REGGE RESIDUES IN THE VENEZIANO MODEL*

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We show how the Veneziano five-point function may be used as a model for Regge residues in quasi–two-body processes. An artificial reaction with many of the properties of $\pi N \to \pi\pi N$ is studied in this model; density matrices for spin-1 and spin-2 resonances produced by pion exchange are calculated. The results are encouraging enough to warrant construction of a more detailed model along these lines.

One of the principal difficulties with the simple Regge-pole model has been the arbitrariness of the Regge residue functions. Residues can be evaluated at the internal particle poles; but in the absence of information about all particles on a given trajectory, the t dependence of these functions is unspecified. Hence extrapolation away from the pole is fraught with uncertainty. This is best illustrated by pion-exchange processes such as $\pi N \to \rho N$, $\pi N \to \rho\Delta$, $\pi N \to f N$, and $\pi N \to f\Delta$: The contribution of pion trajectory exchange to the t-channel zero-helicity amplitude $f_{00,\frac{1}{2}\frac{1}{2}}{}^t$ can be extrapolated from $t=\mu^2$ only by making assumptions about dynamic residue functions, and there is no method for determining the contribution to other helicity amplitudes which do not couple at the spin-zero pole.

The recently proposed generalizations[1,2] of the Veneziano formula to n-point functions provide a model for the t dependence of all these residues. We illustrate this by some calculations for the simplest possible artificial reaction, $\sigma\sigma \to \sigma\sigma\sigma$, where σ is a scalar-isoscalar particle which is taken to have the pion mass. The internal Regge trajectories are taken to be the σ trajectory so that we have a sort of Reggeized version of the $\lambda\Phi^3$ (or $\lambda\sigma^3$) field-theory model. Within this model we calculate the helicity amplitudes for $\sigma\sigma \to \sigma$+(spin 1) and $\sigma\sigma \to \sigma$+(spin 2), where (spin 1) and (spin 2) are the $\sigma\sigma$ resonances analogous to the ρ and f. Although our model lacks many of the features of the real world, the resemblance of predicted density-matrix elements to the data should encourage calculations with a more realistic model.

We begin with the Bardakçi-Ruegg amplitude $B_5(-\alpha_{AB}, -\alpha_{A1}, -\alpha_{12}, -\alpha_{23}, -\alpha_{3B})$ for the graph of $\sigma\sigma \to \sigma\sigma\sigma$ depicted in Fig. 1. For simplicity, we have assumed that the σ particles are nonidentical and that this is the only amplitude which contributes. The five internal trajectories all have the same form: $\alpha_{IJ} = \alpha'(s_{IJ}-m^2)$, where m is the mass of the spin-zero σ particle, $m=0.138$ BeV. In numerical calculations, we have taken $\alpha'=1/\text{BeV}^2$.

Białas and Pokorski[3] noted that B_5 can be written in the form

$$B_5(-\alpha_{AB}, -\alpha_{A1}, -\alpha_{12}, -\alpha_{23}, -\alpha_{3B}) = B(-\alpha_{AB}, -\alpha_{A1}) \sum_{m=0}^{\infty} \frac{(-1)^m}{(m-\alpha_{23})m!} \frac{\Gamma(-\alpha_{B3})}{\Gamma(-\alpha_{B3}-m)}\, {}_3F_2(x_1, x_2, x_3, ; x_4, x_5; 1)$$

with $x_1 = \alpha_{12} - \alpha_{AB} - \alpha_{B3}$, $x_2 = -\alpha_{A1}$, $x_3 = -m$, $x_4 = -\alpha_{A1} - \alpha_{AB}$, and $x_5 = -\alpha_{B3} - m$. This displays explicitly the behavior of the function as a sum over resonances in the s_{23} channel. The amplitude for production and decay of a resonance with spin n, together with all its daughters, is proportional to the residue of the pole at $\alpha_{23}=n$. For each such integer, the generalized hypergeometric function reduces to a finite series depending on the four other energies s_{AB}, s_{A1}, s_{12}, and s_{B3}. Two of these, s_{AB} and s_{A1}, can be selected to describe the quasi–two-body process $\sigma+\sigma \to$ (resonance) $+\sigma$. The other two can be re-expressed in terms of s_{AB}, s_{A1}, and two angles in the Gottfried-Jackson[4] frame describing the resonance decay.

This change of variables allows one to exhibit the decomposition of the amplitude into spherical harmonics in the Jackson frame; the coefficients of these spherical harmonics are the t-channel helicity amplitudes for production of the resonance at $\alpha_{23}=n$ and all its daughters. Knowledge of these amplitudes allows calculation of all density-matrix elements for the resonances produced, and of the differential cross sections for their production. In the absence of any other pure Regge model for most of these quantities, these predictions are particularly interesting. We must bear in mind, however, that the treatment of daughters in the Veneziano amplitude is not unitary in any approximation; thus ampli-

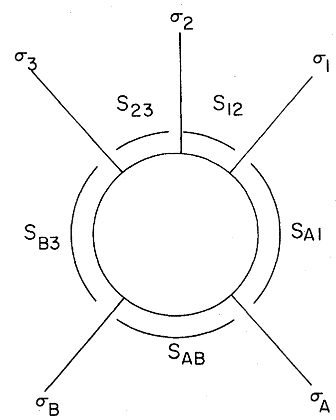

FIG. 1. Labeling of kinematics for the graph considered.

tudes for production of a daughter particle will generally be too large compared with those for production of the parent. This means that the daughter-parent interference density matrices cannot be calculated accurately without improving the model.

Helicity amplitudes for production of the spin-1 and spin-0 resonances at $\alpha_{23}=1$ are displayed in the upper part of Table I; amplitudes for the production of the spin-2 resonance at $\alpha_{23}=2$ are given in the lower part. Because some of the kinematic quantities which are simple in the Jackson frame are moderately complicated functions of s_{AB} and s_{A1}, we have taken the limit $s_{AB}\to\infty$ in the expression multiplying the beta functions; this allows the Regge residues to be read off directly. Note that the factorization property for Regge residues can be used to remove dependence of these amplitudes on the $\sigma\sigma$ vertex. Our results can therefore be applied to obtain a mod-

Table I. Helicity amplitudes for production of $\sigma\sigma$ resonances $(\sigma\sigma)_R$ in the reaction $\sigma\sigma \to (\sigma\sigma)_R\sigma$. The symbols are defined by

$$Q=-\pi^{1/2}\alpha'(s_{23}-4m^2)^{1/2}B(-\alpha_{AB},-\alpha_{A1}),\quad R=(t-4m^2)^{1/2}\sin\theta_t/2s,$$

$$p=\tfrac{1}{2}(s_{23}-4m^2)^{1/2},\quad E=m^2-t-s_{23},$$

$$P_b{}^t=\frac{1}{2}\left[\frac{[t-(m-s_{23})^{1/2}]\,[t-(m+s_{23}{}^{1/2})^2]}{t}\right]^{1/2}.$$

In each case s_{23} is to be evaluated at the resonance in question.

Spin of Resonance	Helicity Amplitude	Prediction
Resonances produced when $\alpha_{23}=1$		
1	$f^t_{00;00}$	$Q\left[\frac{2\sqrt{t}\,p_b^t}{\sqrt{3}\sqrt{s_{23}}}+\frac{\alpha_{A1}E}{2\sqrt{3}\,\alpha'\sqrt{s_{23}}\sqrt{t}\,p_b^t}\right]$
	$f^t_{10;00}$	$QR\sqrt{\frac{2}{3}}\,\frac{\alpha_{A1}}{\alpha'}$
0	$g_{00;00}$	$Q\left[\frac{1+\alpha' m^2-\alpha' s_{23}/2}{\alpha' p}\right]$
Resonances produced when $\alpha_{23}=2$		
2	$f^t_{00;00}$	$Q\,\frac{\alpha' p}{6\sqrt{5}}\left[\frac{8t(p_b^t)^2}{s_{23}}+\frac{4\alpha_{A1}E}{\alpha' s_{23}}+\frac{\alpha_{A1}(\alpha_{A1}-1)E^2}{2\alpha'^2 s_{23}t(p_b^t)^2}+\frac{\alpha_{A1}(\alpha_{A1}-1)}{\alpha'^2(p_b^t)^2}\right]$
	$f^t_{10;00}$	$QR\;2\sqrt{\frac{2}{15}}\;p\left[\frac{\sqrt{t}\,p_b^t\alpha_{A1}}{\sqrt{s_{23}}}+\frac{\alpha_{A1}(\alpha_{A1}-1)E}{4\alpha'\sqrt{s_{23}}\sqrt{t}\,p_b^t}\right]$
	$f^t_{20;00}$	$QR^2\,\frac{2p\,\alpha_{A1}(\alpha_{A1}-1)}{\sqrt{30}\,\alpha'}$

el of the $\pi\rho$ (and πf) helicity-1 (and helicity-2) vertex in experimentally accessible reactions.

Differential cross sections for "ρ" and "f" production computed from these amplitudes are very similar in shape to experimental cross sections[5] for $\pi N \to \rho\Delta$ and $\pi N \to f\Delta$, reactions in which π exchange seems to dominate. The "f" production cross section is slightly broader than the "ρ" cross section, also in agreement with the data. Isospin considerations complicate the comparison of relative magnitudes for "f" and "ρ" production with experiment; this model does, however, give a definite prediction for this ratio without specification of additional coupling constants.

We display in Fig. 2 some density-matrix elements computed with the amplitudes of Table I, for $s_{AB} = 20$ BeV^2. For simplicity, all α's were kept real in evaluating the generalized hypergeometric function. Hence only real density matrices were obtained. This treatment could be modified to include complex α's, if desired.

Figure 2(a) shows the density matrix elements for "ρ" production, after the s-wave $\sigma\sigma$ resonance has been separated off. Because only one vertex has spin, $\sigma\sigma \to$ "ρ"σ has an extra symmetry ($\rho_{11} = -\rho_{1-1}$); thus ρ_{00} and ρ_{10} are the only independent density-matrix elements. Both ρ_{10} and ρ_{00} are similar in behavior to the available data[6] in π exchange reactions. This implies that it will be worthwhile to construct a more physical model of the ρ production amplitude in this formalism. Similar considerations apply to the density matrix elements for $\sigma\sigma \to$ "f"σ, shown in Fig. 2(b).

We have also calculated the density matrices for spin-1 and spin-0 production at $\alpha_{23} = 1$, normalized to the condition $\rho_{00}{}^{11} + 2\rho_{11}{}^{11} + \rho_{00}{}^{00} = 1$ (upper indices refer to the spin of the produced particle; lower ones to the helicity). As expected, the model fails in this sort of daughter-parent comparison. The spin-0 contribution $\rho_{00}{}^{00}$ is much too large compared with the spin-1 contribution $\rho_{00}{}^{11}$. (The model gives, at $t = 0$, $\rho_{00}{}^{00} \sim 0.78$, $\rho_{00}{}^{11} \sim 0.22$, whereas the experimental result[6] is closer to the reverse of this.)

In spite of this difficulty, we conclude that this type of application of the Veneziano five-point amplitude warrants further investigation in more detailed models. It considerably simplifies application of the Veneziano prescription to high-spin quasi-two-body processes, as well as providing a toy model for the Regge residue functions.

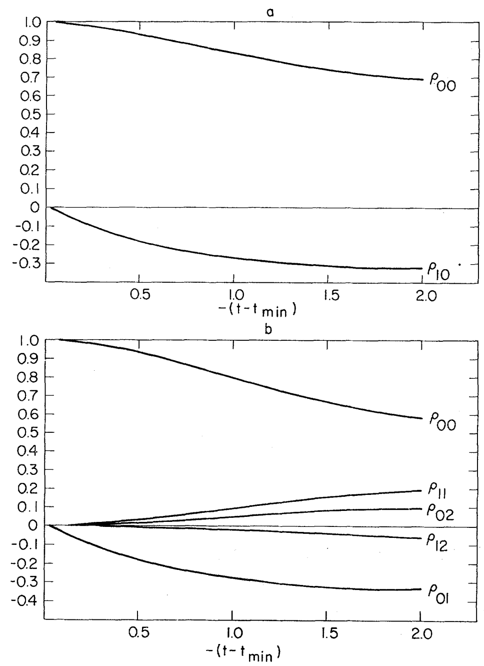

FIG. 2. (a) Density-matrix elements for spin-1 production. (b) Density-matrix elements for spin-2 production.

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