reasons, one of which has already been discussed in some detail.[16]

We are greatly indebted to Professor R. P. Feynman and Professor C. N. Yang for the most instinctive and helpful discussions.

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[1]H. Cheng and T. T. Wu, Phys. Rev. Letters 22, 666 (1969).

[2]H. Cheng and T. T. Wu, Phys. Rev. 182, 1852, 1868, 1873 (1969).

[3]H. Cheng and T. T. Wu, Phys. Rev. 182, 1899 (1969).

[4]T. Regge, Nuovo Cimento 14, 951 (1959).

[5]N. Byers and C. N. Yang, Phys. Rev. 142, 976 (1966).

[6]T. T. Chou and C. N. Yang, Phys. Rev. 170, 1591 (1968), and 175, 1832 (1968), and Phys. Rev. Letters 20, 1213 (1968).

[7]L. I. Schiff, Phys. Rev. 103, 443 (1956); T. T. Wu, ibid. 108, 466 (1957); D. S. Saxon and L. I. Schiff, Nuovo Cimento 6, 614 (1957). The Dirac equation is analyzed in Schiff's paper only.

[8]These particles may or may not be the elementary particles and resonances we know now. For example, they may be members of a set of more "fundamental" particles.

[9]Aside from the explicit results of Refs. 1-3, we do not know any compelling reason why this is so.

[10]By noncovariant perturbation theory, we mean the perturbation theory easily found in most elementary textbooks on quantum mechanics, i.e., the perturbation theory before Feynman invented his rules. Although not manifestly covariant, noncovariant perturbation theory is still consistent with special relativity.

[11]H. Cheng and T. T. Wu, "Impact Factor and Exponentiation in High-Energy Scattering Processes" (to be published).

[12]H. Cheng and T. T. Wu, "Form Factor and Impact Factor in High-Energy Scattering" (to be published).

[13]The difference in scales is, however, interesting and instructive. For example, the corrections to Delbrück scattering and electron Compton scattering are respectively of order p^{-1} and p^{-2} (neglecting logarithms). This is easily understood as the ratio between the most important scale p and the next largest scales, 1 and p^{-1}, respectively.

[14]Note that there are two sets of energy denominators, one set for each incident particle.

[15]B. W. Lee and R. F. Sawyer, Phys. Rev. 127, 2266 (1962).

[16]T. T. Wu, Bull. Am. Phys. Soc. 14, 49(T) (1969).

OMEGA PRODUCTION IN $\pi^+d \to \pi^+\pi^-\pi^0pp$ AT 4.19 GeV/c*

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We report on ~340 events of the reaction $\pi^+n \to \omega^0p$ at a beam momentum of 4.19 GeV/c. The differential cross section shows neither a broad dip for $t < -0.2$ (GeV/c)^2 nor a dip in the region $t \approx -0.6$ (GeV/c)^2. There is a 2-standard-deviation dip in ρ_{00} between $t = -0.2$ and $t = -0.3$ (GeV/c)^2 and a small negative value of Reρ_{10} for $t < -1.0$ (GeV/c)^2. The dip in ρ_{00} is consistent with the vanishing, in that region of t, of an exchanged trajectory with unnatural parity. A simple Regge-model calculation with $\rho + B$ exchange does not reproduce the data.

In a bubble-chamber study of the reaction

$$\pi^+d \to \pi^+\pi^-\pi^0pp \tag{1}$$

at a beam momentum of 4.19 GeV/c, we have identified 338 events which correspond to the reaction

$$\pi^+n \to \omega^0p \tag{2}$$

with the other final-state proton participating as a spectator to the strong interaction. Reaction (2) is particularly interesting because G-parity conservation limits the low-lying exchanges to the ρ and B trajectories. The data are not compatible with ρ-exchange dominance of the reaction. The differential cross section varies smoothly, with no dip apparent at $t \approx -0.6$ (GeV/c)^2 or in the forward direction [$t < -0.2$ (GeV/c)^2]. However, a dip is suggested in ρ_{00} for the ω^0 at $t \approx -0.25$ (GeV/c)^2.

We have measured about 21 000 four-prong events on film taken in the Lawrence Radiation Laboratory 72-in. deuterium bubble chamber to obtain about 2700 events fitting Reaction (1). Each event contains a stopping track whose projected length is greater than 1 mm, which we identify as the spectator to Reaction (2). The remaining proton typically possesses a much higher momentum. Events were classified as belonging to Reaction (1) if this hypothesis possessed the smallest one-constraint χ^2 (and no four-constraint fit was attained), and the fitted momenta of the charged tracks agreed with bubble density when examined on the scanning table. [About 75% of the nonspectator protons in Reaction (2) could be unambiguously identified in this way.] We report here on the 338 events in the $\pi^+\pi^-\pi^0$ invariant-mass spectrum between 650 and 900 MeV/c^2 shown in Fig. 1(a), which we have defined as the ω^0 region. Tests on the data have been performed to check against the presence of biases. The mass squared of the neutral particle for events fitting Reaction (1) was calculated from the measured tracks and is shown in Fig. 1(b). The resulting distribution of mass squared is symmetric, centered at $m_{\pi^0}{}^2$, and agrees well with the resolution function calculated from the errors assigned to the measured tracks by the fitting program. The distribution of χ^2 probability for these events is shown in Fig. 1(c). This distribution is quite flat if we exclude events whose probability is <2%, as shown by the dotted line in the first bin. We have chosen to include all of the events in our analysis; the exclusion of those with $P(\chi^2) < 2\%$ has no effect on our conclusions. The spectator-proton angular distribution (not shown) is isotropic in agreement with the impulse approximation. In Fig. 1(d) we show the spectator momentum distribution and the prediction of the Hulthén wave function scaled to the data. The agreement is excellent.

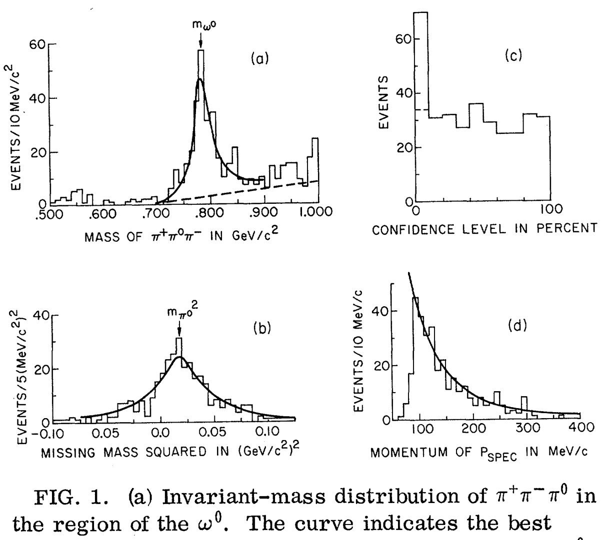

FIG. 1. (a) Invariant-mass distribution of $\pi^+\pi^-\pi^0$ in the region of the ω^0. The curve indicates the best least-squares fit to find the mass and width of the ω^0. The dashed line shows a linear estimate of the background. (b) Missing-mass–squared distribution of π^0. Curve represents resolution function normalized to the number of events. (c) Confidence-level distribution. Dashed line indicates the number of events in the first bin which have confidence level >2%. (d) Spectator-momentum distribution. Curve represents Hulthén wave function normalized to the number of events between 120 and 200 MeV/c.

As a final check on the data, the three-pion mass spectrum was fitted to a linear background curve plus an s-wave Breit-Wigner shape for the ω^0. The best fit, shown in Fig. 1(d), is achieved for a mass of 781 ± 3 MeV/c^2 and a full width of 45 ± 6 MeV/c^2. The Breit-Wigner resolution function for the fitted errors on the ω^0 mass has a full width of 30 ± 3 MeV/c^2, so that we infer a value of 15 ± 7 MeV/c^2 for the intrinsic width of the ω^0. These agree well with the accepted[1] values $m = 783.4 \pm 0.7$ and $\Gamma = 12.6 \pm 1.1$ MeV/c^2. From Fig. 1(a) we conclude that the non-ω^0 events in this region, both background and incorrect assignments to Reaction (2), represent no more than 17% of the data.

Our event sample for Reaction (2) corresponds to a cross section for that reaction of 0.345 $\pm$ 0.050 mb.[2] This number reflects corrections for that fraction of events in which (a) the spectator proton in Reaction (1) was not visible, (b) the ω^0 did not decay into $\pi^+\pi^-\pi^0$, and (c) Reaction (2) was suppressed (in the forward direction) because of the identity of the final-state protons.[3] This last correction is about 4%.

The general features of our data for Reaction (2) agree with those observed in earlier experiments at lower energies.[4-6] In Fig. 2 we show the differential cross section $d\sigma/dt$, where t is the four-momentum transfer squared between the incident π^+ and the outgoing ω^0. This distribution is smooth, with no prominent dips. We note that in a simple Regge model[7] with only ρ exchange, dips are predicted at $t = t_{\min}$ and $t \sim -0.6$ (GeV/c)^2, which are not present in the data.

The production density matrix elements, shown in Fig. 3 as functions of t, have been calculated in the ω^0 rest frame from the angular distribution

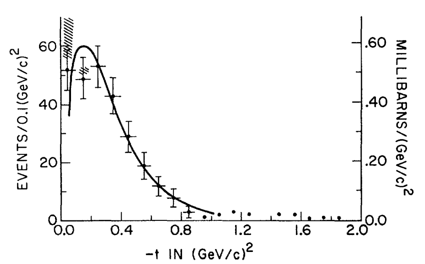

FIG. 2. Differential cross section for $\pi^+n \to \omega^0 p$. The curve represents the predictions of the Regge model and parameters in Ref. 11. The Pauli correction is indicated by the cross hatching in the first two bins. The limits correspond to the values 0.1 and 10.0 for the ratio of spin-flip to spin-nonflip amplitudes.

of the normal to the ω^0 decay plane.[8] We have chosen to display our results in finer bins in t than have previous experimenters and have made many tests to be sure that no spurious features of the data are thereby generated by the small numbers of events in each bin.[9] We have compensated for the presence of incoherent background by studying the values of the moments of the decay angular distribution in mass bins adjacent to the ω^0. The distribution of ρ_{00} after a background subtraction is shown in Fig. 3(d). Although uniformly shifted up and possessing larger errors, this distribution is similar to the distribution of ρ_{00} shown in Fig. 3(a). In particular, the dip near $t\sim-0.2$ $(\mathrm{GeV}/c)^2$ is still present, implying that it does not arise from the behavior of background in the ω^0 mass region.

We have attempted to fit the data using particle-exchange models. If a single particle is exchanged it must have quantum numbers $I^G=1^+$, suggesting among known particles only the ρ or B meson. The absorptive peripheral model[10] (including only ρ exchange) predicts a cross section about six times too large, in contrast to the situation at 2.7 GeV/c [4] where the agreement is good.[11] A simple Regge model[12] with ρ and B exchange, however, does possess a proper s dependence. We varied the parameters of this model, starting at values for the parameters as given in Ref. 10, in a least-squares fit to our differential cross section and density-matrix elements. The results of the fit lessened the importance of the term in the amplitude proportional to $\alpha_\rho(t)$.[13] For $|t|>0.1$ $(\mathrm{GeV}/c)^2$, the model successfully predicts both the shape and magnitude of the differential cross section at this energy as seen in Fig. 2 and also fits the differential cross sections at lower energies (not shown). The Regge model does not match the dip in ρ_{00} in our data. However, it does predict negative values for $\mathrm{Re}\rho_{10}$ in contrast to the absorptive peripheral model, another Regge model,[14] and an absorptive Regge model,[15] which all give the wrong sign at small values of t. Negative values for $\mathrm{Re}\rho_{10}$ have also been found at 3.65 GeV/c.[6]

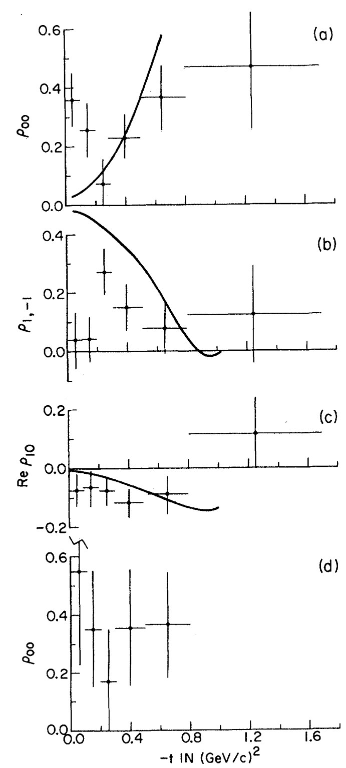

FIG. 3. Density matrix elements (a) ρ_{00}, (b) $\rho_{1,-1}$, (c) $\mathrm{Re}\rho_{10}$, and (d) ρ_{00} after background subtraction described in the text. The solid curves are the prediction of the Regge model.

Högaasen and Lubatti[16] have suggested that a dip in $\rho_{00}(d\sigma/dt)$ indicates the vanishing of unnatural-parity exchange (in our case the B meson) because natural-parity exchange (ρ) without absorption predicts $\rho_{00}=0$ for all t. We observe a 2-standard-deviation dip in ρ_{00} and a corresponding dip in $\rho_{00}(d\sigma/dt)$ between $t=-0.2$ and $t=-0.3$ $(\mathrm{GeV}/c)^2$. One might expect similar behavior in the reaction

$$\pi^+n\rightarrow\omega^0N^{*++}(1236).$$

We have examined data at nearby energies[17] but find no corresponding dip in ρ_{00} that is statistically significant.

In conclusion we point out that the simple Regge model does not correctly predict the angular behavior of (1) $d\sigma/dt$, (2) ρ_{00}, or (3) $\rho_{1,-1}$ near $t=0$. This is because the unnatural-parity trajectory is not correctly described ($f_{++}{}^0$ must be large near $t=0$ to fit the data).

We express our appreciation to Professor L. Jones for many illuminating conversations about the Regge model. We wish to thank Dr. G. E. Hite for the program used to calculate the predictions of the Regge model. Professor U. E. Kruse made many contributions during the early stages of the experiment. Finally, we are pleased to thank members of the Alvarez group at the Lawrence Radiation Laboratory who participated in the experimental run, and the efforts of the scanning, measuring programming, and engineering staff at the University of Illinois.

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[1]Particle Data Group, Rev. Mod. Phys. 41, 109 (1969).

[2]Path length was determined by counting beam tracks. Contamination was found from a δ-ray analysis.

[3]I. Butterworth, J. Brown, G. Goldhaber, S. Goldhaber, A. Hirata, J. Kadyk, B. Schwarzchild, and G. Trilling, Phys. Rev. Letters 15, 734 (1965).

[4]2.7 GeV/c: R. J. Miller, S. Lichtman, and R. B. Willman, Phys. Rev. 178, 2061 (1969).

[5]3.25 GeV/c: A. Cohn, W. Bugg, and G. Condo, Phys. Letters 15, 344 (1965).

[6]3.65 GeV/c: G. Benson, University of Michigan Technical Report No. COO-1112-4, 1966 (unpublished).

[7]L.-L. Wang, Phys. Rev. Letters 16, 756 (1966).

[8]The decay distribution can be described by

$$W(\Omega_n)=(3/4\pi)\{\tfrac{1}{2}(1-\rho_{00})+\tfrac{1}{2}(3\rho_{00}-1)\cos^2\theta_n-\rho_{1,-1}\sin^2\theta_n\cos 2\varphi_n-\sqrt{2}\,\mathrm{Re}\rho_{10}\sin 2\theta_n\cos\varphi_n\}.$$

We have evaluated this expression in the (right-handed) Gottfried-Jackson frame where the z axis is the direction of the incident pion in the rest system of the ω^0, and where the y axis is in the direction of the normal to the production plane in the rest frame of the outgoing ω^0-p system as suggested by G. Benson, B. Roe, D. Sinclair, and J. Vander Velde, Phys. Rev. Letters 22, 1074 (1969). We evaluated the density matrix elements as moments of this distribution. The errors shown in Fig. 3 derive from the standard deviations of these moments.

[9]More stringent cuts on the ω^0 mass, the exclusion of events with the lowest χ^2 probability, and the exclusion of events for which the measured π^0 missing-mass squared is furthest from its nominal value do not appreciably alter the behavior of the density matrix elements.

[10]J. D. Jackson, J. T. Donohue, K. Gottfried, R. Keyser, and B. E. Y. Svennson, Phys. Rev. 139, B428 (1965).

[11]Initially, we chose parameters that were used at 2.7 GeV/c, Ref. 4. By varying the parameters at the expense of increasing the cross section even more we were able to get better agreement with ρ_{00}.

[12]G. E. Hite, thesis, University of Illinois, 1967 (unpublished). The calculation assumes that the target neutron is at rest in the laboratory. The helicity amplitudes $f_{NN}{}^V$ used in this model are

$$f_{++}{}^1=S^J[a_+\alpha_\rho+b_+(t/4m_n{}^2-1)]\frac{m_n\tau_{\pi\omega}}{2s_0}\left(\frac{s-u}{2s_0}\right)^{\alpha_\rho-1}\sin\theta_t\left(\frac{\alpha_\rho}{\alpha_\rho{}'}\right),$$

$$f_{+-}{}^1=S^Ja_+t^{1/2}(1+\cos\theta_t)\frac{\tau_{\pi\omega}}{2s_0}\left(\frac{s-u}{2s_0}\right)^{\alpha_\rho-1}\frac{\alpha_\rho{}^2}{2\alpha_\rho{}'},$$

$$f_{+-}{}^1=S^Ja_+t^{1/2}(1-\cos\theta_t)\frac{\tau_{\pi\omega}}{2s_0}\left(\frac{s-u}{2s_0}\right)^{\alpha_\rho-1}\frac{\alpha_\rho{}^2}{2\alpha_\rho{}'},$$

$$f_{++}{}^1=S^Ja_-\left(\frac{\tau_{np}}{2s_0}\right)\left(\frac{s-u}{2s_0}\right)^{\alpha_B-1}\frac{\alpha_B}{2\sqrt{2}\,\alpha_B{}'}\sin\theta_t,$$

$$f_{++}{}^0=S^J[\alpha_Ba_-(t+m_\omega{}^2-m_p{}^2)/2m_\omega+b_1(1-\alpha_B)\tau_{\pi\omega}{}^2]\frac{\tau_{np}t^{-1/2}}{4s_0\alpha_B{}'}\left(\frac{s-u}{2s_0}\right)^{\alpha_B-1}\cos\theta_t,$$

where

$$S^J=\frac{\pi\alpha'}{z}\left(\frac{e^{i\pi(1-\alpha)}-1}{\sin\pi(1-\alpha)}\right),\qquad \alpha=1+\alpha'(t-m_E{}^2),\qquad \tau_{ab}=[t-(m_a+m_b)^2][t-(m_a-m_b)^2]^{1/2},$$

$$\cos\theta_t=[t(s-u)+(m_\pi{}^2-m_\omega{}^2)(m_n{}^2-m_p{}^2)]/\tau_{\pi\omega}\tau_{np}.$$

[13]The parameters of the best fit are $a_+=2.9$, $b_+=11.1$, $a_1=50.4$, $b_1=50.5$, $\alpha_\rho{}'=0.65$, $\alpha_B{}'=1.01$, and $s_0=0.62$. The term containing to $\alpha_\rho(t)$ goes to zero at $t\sim 0.6$. Because of the small value of a_+, the numerical coefficient of this term, the resulting dip in the cross section is very small.

[14]M. Barmawi, Phys. Rev. 166, 1857 (1968).

[15]F. Henyey, K. Kajantie, and G. L. Kane, Phys. Rev. Letters 21, 1782 (1968).

[16]H. Högaasen and H. J. Lubatti, Phys. Letters 26B, 166 (1968).

[17]D. G. Brown, thesis, University of California, Berkeley, 1968 (unpublished); Bonn-Durham-Nijmegen-Paris-Strasburg-Jurin Collaboration, Nucl. Phys. B7, 687 (1968).