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DEUTERON-DEUTERON INTERACTIONS AT SIX LABORATORY MOMENTA FROM 680 TO 2120 MeV/c*

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The elastic and total cross sections for deuteron-deuteron interactions at 680, 920, 1180, 1500, 1750, and 2120 MeV/c are presented. The data are compared with theoretical calculations and shown to be in qualitative agreement with predictions based on the Glauber multiple-scattering formalism. In particular the existence of large-angle deuteron-deuteron elastic scattering provides evidence for the "simultaneous" scattering amplitudes predicted by the Glauber theory.

Deuteron-deuteron scattering is the simplest example of a collision between two composite nuclear systems. Its experimental study provides a test of features predicted by multiple nuclear scattering theories that can be made only when both projectile and target are composite. There are several theoretical approaches to this problem[1-5] and we compare our data with some of these that have been applied specifically to elastic dd scattering.[6-8] Previous studies have been made of particle-deuteron collisions ($pd, \bar{p}d, \pi d, Kd$) where the Glauber multiple-scattering theory,[2] with a recent extension which takes into account the quadrupole deformation of the deuteron,[9] has proved quite successful.[10,11] We find that the Glauber theory is also qualitatively correct in interpreting dd elastic scattering, but more detailed theoretical calculations must be done before quantitative success can be claimed. Such studies, applied to composite systems of familiar particles, may help in formulating theoretical models which treat hadron-hadron scattering as collisions between composite systems of unknown subparticles.[12]

This report includes the results of an experimental survey we have made of dd interactions at lab momenta of 680, 920, 1180, 1500, 1750, and 2120 MeV/c. The purpose is to present the general features of the total and elastic cross sections in the transition region between low and high energies.

A total of 500 000 pictures were taken in the Princeton-Pennsylvania 15-in. rapid-cycling bubble chamber filled with deuterium. This is a 0.36-event/μb exposure. The deuterons were obtained from a mass-separated secondary beam with 1% momentum resolution. The beam flux was determined by counting beam tracks and measuring a sample uniformly spaced throughout the film. The measured sample was used to correct the counted beam tracks so that the same angle and spatial cuts were made as for the accepted dd interactions. The film was scanned for all events. Events which clearly did not satisfy elastic kinematics were eliminated using a three-dimensional projector which permitted rough (±5°) angle measurements in the scattering plane. This resulted in a sample of 20 000 events (30%

of all events found in scanning) which was measured on conventional image-plane digitizers and processed through the THRESH-GRIND analysis programs. The resulting $dd \to dd$ fits were then corrected for losses and background contamination. The scanning efficiency was determined by double-scanning 20% of the film and correcting for dip losses from the distribution of elastic events versus the azimuth angle of the scattering plane. This correction depends on the scattering angle and varied from $(35 \pm 6)\%$ at $t = 0.01$ $(\mathrm{GeV}/c)^2$ to $(5 \pm 5)\%$ above $t = 0.15$ $(\mathrm{GeV}/c)^2$. In the $dd \to dd$ kinematic fitting there is a background at low momentum transfers due to the process $dd \to dpn$ in which the target deuteron breaks up. This has been studied by generating a Monte Carlo set of $dd \to dpn$ events with the same measurement errors as our actual data and processing them with our kinematic fitting program. The correction is a maximum of $-(12 \pm 2)\%$ and goes to zero for $t > 0.3$ $(\mathrm{GeV}/c)^2$.

The fully corrected elastic differential cross sections at all six momenta are displayed in Fig. 1. They are quantitatively quite different, but qualitatively each has a very sharp forward peak followed by a relatively flat tail at large four-momentum transfers. In order to display these features better we have plotted the differential cross sections at small t values on an expanded scale at the left of graphs showing the complete scattering range. The errors are mostly statistical but include systematic effects. The integrated elastic cross sections are tabulated in Table I and plotted in Fig. 2. The corrections for unmea-

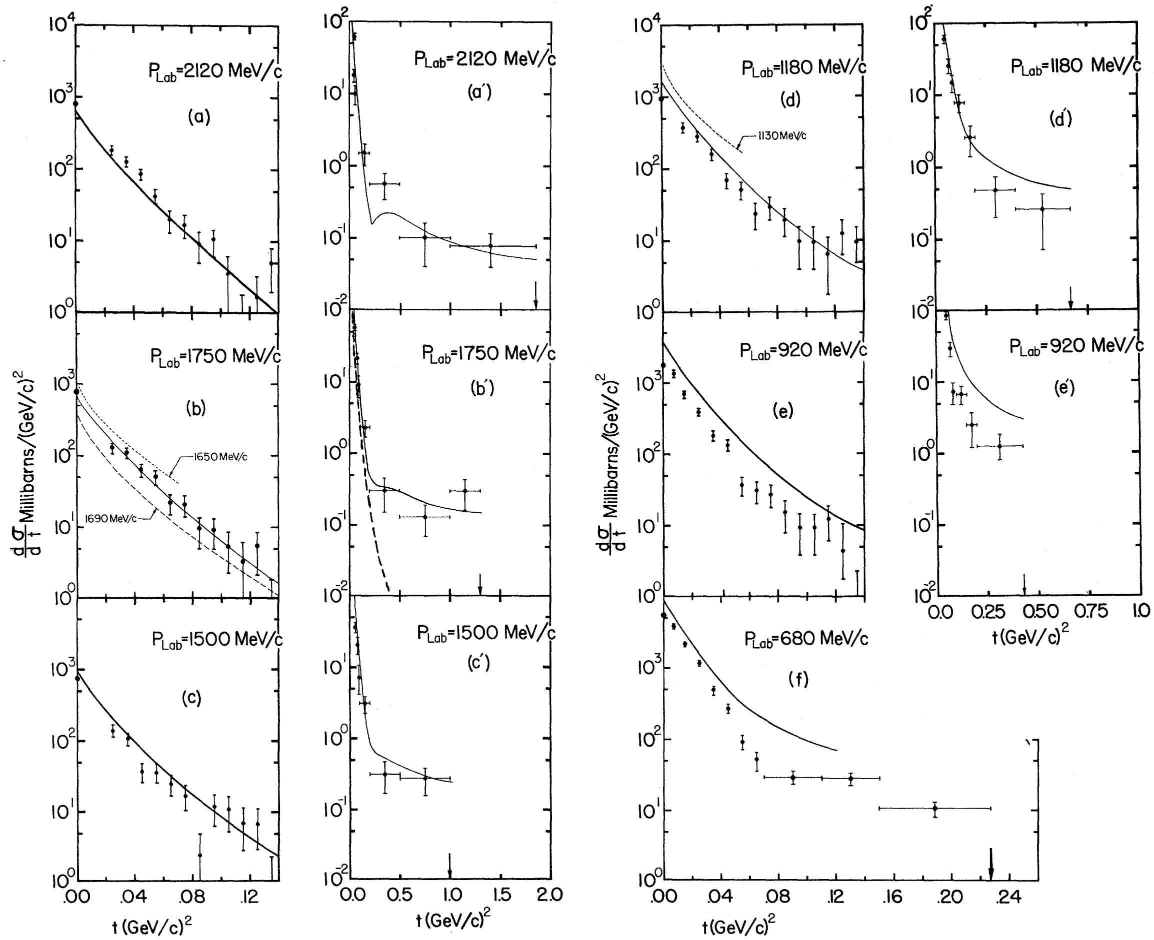

FIG. 1. Deuteron-deuteron elastic differential cross sections for incident laboratory momenta from 680 to 2120 MeV/c. (a)-(e) The forward peak on an expanded t scale. (a′)-(e′), (f) The complete scattering range. The arrows indicate 90° scattering in the c.m. system. The short-dashed curve in (b) and (d) is the calculation of Tubis and Chern using the impulse approximation. The long-dashed curve in (b) is the calculation of Queen including some multiple-scattering effects. The solid curves are our calculations using Franco and Harrington's extension of the Glauber formalism. Only single and double scattering are included.

Table I. Total and elastic deuteron-deuteron cross sections in mb. The total cross sections have not been corrected for small-angle inelastic scatters whose effect is small compared with the errors given (see the text).

Deuteron lab momentum (MeV/c)	Center-of-mass momentum (MeV/c)	$\sigma_{dd}{}^{\text{elastic}}$	$\sigma_{dd}{}^{\text{total}}$
680	335	91.0 ±5.4	329 ±13
920	445	31.5 ±3.0	188 ±8
1180	565	17.4 ±1.7	137 ±5
1500	700	11.7 ±1.2	122 ±5
1750	805	12.4 ±1.2	123 ±5
2120	940	13.8 ±1.4	127 ±5

sured small-angle scattering were made by an extrapolation of the data using the shape predicted by the theoretical calculation to be discussed below.

The total strong-interaction cross sections are tabulated in Table I and plotted in Fig. 2. They were calculated from an event count in a complete scan of all the film and checked for systematic errors by a second scan done especially for a total cross-section measurement. Corrections for scanning efficiency, small-angle elastic scatters, and beam contamination were made. No corrections were made for small-angle inelastic scatters, but we estimate this effect to be less than 3 % at all momenta. Optical points have been calculated from each total cross section and these are plotted as solid circles in Figs. 1(a) to 1(f) (the errors are smaller than the plotted points).

These data can be used to test several theoretical calculations which have been applied to dd scattering with different approximations concerning the nature of multiple nuclear scattering. The simplest approximation is to include only the effects of a single nucleon-nucleon interaction in each dd collision. This is the impulse approximation and its most immediate prediction is that the total dd cross section is $2(\sigma_{pp}+\sigma_{np})$. In this case the cross-section defect,

$$\Delta\sigma=\sigma_{dd}-2(\sigma_{pp}+\sigma_{np}), \tag{1}$$

could theoretically be used as a direct measure of the presence of multiple scattering in the dd collision. In Fig. 2 we have plotted $2(\sigma_{pp}+\sigma_{np})$ as a solid curve[13] and find agreement with the total dd cross section within our experimental errors. On the basis of this it can be said that total cross sections would have to be measured to at least 1% accuracy before it would be possible to measure $\Delta\sigma$ to 10% accuracy. This assumes the maximum possible $\Delta\sigma$ allowed by our data. Consequently, this is a rather insensitive way to study multiple scattering within the deuteron.

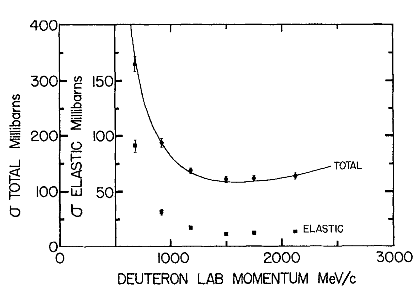

FIG. 2. Total (solid circles) and elastic (solid squares) deuteron-deuteron cross sections versus incident laboratory momentum. The corrections made to the data are described in the text. The solid curve is $2(\sigma_{pp}+\sigma_{np})$.

In contrast, the elastic cross section in this momentum range is strongly influenced by multiple scattering. Here the impulse approximation fails to predict both the shape and magnitude of. the experimental data. The contribution to the dd elastic scattering amplitude from single nucleon-nucleon scattering can be written

$$F_{dd}{}^1(\vec{\Delta})=4f(\vec{\Delta})S^2(\tfrac{1}{2}\vec{\Delta}), \tag{2}$$

where $\vec{\Delta}$ is the three-momentum transfer in the dd c.m. system, $f(\vec{\Delta})$ is the nucleon-nucleon elastic scattering amplitude,

$$S(\vec{\Delta})=\int e^{i\vec{\Delta}\cdot\vec{r}}\psi_d{}^*(\vec{r})d^3\vec{r}$$

is the deuteron form factor, and

$$d\sigma/dt=|F_{dd}(\vec{\Delta})|^2 \text{ with } t=|\vec{\Delta}|^2\hbar^2. \tag{3}$$

In order to most clearly display the structure of the dd scattering amplitude we write the constituent nucleon-nucleon amplitudes without spin or charge labels. The spherical and quadrupole parts of the deuteron form factor are not displayed explicitly either. The sticking factor $S^2(\frac{1}{2}\vec{\Delta})$ behaves approximately as $\exp(-25t)$ at small momentum transfers and expresses the relative probability of a deuteron sticking together if one of its constituents receives an impulse $\vec{\Delta}$. It is this factor that is mainly responsible for the sharp forward peak in $d\sigma/dt$ at all six momenta. The form factor decreases smoothly with increasing t and therefore single scattering cannot explain the observed sudden break in $d\sigma/dt$. In addition the impulse approximation predicts much too large a cross section in the forward direction over most of our energy range as can be seen in Figs. 1(b) and 1(d) by comparing our data with a calculation done by Tubis and Chern.[14] Queen[6] has applied the Watson multiple-scattering theory[1] and made a partial summation of multiple-scattering corrections to the impulse approximation. An important approximation he makes is that the deuterons always remain in their ground state during each collision contributing to the multiple-scattering process. The results of the calculations are plotted as long-dashed curves in Figs. 1(b) and 1(b′). A comparison with the impulse approximation in Fig. 1(b) shows that the magnitude of the cross section is greatly reduced at low t,[15] and now falls below our data. At large four-momentum transfers the calculated $d\sigma/dt$ is increased but there is not even qualitative agreement with experiment. Queen concluded that the large four-momentum transfer region is extremely sensitive to the quadrupole deformation of the deuteron and that the neglected unbound deuteron intermediate states were negligible. We will now present evidence that suggests that just the opposite is true.

Franco[7] and Harrington[8] have independently applied the Glauber multiple-scattering formalism to dd elastic scattering. The structure predicted for the scattering amplitude is

$$F_{dd}(\vec{\Delta}) = 4f(\vec{\Delta})S^2\Big(\frac{\vec{\Delta}}{2}\Big)+\frac{2i\hbar}{\pi^{3/2}}S\Big(\frac{\vec{\Delta}}{2}\Big)\int f\Big(\frac{\vec{\Delta}}{2}+\vec{q}\Big)S(\vec{q})f\Big(\frac{\vec{\Delta}}{2}-\vec{q}\Big)d^2\vec{q}+\frac{i\hbar}{\pi^{3/2}}\int f\Big(\frac{\vec{\Delta}}{2}+\vec{q}\Big)S^2(\vec{q})f\Big(\frac{\vec{\Delta}}{2}-\vec{q}\Big)d^2\vec{q} + \text{higher order scattering terms}, \qquad (4)$$

where f and S are defined in (2). The second and third terms in Eq. (4) represent two distinct types of double-scattering processes. The sequential-scattering amplitude corresponds physically to one nucleon of a deuteron hitting successively each nucleon of the other deuteron. The requirement that the first deuteron holds together brings in the form factor $S(\frac{1}{2}\vec{\Delta})$ and the double collision within the second deuteron brings in an integral over the internal momentum transfer as shown. The simultaneous-scattering amplitude represents a process in which the two nucleons in one deuteron each scatter once off the two nucleons in the second deuteron. This allows the transfer of a large momentum to each deuteron but a small relative momentum transfer to the nucleons within a deuteron. At large t values the only surviving term will be the simultaneous- (and higher order) scattering amplitude because the first two amplitudes decrease exponentially as S^2 and S, respectively. At small t values the cross section is reduced from the impulse-approximation prediction because of the destructive interference between the first two amplitudes.

We have used the above formalism to calculate the dd elastic differential cross sections for our energies. The nucleon-nucleon amplitudes were parametrized as

$$F_{NN'}(\vec{\Delta}) = \frac{\sigma_{NN'}}{4\pi^{1/2}\hbar}(\alpha_{NN'}+i)\exp\Big(-\frac{B_{NN'}}{2}\Delta^2\Big), \qquad (5)$$

where N and N' refer to a neutron or proton. The total cross sections σ_{pp} and σ_{np} and the slopes B_{pp} and B_{np} were extracted from data compiled by Wilson[16] and Hess.[17] The ratios of real to imaginary parts were obtained from a summary in a paper by Dutton et al.[18] Spin dependence was neglected and charge independence was assumed throughout. The Moravcsik-III deuteron wave function[19] was used and the effects of a 7% D-wave contribution were included in the calculation. The nucleon-nucleon amplitudes were extended up to a momentum transfer squared which lies beyond 90° in nucleon-nucleon scattering but at which the numerical evaluation of the multiple-scattering integrals has converged. In practice we let Δ^2 extend to the full maximum value of t in dd scattering.

The results, neglecting triple and higher order scattering, are plotted as solid curves in Fig. 1. At the higher momenta the agreement with our data is quite good. In particular, the sharp break and flat tail are reproduced and demonstrate the importance of the simultaneous scattering term. However, any quantitative agreement between our calculations and experiment cannot be taken too seriously until the effects of higher order multiple-scattering terms and of the spin dependence of the nucleon-nucleon amplitudes are examined. A surprising fact is the quality of the agreement between the data and the Glauber theory in the region of large angles where the basic theoretical assumptions may not hold and the formal extension which we made for computing it is difficult to justify. We are currently completing experiments at 2.2 and 7.9 GeV/c designed to provide data with much higher statistics both at large scattering angles and in the interesting region at the break in $d\sigma/dt$. A more detailed calculation is being done to include multiple-scattering effects neglected in the present work.

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OBSERVATION OF REGGE EFFECTS IN THE REACTION $\pi^+p \rightarrow \pi^0\Delta^{++}$

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We report a measurement of the total and differential cross sections and Δ^{++}-decay angular distributions for the reaction $\pi^+p \rightarrow \pi^0\Delta^{++}$ at incident momenta between 2.67 and 4.08 GeV/c. A dip near $t=-0.5$ and a backward peak are clearly observed. In a Regge-pole exchange model we determine the parameters of the ρ trajectory.

The Regge-pole model is most easily tested in reactions in which only a single trajectory can be exchanged in the t channel. The success of the ρ-trajectory model in describing the π^-p charge-exchange reaction[1] makes it important to see if the same trajectory also describes other reactions dominated by ρ exchange. The reaction $\pi p \rightarrow \pi\Delta$ is the only other such example. The dip observed near $t=-0.5$ in the differential cross section for the π^-p charge-exchange reaction