are given by

$$\frac{v_\theta}{c} = f\frac{\gamma M}{N\sqrt{u}}, \tag{14}$$

$$\frac{v_\xi}{c} = \frac{\gamma M}{N\sqrt{u}} - NE. \tag{15}$$

Thus it can be seen that while the first bracket in Eq. (13) varies as ι^{-2} for small ι, the other terms involve a ι^{-4} dependence. Formally it would appear that for small ι we do have a large increase in plasma loss over the Pfirsch-Schlüter result. However, in the expansion procedure, we have still to determine the mass fluxes exactly. This is done by requiring that the solution in first order be periodic in θ. Closer examination reveals that from this we obtain two complicated differential equations with respect to r, for Γ and ψ. The solution of these equations with the appropriate boundary conditions would enable us to evaluate $W^{(1)}$ exactly. Rather than carry out this full prescription, we have made the plausible assumption that the zeroth-order angular momentum through each magnetic surface is zero. Such an assumption relates M and E:

$$\gamma M = E\langle N^3\sqrt{u}\rangle. \tag{16}$$

This is in addition to the zeroth-order limitations imposed on Γ and ψ which were mentioned above. Equation (16) and the solvability condition (12) lead directly to restrictions on the components of flow velocity:

$$|v_\theta/c| \leq f\gamma, \tag{17}$$

$$\left|\frac{v_\xi}{c}\right| \leq \gamma\left[1 - \frac{1-\kappa}{\langle N^3\rangle}|M| \exp\left\{-\tfrac{1}{2}M^2\left(1 + \frac{\gamma^2}{\langle N^3\sqrt{u}\rangle}[(1+\kappa)^2 - 1]\right)\right\}\right] \leq 1, \tag{18}$$

where for representative ι, κ values, $\langle N^3\sqrt{u}\rangle \sim 1$.

One further assumption is required before we can estimate the orders of magnitude of the different terms in expression (13). This concerns the "smoothness" of the radial variation of quantities. We require that all functions be well behaved in the sense that $\partial A/\partial r \sim A/r$ is a reasonable approximation.

With this information it is easy to show that each of the three brackets in expression (13) is of the same order of magnitude. In other words the contributions to plasma loss due to inertia are, at most, several times the value of the term describing classical diffusion.[1] This result seems to agree with preliminary calculations involving the numerical integration of the fluid equations. For representative situations, a total plasma loss of approximately five times the Pfirsch-Schlüter result is found.

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REVERSIBILITY AND THE DAMPING OF A DOUBLE BEAM INSTABILITY*

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An experimental and theoretical investigation has been made of the absolute instability that occurs near $\frac{1}{2}\omega_c$ when two opposing drifting electron beams interpenetrate in a static magnetic field. The theory allows determination of the beam temperature by measurement of cutoff magnetic field or cutoff beam velocity. The result of the measurement is that the beam temperature is equal to the cathode temperature and we conclude that no irreversible change takes place.

It is well known that the Vlasov equation describes reversible phenomena in a collisionless plasma and it has been decisively proved by Malmberg et al.[1] by their echo experiment that under proper experimental conditions the information left behind after phase mixing can be recovered by application of a second signal. This implies that the temperature of interacting beams which give rise to a growing instability should not change. On the other hand Etievant and Perulli[2] and Maxum and Trivelpiece[3] have investigated the oscillation limits of a counterstreaming beam instability in a magnetic field and found that oscillation ceases when the wavelength is lowered and approaches the Debye length. On the assumption that Landau damping sets in when the wavelength of the instability becomes approximately equal to the Debye wavelength λ_D, they determine the beam temperature and find it up to 700 times higher than the cathode temperature, i.e., the beam temperature before interaction. Maxum and Trivelpiece invoke a "beam thermalization" phenomenon involving ions to explain these results, but we shall show that this is not necessary. We wish to report on our experimental and theoretical investigation of the same instability in order to show that the beam temperature remains close to the original cathode temperature. Thus we show that no irreversible increase of the beam temperature occurs and that this class of phenomena is reversible in the thermodynamic sense as long as the mean free path is long compared with the tube dimensions.

The experimental setup consists of two Pierce-type electron guns with oxide-coated cathodes situated at opposite ends of a conducting drift tube of 6.0 mm internal diameter and 28.0 cm length. The beams fill the drift tube. A 2.0-mm gap between each gun anode and the drift tube allows oscillations to be coupled out by suitable means, e.g., external resonators. The anode-cathode separation of each gun is 1.0 cm with a current-controlling electrode situated between. The whole setup is sealed off under very good vacuum in a glass envelope. A longitudinal magnetic field is applied. It has been shown, e.g., by Lazarus[4] as well as in Refs. 2 and 3 that interactions arise between the fast cyclotron and slow space-charge modes and occur near half the cyclotron frequency. We have confirmed earlier observations of the resulting instability and shown by considerations to be published elsewhere that it is an absolute one. Our measurements of the oscillation limits are shown in Fig. 1 where the cutoff beam energy is shown as a function of magnetic field and beam density.

The theoretical analysis of this bounded system is based on a paper by Lichtenberg and Jayson[5] who showed that a good approximation is obtained by endowing the beams with random (Maxwellian) energy in the direction of propagation only. The dispersion equation becomes

$$\lambda_D^2(\rho^2 n\nu/a^2+\beta^2)-S^{(1)}(\omega)=S^{(2)}(\omega),$$

where

$$S^{(k)}=1+\xi_0^{(k)}Z(\xi_0^{(k)})+\frac{\lambda_D\rho_{n\nu}^2\omega_p}{a^2 2\sqrt{2}\beta\omega_c}[Z(\xi_-^{(k)})-Z(\xi_+^{(k)})]$$

and

$$\xi_0^{(k)}=(\omega-v\beta)/\beta v_{\mathrm{Th}}\sqrt{2},$$

$$\xi_\pm^{(k)}=(\omega-v\beta\pm\omega_c)/\beta v_{\mathrm{Th}}\sqrt{2},$$

and $v=+|v|$ for the first beam with $k=1$, $v=-|v|$ for the second beam with $k=2$. $\rho_{n\nu}$ is the νth zero of the nth-order Bessel function, a the drift tube radius, β the propagation constant, v_{Th} the beam thermal velocity, λ_D the Debye length, and $Z(\xi)$ the plasma dispersion function. Solutions of this equation are obtained by plotting the right- and left-hand sides as a function of real ω (putting imaginary ω equal to zero) near $\frac{1}{2}\omega_c$ with the velocity put equal to the experimentally determined cutoff velocity, and looking for values of $\lambda_D = (kT/4\pi ne^2)^{1/2}$, i.e., the temperature T for which intersection of the two curves just occurs. For this temperature, then, the instability just disappears, whereas for lower temperature there is no intersection. The roots are complex corresponding to an absolute instability. Using this theory, beam temperature is determined by measurement of the cutoff velocity for constant magnetic field. The result of our temperature measurement is that the beam temperature is near 0.1 V over a range of drift energies from 50 to 600 eV. This is exactly the temperature of the cathodes, thus measurement shows that no temperature rise takes place. Our theory, valid in bounded geometry, is a radical improvement on the crude criterion $\lambda=\lambda_D$ for cutoff, from which

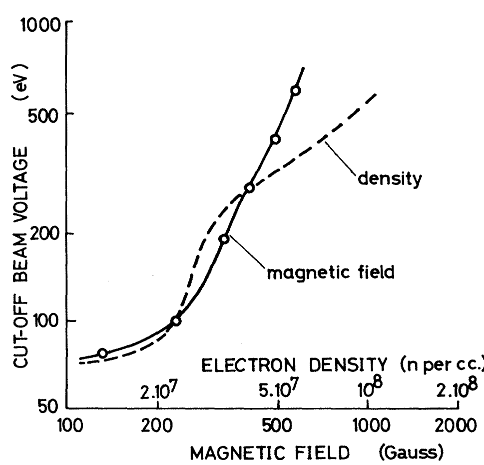

FIG. 1. Oscillation cutoff conditions.

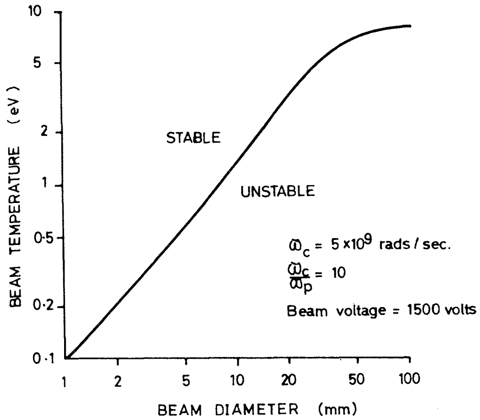

FIG. 2. Stability limit variation with beam diameter.

one would deduce a "thermalization process" leading to a twenty- to thirtyfold temperature rise from our data.

To show the improtance of the drift-tube diameter for the theory of our measurement, we reproduce a curve of the stability limit, i.e., a curve of beam temperature versus beam diameter for the magnetic field, plasma frequency, and cutoff beam energy given (Fig. 2). It is seen that it would take a beam temperature an order of magnitude higher than that obtained for our beam diameter of 6.0 mm to obtain cutoff in a drift tube with a diameter of 10 cm. The theoretical curve is very sensitive to wave numbers. We used the wave number calculated by solving a cubic approximation to the despersion equation in the interaction region.

We have recalculated the stability limit according to our theory for Etievant's parameters. Figure 3 shows curves of beam temperature versus cutoff voltage for the parameters of our experiment and for those of Etievant. Damping sets in at 0.07 eV in ours and at 0.25 eV in Etievant's experiment. It is seen that we deduce a temperature about 0.3 eV for his case. Since he used a hot tungsten source, while we used an indirectly heated Ba-oxide cathode, the higher temperature should be expected. It is unfortunately not possible to recalculate results from the experiment of Maxum and Trivelpiece in the same way, as in their case the geometry is that of concentric beams which would need a much more complicated analysis involving Bessel and

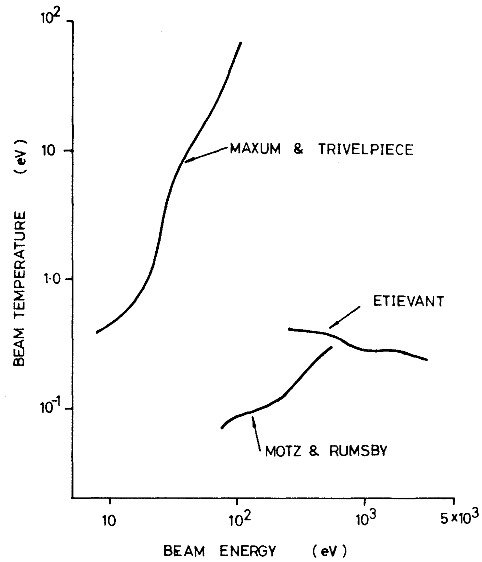

FIG. 3. Experimental stability limits.

Neumann functions. Their results obtained by equating the oscillation wavelength to the Debye length are also shown in Fig. 3.

The details of the cathode and anode gun dimensions, etc., are irrelevant in connection with the experiments described. They are, however, relevant in connection with other oscillations which were also observed, such as transit-time oscillations due to cathode sheaths, etc. We have satisfied ourselves that all the other oscillations observed at different frequencies superpose linearly and do not interact with the instability under review.

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