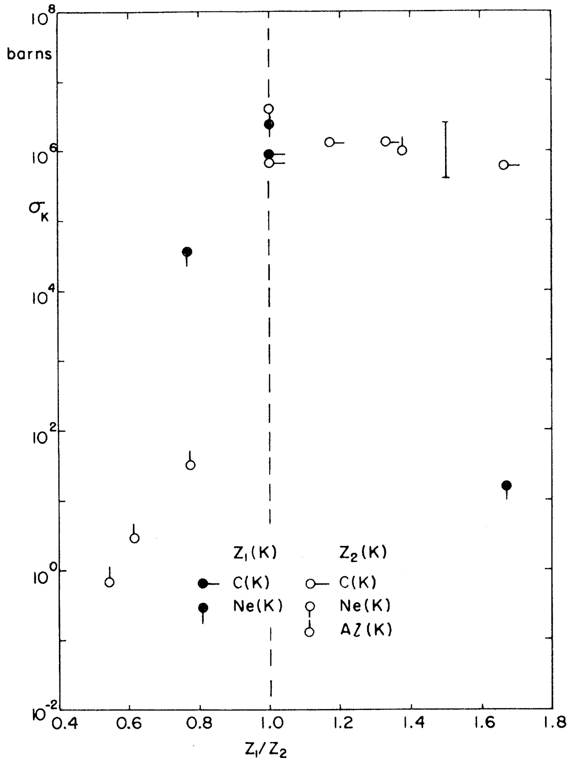

FIG. 2. K-shell ionization cross sections σ_K for projectiles $Z_1(K)$ and targets $Z_2(K)$ as a function of Z_1/Z_2. The cross sections are compared at energies such that $d=5\times10^{-2}(a_{1K}+a_{2K})$ is the same for all projectile-target pairs. The carbon data are taken from Ref. 7.

dence is clear: The Pauli ionization cross sections for the K shells of Z_2 jump by four to five orders of magnitude in traversing $Z_1/Z_2=1$. The cross section for K-shell Pauli ionization of Z_1 drops concurrently. Highly selective excitation of inner shells will occur in a composite target if a projectile Z_1 impinges on a mixture of Z_2 and Z_2' such that $Z_2 \leq Z_1 < Z_2'$.

At low energies Coulomb excitation proceeds at internuclear distances $\sim v_1/\omega_{2K} \ll a_{2K}$,[16] while Pauli excitation happens at $\sim a_{2K}$. Therefore, one can differentiate between the two excitation mechanisms in a coincidence experiment between the scattered primary particle and the emerging characteristic x ray or the equivalent Auger electron. Experiments along these lines are in progress.[21]

We benefited from discussions with A. Schwarzschild. The 0.4- to 3.2-MeV Ne data were taken with the Van de Graaff at the Department of Terrestrial Magnetism, Carnegie Institution of Washington. We are grateful to L. Brown and his colleagues for their help and hospitality.

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*Work supported by U. S. Atomic Energy Commission.

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