between the data and the Glauber theory in the region of large angles where the basic theoretical assumptions may not hold and the formal extension which we made for computing it is difficult to justify. We are currently completing experiments at 2.2 and 7.9 GeV/c designed to provide data with much higher statistics both at large scattering angles and in the interesting region at the break in $d\sigma/dt$. A more detailed calculation is being done to include multiple-scattering effects neglected in the present work.

We wish to acknowledge the contributions made to this work by H. Blumenfeld in the early stages of the experiment and the encouragements received from V. Franco, R. Glauber, and D. Harrington.

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OBSERVATION OF REGGE EFFECTS IN THE REACTION $\pi^+p \to \pi^0\Delta^{++}$

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We report a measurement of the total and differential cross sections and Δ^{++}-decay angular distributions for the reaction $\pi^+p \to \pi^0\Delta^{++}$ at incident momenta between 2.67 and 4.08 GeV/c. A dip near $t=-0.5$ and a backward peak are clearly observed. In a Regge-pole exchange model we determine the parameters of the ρ trajectory.

The Regge-pole model is most easily tested in reactions in which only a single trajectory can be exchanged in the t channel. The success of the ρ-trajectory model in describing the π^-p charge-exchange reaction[1] makes it important to see if the same trajectory also describes other reactions dominated by ρ exchange. The reaction $\pi p \to \pi\Delta$ is the only other such example. The dip observed near $t=-0.5$ in the differential cross section for the π^-p charge-exchange reaction has been interpreted as evidence that the ρ trajectory passes through zero at this point, and so we expect to observe the same dip in the $\pi\Delta$ final state if the same trajectory is exchanged.[2]

We report here a study of the final state[3] $\pi^0\Delta^{++}$ as observed in the reaction

$$\pi^+p \to \pi^+p\pi^0. \qquad (1)$$

An exposure of the 72-in. hydrogen bubble chamber to a separated π^+ beam at the Bevatron yielded 96 000 two-prong events distributed among the five incident momenta 2.95, 3.20, 3.53, 3.74, and 4.08 GeV/c. A similar exposure of the 25-in. bubble chamber has yielded to date 45 000 two-prong events at 2.67 GeV/c.[4] The events were measured on the flying-spot digitizer (FSD) and constrained to hypothesis (1). Extensive use was made of the automatic ionization measurements available from the FSD[5] in order to discriminate against conflicting hypotheses.

A major problem in the 3- to 4-GeV/c experiment was the proton contamination in the beam, which varied from 3 to 10% for most of the film but was 22% for 4.08-GeV/c exposure. This is a particularly severe problem in the small-momentum-transfer region, where the process $pp \to n\Delta^{++}$ is difficult to distinguish from the process $\pi^+p \to \pi^0\Delta^{++}$ with the usual kinematic constraint and ionization method.[6] The former process has a cross section approximately 15 times the latter in our energy range, making even a small beam proton contamination manifest. However, the mass difference between the π and p produces an upward shift in the missing mass when a real proton event is treated as a pion event. By measuring small samples of film with incident protons at 2.95, 3.65, and 4.0 GeV/c and treating $pp \to n\Delta^{++}$ events as $\pi^+p \to \pi^0\Delta^{++}$ events, we have determined the cuts on the missing mass squared required to remove incident-proton events.[7] The 2.67-GeV/c film was taken with a beam proton contamination of less than 0.5%.

Examination of the Dalitz plot for Reaction (1) shows that this final state is dominated by the reactions

$$\pi^+p \to \pi^0\Delta^{++}, \qquad (2)$$

$$\pi^+p \to \rho^+p, \qquad (3)$$

and

$$\pi^+p \to \pi^+\Delta^+. \qquad (4)$$

If one evaluates the $\pi^+\pi^0$ invariant-mass squared from Reaction (1) in the Δ^{++} rest frame, one finds that there is a linear relationship between the mass squared and the cosine of the angle that the decay proton makes with the Δ^{++} line of flight. The distribution of this decay angle is such that Reaction (3) overlaps Reaction (2) in a region of angles lying entirely in the forward hemisphere. This allows us to use the method of Eberhard and Pripstein[8] to remove the overlap events and repopulate the sample with events from the corresponding part of the backward decay hemisphere. These repopulated events represent 15% of our sample [with a ρ-band cut of $0.64 < M(\pi\pi) < 0.90$] at 3-4 GeV/c, and 30% of our sample at 2.67 GeV/c.[9] The peripheral nature of the production process (3) reflects itself as very small momentum transfers to the Δ, so that the removal of these overlap events is essential for studying Reaction (2). The Δ^{++} band is defined by the mass interval $1.12 < M(\pi^+p) < 1.32$.

We then present results for 385 events between 3 and 4 GeV/c and 420 events at 2.67 GeV/c. The Dalitz-plot projections show less than 10% background for most production angles. This may be somewhat higher in the backward hemisphere where small signals make background estimates difficult. The cross section for Reaction (2) is presented in Fig. 1 together with some previous measurements from the literature.[10] We note the power-law behavior with incident pion momentum; a fit to our data and the 8-GeV/c point gives a $p^{-1.6}$ behavior.

Figure 2(a) shows the differential cross section, $d\sigma/dt$, for the two energy regions. A fit of the forward peak by $d\sigma/dt = Ae^{Bt}$ yields $B = 8.8$

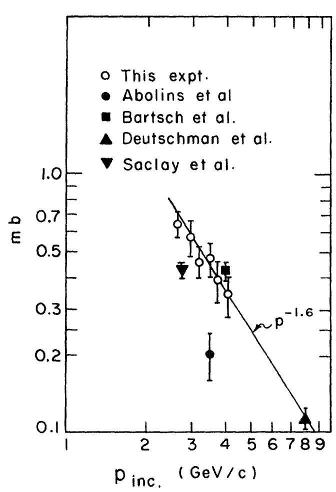

FIG. 1. Total cross section as a function of incident momenta for $\pi^+p \to \pi^0\Delta^{++}$. The solid curve is $p_{\rm inc}^{-1.6}$ from a fit to this experiment and the 8.0-GeV/c data.

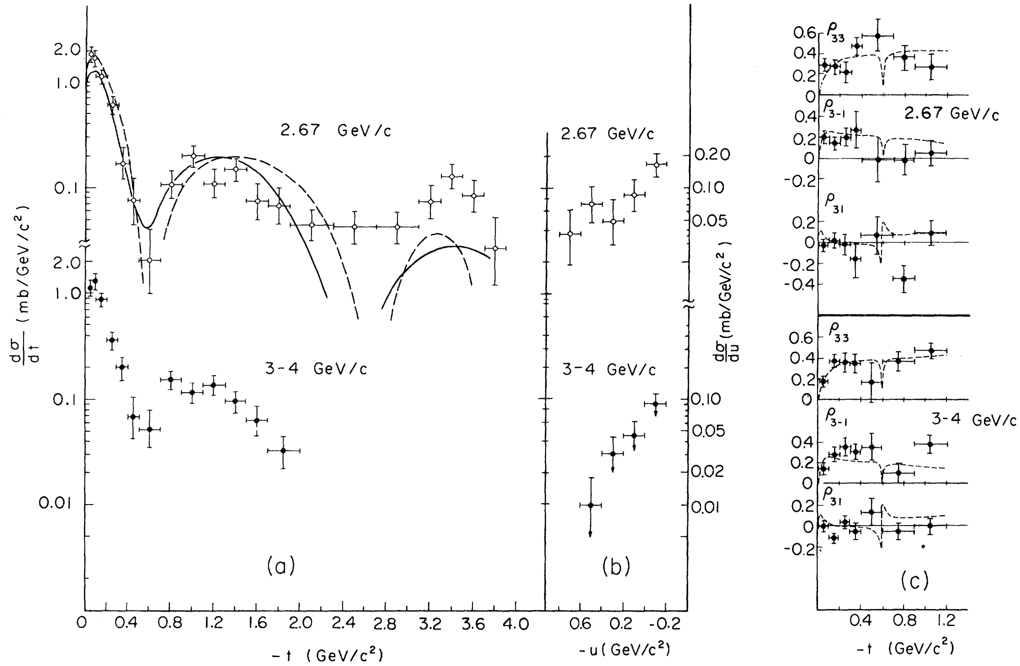

FIG. 2. (a) $d\sigma/dt$ vs t at 2.67 and 3-4 GeV/c. The curves are discussed in the text. (b) $d\sigma/du$ vs u in the small-$|u|$ region. (c) Density matrix elements as a function of t. The curves are discussed in the text.

± 1.0 at 2.67 GeV/c and $B = 8.0 \pm 0.9$ at 3-4 GeV/c. The dip near $t = -0.5$ is clearly observed at both energies.[11] A broad, second minimum near $t = -2.5$ is observed in the 2.67-GeV/c data. A backward peak is observed at both energies, and is shown as a function of u in Fig. 2(b).[12]

In the ρ Regge-pole description the helicity-flip amplitudes for $\pi^+p \to \pi^0\Delta^{++}$ go as

$$g(\alpha)F(s-u,\alpha)\beta(t)\frac{1-e^{-i\pi\alpha}}{\Gamma(\alpha+1)\sin\pi\alpha}$$

[where $g(\alpha) \to 0$ as $\alpha \to 0$] and, for a canonical ρ trajectory of $0.5+t$, have zeros at $t = -0.5$, -2.5, etc. Thus the minimum at -0.5, and that at -2.5 in the 2.67-GeV/c data, may be taken as evidence for exchange of the ρ trajectory in this reaction. This second minimum could be confirmed by its observation at constant t, independent of incident momentum.

As a representation of the available π^-p charge-exchange data, we show a best fit of Barger and Phillips[13] to those data (with a particular Ansatz for the large-t region), normalized to this experiment [solid curve on Fig. 2(a)]. The similarity of the shapes is evident and suggests that both processes are dominated by exchange of the same trajectory.

Krammer and Maor[14] have done a simultaneous fit to the reactions $\pi^+p \to \pi^0\Delta^{++}$, $\pi^+p \to \eta^0\Delta^{++}$, and $K^-p \to K^0\Delta^{++}$ with a $\rho + A_2$ Regge-pole model; the fit included our 3- to 4-GeV/c data, so the dashed curve in Fig. 2(a) shows the predictions of this model for 2.67 GeV/c only. We have extended the model past the t region considered valid by the authors, and in this region the model should be taken as indicating the dip expected near $t = -2.5$, rather than as a quantitative prediction. The spin-density matrix elements in the Jackson frame for the decay are shown in Fig. 2(c) as a function of t, together with the predictions by Krammer and Maor. Within the limited statistics the two are consistent, with the possible exception of ρ_{33} near $t = -0.6$ for the 2.67-GeV/c data. We keep in mind that the predictions of the $M1$ dominance model[15] at the $\rho\Delta p$ vertex are $\rho_{33} = 0.373$, $\rho_{3-1} = 0.216$, and $\rho_{31} = 0$.

Mathews[16] has discussed a linear fit of the available data with

$$\frac{d\sigma}{dt} = \left[\frac{G(t)}{P_{1\mathrm{ab}}^2}\right]\left(\frac{s-u}{2}\right)^{2\alpha(t)}. \qquad (5)$$

As a necessary condition for the use of this equation the ρ_{ij} for Reaction (2) at a given t should be independent of s.[16] Comparison of the ρ_{ij} at 2.67 and 3-4 GeV/c [Fig. 2(c)] and at 8 GeV/c[17] shows that this condition is at least approximately satisfied experimentally. We have then performed a least-squares fit by Eq. (5) for $|t| \le 1.2$ GeV/c^2

which gives

$$\alpha_\rho(t) = (0.56 \pm 0.04) + (1.34 \pm 0.12)t,$$

from $\pi^+p \to \pi^0\Delta^{++}$, using the 3- to 4- and 8-GeV/c data, and

$$\alpha_\rho(t) = (0.49 \pm 0.03) + (0.82 \pm 0.07)t,$$

from $\pi^+p \to \pi^0\Delta^{++}$, using the 2.7-GeV/$c$ data as well, compared with

$$\alpha_\rho(t) = (0.57 \pm 0.01) + (0.91 \pm 0.06)t$$

from $\pi^-p \to \pi^0 n$. These results are taken as evidence that the same trajectory is exchanged in both reactions. The flattening of the trajectory obtained by including the 2.67-GeV/c data is due to the steeper low-t behavior at this momentum, a possible consequence of an s-channel resonance contribution, as discussed below.

In the π^-p charge-exchange reaction a polarization has been observed[18]—something which is impossible if only a single trajectory is exchanged. Many theoretical models have been proposed to explain this polarization, and we can ask if a similar phenomenon is present in Reaction (2). Ringland and Thews[19] have proposed a test for the exchange of a single trajectory which asks if the contributing amplitudes are relatively real. The test requires that the combination

$$\rho_{33}\rho_{11} - (\mathrm{Re}\rho_{31})^2 - (\mathrm{Re}\rho_{3-1})^2$$

be zero. A violation of this condition implies the exchange of more than one trajectory, but satisfaction gives no information. Figure 3 shows this sum as a function of momentum transfer for our data and the 8-GeV/c CERN data.[17] This highest energy indicates a violation at small t which is no longer evident at the other energies.[20]

We have shown that a t-channel ρ-exchange description accounts for most of the features of Reaction (2). However, we cannot rule out s-channel resonance or u-channel baryon-exchange interpretations of some of these features. For completeness we briefly discuss these interpretations.

The 2.67-GeV/c incident momentum is at the peak of the $I=\frac{3}{2}$, $J^P = \frac{11}{2}^+$ $\Delta(2420)$. The locations in $\cos\theta$ of the minima and the backward peak are consistent with h-wave decay of this resonance into $\Delta\pi$. That the minima from an s-channel resonance should coincide with those from a t-channel exchange is an expectation of duality.[21] Thus the t-channel and s-channel interpretations may be equivalent.

The backward peak observed at both energies

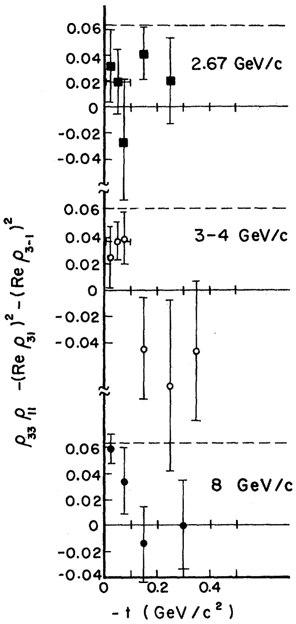

FIG. 3. Test of the Ringland-Thews conditions: $\rho_{33}\rho_{11} - (\mathrm{Re}\rho_{31})^2 - (\mathrm{Re}\rho_{3-1})^2$ vs t for 2.67, 3-4, and 8 GeV/c. The dashed line at 0.0625 represents the maximum violation of the condition.

[Fig. 2(b)] can be interpreted as evidence for u-channel baryon exchange. The ratio of the cross sections in the backward interval $\Delta u = 0.2$ GeV/c^2 between 2.67 and 3-4 GeV/c is approximately 1.8, corresponding to a p^{-3} behavior. A fit with $d\sigma/du = Ae^{Bu}$ for the interval $-0.4 \le u \le 0.2$ gives $B = 3 \pm 1$ at both energies. Both this value of B and p^{-3} behavior are consistent with those observed in processes thought to go by baryon exchange.[22,23] Again, allowing for duality, the u-channel exchange interpretation may be an equivalent rather than an alternative description.

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ARE REGGE TRAJECTORIES STRAIGHT LINES?

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For several Regge trajectories of elementary particles, we exhibit formula fits to the experimental masses. We find that it may be premature to conclude that even the existing trajectories form straight lines when the squares of the particle masses are plotted against the particle spins.

It is widely accepted that the elementary particles which belong to the same Regge trajectory lie on a straight line when the squares of the particle masses M are plotted against the particle spins J. In this Letter, we point out that this acceptance is premature, that is, that other simple formulas describe the masses as well as does a straight line.

For example, Table I shows fits to the experimental masses[1] for the three-parameter formulas

$$M(J)=A-B/(c+J)^2 \tag{1}$$

and

$$M(J)=A-B/(c+J), \tag{2}$$

as well as for the two-parameter straight line

$$[M(J)]^2=A+BJ. \tag{3}$$

The parameters listed in Table I are determined by minimizing χ^2. We show those Regge trajectories which presently contain four or more particles–i.e., $N^*(\frac{3}{2})$, $\Delta(\frac{3}{2})$, ρ, $Y_0^*(\frac{1}{2})$, and $Y_1^*(\frac{3}{2})$. When the error of an experimental mass is not available, we assign a value $X=10$ MeV. Because of this arbitrary choice, the absolute val-