STRUCTURE OF F^{18} AND O^{18}†

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A detailed shell-model description of F^{18} and O^{18} is given for all levels of known spin and parity. New assignments are suggested. It is shown that a weak-coupling model is highly successful in reproducing the exact results.

A recent calculation[1] showed the possibility of using a very simplified but realistic model to describe O^{16}. It was therefore interesting to extend the results to other nuclei and investigate the validity of the approach. Preliminary calculations[2] showed reasonable features but also important discrepancies in $A = 18$. Since changes in the model space would result in enormous complications, but varying the interaction would be simple, a careful study of the matrix elements was undertaken. The idea was not to fit energy levels but to stay close to realistic calculations[3] as far as possible, or to Talmi-type fits in cases where pure configuration assignments are reliable (as in the lowest negative-parity states in N^{14}).

The ZBM model omits $d_{3/2}$ and $p_{3/2}$ orbitals and therefore requires renormalizations that are not included in the usual reaction-matrix calculations. To understand their nature we used the Bloch-Horowitz[4] approach to reduce the complete *sd*-shell problem for two particles to the $d_{5/2}s_{1/2}$ model subspace adopted by Arima et al.[5] The resulting (renormalized) KLS and Kuo matrix elements agree quite nicely with the fitted ones for $T = 1$, and for $J = 4, 5$, $T = 0$. For $J = 3$, $T = 0$ there is disagreement, but it can be corrected. Replacing the three renormalized values in the set of ACLM produces no significant change[6] except for the appearance of a second 3^+ state in F^{18} at about 4 MeV where an excellent experimental candidate is seen.[7] For $J = 2$, $T = 0$ there is only one matrix element in the space, and the disagreement is only ficticious since ACLM included in their search the 2^+ state at 2.52 MeV in F^{18} which was later shown[8] to be 4p-2h (four-particle, two-hole) and, therefore, out of the model space. In the case of $J = 1$, $T = 0$ the discrepancy is more real. Of all the lowest two-particle states given by realistic calculations, the ground state of F^{18} is the only one containing strong $d_{3/2}$ components. The Bloch-Horowitz prescription seems to fail in this case, and the fact that very good results can still be obtained by using the restricted space deserves further study. The price one has to pay for the truncation of the basis is relatively low: The second two-particle 1^+0 state is 2-3 MeV higher than the experimental candidates at about 5 MeV. If the realistic renormalized values for $J = 1$, $T = 0$ are used, rather bad shifts occur in the 3- and 4-particle ACLM spectra but they produce only small variations in the present (6-particle) calculation. The reasonable overall agreement between fitted and realistic values is encouraging, but it should be remembered that the corrections associated with the omission (not neglect!) of the $d_{3/2}$ shell and the usual core polarization diagrams are not the only ones. The correlations of the core (included explicitly in this paper) are very important. The presence of a strong 2p-2h component[1] in the ground state of O^{16} leads to coupling effects which tend to depress the ground state of O^{18} by as much as 2 MeV. The agreement with experiment of reaction matrix calculations that neglect this fact seems, therefore, rather fortuitous. Another effect was demonstrated by Kahana, Lee, and Scott[9] who showed that the use of Woods-Saxon wave functions gives rise to reductions of as much as 40% in $\langle s^2|\bar{v}|s^2JT\rangle$ matrix elements, thus worsening the simple two-particle picture. In our larger calculation, however, such changes produce positive results.

For matrix elements containing p particles a Bloch-Horowitz program is much harder to carry out. For the negative-parity ones we rely on the Talmi-Unna[10] results obtained from the spectrum of N^{14}. They are expected to differ markedly from realistic values through the presence of deformation in the C^{12} core. In ZBM (case II) the Talmi-Unna matrix elements were changed arbitrarily to improve the position of the $T = 1$ states of O^{16}. This seemingly innocuous operation turned out to have unexpected consequences that are at the origin of many problems. They can be understood in terms of the coupling scheme of Bansal, French, and Zamick,[11] in which a monopole force is introduced as an average over negative-parity matrix elements to account for the unperturbed positions of many-particle,

Table I. Antisymmetrized two-body matrix elements and single-particle energies (MeV). Here, $p=(1p_{1/2})$, $s=(2s_{1/2})$, and $d=(1d_{5/2})$. Only values differing from ZBM, case II are shown.

$\langle l_1 l_2 \vert \bar{v} \vert l_3 l_4 JT\rangle$			$\langle l_1 l_2 \vert \bar{v} \vert l_3 l_4 JT\rangle$			$\langle l_1 l_2 \vert \bar{v} \vert l_3 l_4 JT\rangle$		
dd	*dd* 01	−2.41	*dp*	*dp* 30	−2.59	*ss*	*pp* 10	+0.83
dd	*pp* 10	−1.20	*dp*	*dp* 31	−0.35	*sp*	*sp* 00	−3.48
ds	*ds* 20	−1.70	*ss*	*ss* 01	−1.67	*sp*	*sp* 01	+1.67
dp	*dp* 20	−3.93	*ss*	*ss* 10	−3.17	*sp*	*sp* 10	−1.28
dp	*dp* 21	+0.71	*ss*	*pp* 01	+0.82	*sp*	*sp* 11	−0.25

many-hole levels. The upward shift of the $T=1$ states of O^{16} affects the monopole force and the net result is to depress states in which the particles and the holes are coupled to high T (it increases the coefficient b of Zamick), crowding the spectra and overemphasizing configuration mixing. A much healthier situation for $A=15$ to 18 is achieved if we do not insist on getting the $T=1$ states of O^{16} at the right positions. This is, at present, the worst difficulty of the ZBM model. We are left with the comfort that the 0^-0 state in O^{16}, which comes much too low in ZBM, can be moved up without causing any trouble by slight changes in the Talmi-Unna matrix elements.

In Table I we list the interaction used to obtain the results summarized in Table II. The only attempt to improve level positions was made for the ground state of O^{18} and the 0^-0 state of O^{16}. Other modifications that lead to better looking spectra, such as the ones mentioned for the $J=1, 3$; $T=0$ states, are not included, but it was checked that the third 3^+0 and fourth 1^+0 states could be made to come at the right positions, with little effect on the remaining states, by using the realistic renormalized values (the corresponding entries have been left blank in Table II). The s^2 matrix elements were corrected in line with the results of Ref. 9. The off-diagonal matrix elements $\langle d^2, s^2 \vert \bar{v} \vert p^2 JT\rangle$ had been taken from miscellaneous sources in ZBM. A consistent set due to T. Kuo was chosen in this calculation. It is interesting to point out a change in sign in $\langle s^2 \vert \bar{v} \vert p^2 JT\rangle$ that leads to a 1.5 MeV reduction in excitation energy for the 0^-0 state in F^{18}.

We may summarize this review of the model

Table II. States in F^{18} and O^{18}. For $T=0$, levels are included up to 5 MeV, and up to 7 MeV for $T=1$. Only F^{18} excitation energies are given. Parentheses indicate uncertain or very uncertain (double parentheses) correspondence. The notation O^{18} and F^{18} is used for the *sd* shell two-particle states (obtained directly from Table I) and should cause no confusion.

$J^\pi T$	E(th)	E(exp)	Type	$J^\pi T$	E(th)	E(exp)	Type
1^+0	0	0	$O^{16}xF^{18}$	2^-0	4.09	((4.23))	$O^{15}xF^{19}(3/2^+,5/2^+)$
3^+0	1.01	0.94	$O^{16}xF^{18}$	1^-0	4.77	((4.36))	$C^{13}xNa^{21}(3/2^+)$
0^+1	0.57	1.04	$O^{16}xO^{18}$	2^-0	5.06	((4.40))	$C^{13}xNa^{21}(3/2^+)$
0^-0	1.32	1.08	$O^{15}xF^{19}(1/2^+)$	4^+1	5.18	4.65	$O^{16}xO^{18}$
5^+0	1.09	1.13	$O^{16}xF^{18}$	0^+1	4.43	4.74	$O^{16}xO^{18}$
1^+0	1.71	1.70	$N^{14}xNe^{20}$	1^+0		((4.84))	$O^{16}xF^{18}$
2^-0	2.46	2.10	$O^{15}xF^{19}(3/2^++5/2^+)$	2^+1	4.96	4.97	$O^{16}xO^{18}$
2^+0	3.18	2.52	$N^{14}xNe^{20}(2^+)$	1^-1	4.61	5.60	$O^{15}xF^{19}(1/2^+)$
2^+1	3.07	3.06	$O^{16}xO^{18}$	3^-1	5.18		$O^{15}xF^{19}(5/2^+)$
1^-0	3.55	3.13	$O^{15}xF^{19}(1/2^+)$	0^+1	5.61	6.47	$C^{14}xNe^{20}$
3^+0	3.16	3.36	$N^{14}xNe^{20}(2^+)$	2^-1	6.23	5.77	$O^{15}xF^{19}(5/2^+)$
1^+0	3.51	(3.72)	$N^{14}xNe^{20}(2^+)$	0^-1	6.43		$O^{15}xF^{19}(1/2^+)$
3^-0	3.23	(3.79)	$O^{15}xF^{19}(5/2^+)$	2^+1	6.42	6.37	$C^{14}xNe^{20}(2^+)$
2^+0	3.77	3.84	$O^{16}xF^{18}$	3^+1	6.54	6.56	$O^{16}xF^{18}$
3^+0		4.12	$O^{16}xF^{18}$				

Hamiltonian by saying that the negative-parity matrix elements control the global properties of the states and are critical, while those of positive parity only affect levels selectively and may be varied quite freely without much harm. The interaction presented here is the result of a somewhat arbitrary choice among several that produce similarly good results. It gives a satisfactory description for most of the known levels from $A = 15$ to 18.

We shall now discuss the calculation for $A = 18$. Considerable theoretical attention has been devoted to the description of levels beyond the simple two-particle configurations. The most successful efforts are associated with the states of positive parity.[12] For negative parity[13] no quantitative agreement with experiment could be achieved. Although it is clear that a spherical shell-model basis can provide a unified and rigorous framework, there are two objections to such a project: It is technically very difficult, and the results would be conceptually meaningless because of their complexity. The answer to the first objection was brilliantly given by the shell model code of French, Halbert, McGrory, and Wong.[6] Once this machinery is available the second objection can be disposed of rather trivially. It is just a question of examining the huge wave functions, and we shall show that the shell model provides not only a good, unified explanation of levels in F^{18} and O^{18}, but also a very simple one.

The starting point is the weak-coupling model of Arima, Horiuchi, and Sebe[14] which suggests simple factorizations in the wave functions. Let us consider as a first example the 1^+0 ground state. Conventionally it is described as a two-particle state outside of a closed shell. Using the matrix elements of ZBM case II to solve this simple problem we get

$$|1^+0\,2p\rangle = 0.65d^2 + 0.76s^2. \quad (1)$$

The more complete result for six particles in three orbitals has 47 components but the four biggest terms account for more than 90% of the total (the coupling notation is given in the caption to Table III):

$$|1^+0\,6p\rangle = 0.51d^2p^4 + 0.27d^4(11)p^2(01) + 0.65s^2p^4 + 0.39d^2(01)s^2(10)p^2(01) + \cdots. \quad (2)$$

The wave functions do not look very similar but we have to remember that the core in Eq. (1) is not just the closed shell (p^4) but has some correlations. Therefore we are tempted to take the result of the 4-particle calculation (given in Table III) and couple weakly (multiply simplemindedly!) to $|1^+0\,2p\rangle$. We get[15]

$$O^{16} \times F^{18} = 0.53d^2p^4 + 0.33d^4(11)p^2(01) + 0.62s^2p^4 + 0.38d^2(01)s^2(10)p^2(01) + \cdots. \quad (3)$$

The similarity between the exact and weak-coupling results is amazing. If we include compo-

Table III. Wave functions used as factors in the weak-coupling scheme. Numbers in parentheses indicate partial *JT* coupling followed (only when needed) by seniority and reduced isospin. Asterisks play the role of additional quantum numbers. Only amplitudes larger than 0.2 have been kept. The single-particle spacing for the *sd* shell calculation is $\epsilon_s - \epsilon_d = 0.87$ MeV.

X^A	$J^\pi T$	Wavefunction
O^{15}	$1/2^-, 1/2$	$0.88p^3 + 0.44d^2(01)p$
O^{16}	$0^+, 0$	$0.81p^4 + 0.50d^2(01)p^2(01)$
F^{19}	$1/2^+, 1/2$	$-0.53s^3 + 0.56d^2(01)s - 0.47d^2(10)s - 0.42d^3$
F^{19}	$3/2^+, 1/2$	$0.71ds^2(10) + 0.41d^2(21)s - 0.55d^3$
F^{19}	$5/2^+, 1/2$	$0.35ds^2(01) - 0.40ds^2(10) - 0.32d^2(21)s + 0.31d^2(30)s + 0.70d^3(5/2,1/2,1)$
Ne^{20}	$0^+, 0$	$-0.39s^4 + 0.44d^2(01)s^2(01) - 0.45d^2(10)s^2(10) - 0.43d^3(1/2,1/2)s - 0.50d^4(000)$
Ne^{20}	$2^+, 0$	$0.45ds^3 + 0.27d^2(21)s^2(01) - 0.28d^2(30)s^2(10) + 0.27d^3(3/2,1/2)s - 0.56d^3(5/2,1/2,1)s - 0.43d^4(202) - 0.23d^4(204)^{**}$
Ne^{20}	$4^+, 0$	$0.28d^2(41)s^2(01) - 0.49d^2(50)s^2(10) - 0.49d^3(9/2,1/2)s - 0.48d^4(402) - 0.25d^4(404)^{*} - 0.27d^4(404)^{**}$
Ne^{21}	$3/2^+, 1/2$	$0.34d^2(21)s^3 + 0.29d^3(3/2,3/2)s^2(01) - 0.23d^3(5/2,1/2,3)s^2(10) - 0.25d^4(112)s - 0.27d^4(202)s - 0.41d^4(212)s - 0.51d^5(3/2,1/2,3,3/2) + 0.22d^5(3/2,1/2,5,1/2)$

nents in O^{16} (11 in total) we can account for even finer details. It should be borne in mind that the "multiplication" can be done as a first step, but we have to be careful with the angular-momentum couplings and remember to renormalize after coupling equivalent particles.

As another example we consider the exact wave function for the first 0^-0 state,

$$|0^-0\rangle = -0.52s^3p^3 + 0.49[d^2(01)s](\tfrac{1}{2}\tfrac{1}{2})p^3 - 0.40[d^2(10)s](\tfrac{1}{2}\tfrac{1}{2})p^3 + 0.28[d^2(01)s^3](\tfrac{1}{2}\tfrac{1}{2})p$$
$$-0.36d^3(\tfrac{1}{2}\tfrac{1}{2})p^3(\tfrac{1}{2}\tfrac{1}{2}) - 0.24d^4(000)sp + \cdots. \quad (4)$$

It can be verified easily by using Table III that $O^{15}\times F^{19}$ $(\frac{1}{2})$ reproduces this wave function very well.

This situation is quite general, and good wave functions can be obtained for most of the states by multiplying with a little care. The exceptions are rather mild since they do not involve coupling to new states. They will be mentioned in the comments that follow.

(i) The 1^+0 states are quite pure and come at the right places with the exception of the last one, as was pointed out earlier.

(ii) For 3^+0 the situation is similar except that it is the third state that we miss. When it is restored to its right place using realistic matrix elements, it is found that the fourth state is dominated by $Na^{22}\times C^{12}$, the fifth being $Ne^{20}(4^+)\times N^{14}$. Both come between 5 and 6 MeV.

(iii) The lowest 2^+ states are rather mixed. A third state of the form $F^{20}\times C^{14}$ comes around 6 MeV.

(iv) The 0^+1 and 2^+1 states mix among themselves more strongly than the others. The results are in fair agreement with the results in the first paper of Federman and Talmi.[12] The reason for the stronger mixing is simple: There are some common components in $O^{16}\times F^{18}$ and $C^{14}\times Ne^{20}$ arising from the correlation term in O^{16} $[d^4(000)p^2(01)$ is an example].

(v) The negative-parity states of $T=0$ are not very well known experimentally, with the exception of the first three. We predict quite a number of such states but our assignments above 4 MeV are completely speculative. The first and second 2^-0 states are equal-weight mixtures of $O^{15}\times F^{19}$ ($\frac{3}{2}^+$ and $\frac{5}{2}^+$).

(vi) The negative-parity states of $T=1$ come rather low in energy and the worst discrepancy of the calculation occurs for 1^-1. Similarly, the 3^-1 we predict at 5.18 MeV is more likely to come somewhat higher up.

The calculation is seen to be rather successful in describing the structure of F^{18} and O^{18} and in providing insight into a very powerful coupling scheme. It is unlikely that the regularities observed are accidental and they suggest that complex problems can be tackled by first performing diagonalizations in smaller spaces to define convenient bases. Furthermore the model seems to be providing a conjecture to justify its success: The inclusion of more configurations would refine the description of the "building blocks" (even fragment them into new ones) without affecting their coupling schemes.

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[1]A. P. Zuker, B. Buck, and J. B. McGrory, Phys. Rev. Letters 21, 39 (1968) (to be referred to as ZBM).

[2]A. P. Zuker, B. Buck, and J. B. McGrory, Bull. Am. Phys. Soc. 14, 35 (1969),

[3]Our source of realistic matrix elements is a large number of private communications from T. Kuo and an almost infinite number from S. Kahana, H. C. Lee, and K. Scott (KLS).

[4]C. Bloch and J. Horowitz, Nucl. Phys. 8, 91 (1958); B. Brandow, Rev. Mod. Phys. 39, 771 (1967).

[5]A. Arima, S. Cohen, R. D. Lawson, and M. H. Macfarlane, Nucl. Phys. A108, 94 (1968) (ACLM).

[6]All the shell-model matrices were constructed and diagonalized using the Oak Ridge–Rochester shell-model code of J. B. French, E. C. Halbert, J. B. McGrory, and S. S. M. Wong, to be published.

[7]L. M. Polsky, C. M. Holbrow, and R. Middleton, Phys. Rev. (to be published).

[8]R. Middleton, L. M. Polsky, C. M. Holbrow, and K. Bethge, Phys. Rev. Letters 21, 39 (1968).

[9]S. Kahana, H. C. Lee, and K. Scott, Phys. Rev. 180, 956 (1969).

[10]I. Talmi and I. Unna, Ann. Rev. Nucl. Sci. 10, 353 (1960).

[11]R. K. Bansal and J. B. French, Phys. Letters 11, 145 (1964); L. Zamick, Phys. Letters 19, 580 (1965).

[12]G. E. Brown, in International Congress on Nuclear Physics, Paris, 1964, Proceedings, edited by P. Gugenberger (Centre National de la Recherche Scientifique, Paris, France, 1964), Vol. I, p. 129; P. Feder-

man and I. Talmi, Phys. Letters 15, 165 (1965), and 19, 490 (1965); T. Engeland, Nucl. Phys. 72, 68 (1965); H. G. Benson and B. H. Flowers, Nucl. Phys. A126, 332 (1969).

[13]T. Engeland and P. J. Ellis, Phys. Letters 25B, 57 (1967); M. Harvey, Phys. Letters 3, 209 (1963).

[14]A. Arima, H. Horiuchi, and T. Sebe, Phys. Letters 24B, 129 (1967).

[15]The coupling always takes place between a state obtained from a full model calculation for up to four particles and a "conventional" *sd* state of up to 6 particles. It will be clear from the context whether we refer to F^{18} $(2p)$ or F^{18} $(6p)$ and we shall use the same notation for both.

LEPTON NONCONSERVATION AND THE EARLY UNIVERSE*

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It is pointed out that the existence of a neutrinoless β-decay process would provide a basis for the customary assumption that the number of neutrinos and antineutrinos is equal in the universe.

Speculations on the existence of processes that do not conserve lepton numbers, for instance the neutrinoless β decay of ^{128}Te, or the process $\mu^+ \to e^+ + \gamma$, have recently been discussed in the literature.[1] It is the purpose of this note to point out that this type of process has a direct bearing on some current work in nuclear astrophysics and cosmology.

In a recent paper, the author[2] has explained a method using the principle of detailed balance to arrive at the equilibrium distributions of particle energies. This method is particularly suited to deriving the relations that exist between the distribution functions of elementary particles in equilibrium, a situation which is thought to arise in supernova explosions and is postulated for the very early times of an evolutionary universe.

In brief, the principle of detailed balance between a reaction that transforms a set of particles $a, b, \cdots, n$ into a set $a', b', \cdots, n'$, and its reverse, asserts that

$$N(a)N(b)\cdots N(n)[1\pm N(a')Q^{-1}(a')][1\pm N(b')Q^{-1}(b')]\cdots[1\pm N(n')Q^{-1}(n')]Q(a')Q(b')\cdots Q(n')$$
$$= N(a')N(b')\cdots N(n')[1\pm N(a)Q^{-1}(a)][1\pm N(b)Q^{-1}(b)]\cdots[1\pm N(n)Q^{-1}(n)]Q(a)Q(b)\cdots Q(n). \quad (1)$$

In Eq. (1) the symbol N denotes the number density of the respective particle type at any arbitrary energy (with energy and momentum conservation imposed as restraints on the energy and momentum sums on the left and right). Q normalizes the energy state of any of the particles to single quantum states, so that, for instance, for free particles

$$Q = q[\epsilon(\epsilon^2-\epsilon_0^2)^{1/2}/2\pi^2\hbar^3c^3], \quad (2)$$

where q is the number of spin directions, ϵ the particle's energy, and ϵ_0 its rest energy. The brackets of the form $[1+NQ^{-1}]$ take account of either degeneracy or stimulation, depending on whether the particle involved is a fermion $(1-NQ^{-1})$, or a boson $(1+NQ^{-1})$.

By straightforward manipulation, Eq. (1) can be transformed into a set of differential equations for each particle type involved, with the obvious solutions

$$N = Q/[e^{-\mu}e^{\epsilon/kT}\pm 1] \quad (3)$$

for fermions and bosons, respectively (μ is the chemical potential in conventional notation).

It is now commonly assumed that in the early stages of an evolutionary universe temperature and density were high enough to ensure equilibrium conditions for elementary particles.[3] We then can write a balance relation for any process involving elementary particles which results, with the aid of the distributions (3), in relations between the chemical potentials. For example, the β-decay of the neutron,

$$n \to p + e^- + \bar{\nu}_e, \quad (4)$$

leads to the relation

$$\mu(n) = \mu(p) + \mu(e^-) + \mu(\bar{\nu}_e). \quad (5)$$

Particle-antiparticle processes yield immediately

$$\mu(p) + \mu(\bar{p}) = 0, \quad \mu(e^+) + \mu(e^-) = 0, \quad (6)$$